10.1.3 Characteristic curves of the triode

The two circuits shown in Fig. 10.1.1 include a number of similarities; however, this must not lead to the conclusion that their behavior is equivalent. Already simple standard models indicate differences: for the tube, the correspondence between plate-current and control voltage is described via a power function with an exponent of 1,5 while for the FET, the corresponding exponent is 2. In reality, the characteristic curves of both amplifiers do deviate from this idealization – but not in the sense that they would become more equal.

Often, simple model-calculations for the triode start from the **Child-Langmuir-law**♣:

$$I_a = K_1 \cdot \left(U_a + \mu \cdot U_{gk}\right)^{3/2} = K_2 \cdot \left(U_{gk} + U_a/\mu\right)^{3/2} \qquad \text{Triode characteristic}$$

In this equation, U_{gk} designates the voltage between grid and cathode, and U_a designates the voltage between plate and cathode. K and μ are constants relating to the specific tube while I_a is the plate-current. As simple as this law is: it is as inappropriate for guitar amplifiers. Differences relative the real triode already show up in the range of the characteristic curve that could be seen as reasonably linear; for the overdrive range, the Child-Langmuir-law utterly fails (it was not put together for this scenario, anyway). **Fig. 10.1.6** compares idealized and real triode characteristics – the differences are significant. In literature (e.g. JAES), we find several improvements of the above equation that brings it closer to reality (i.e. closer to the characteristics given in data books), but the resulting complex equations do not only require two but six or even more modeling parameters. If the latter are optimized to model the linear and the weakly non-linear drive range, we may still not assume that the extreme overdrive conditions♥ in guitar amplifiers are also suitably modeled. The following depictions therefore do not orient themselves according to tube models but are based on actual precision-instrumentation-measurements taken from amplifier-typical circuits. This included all the associated uncertainties … whether this exact circuit or this tube-specimen was typical enough, whether the capacitors had been run-in long enough, whether the moon had already risen (or set, or was in the correct house) …

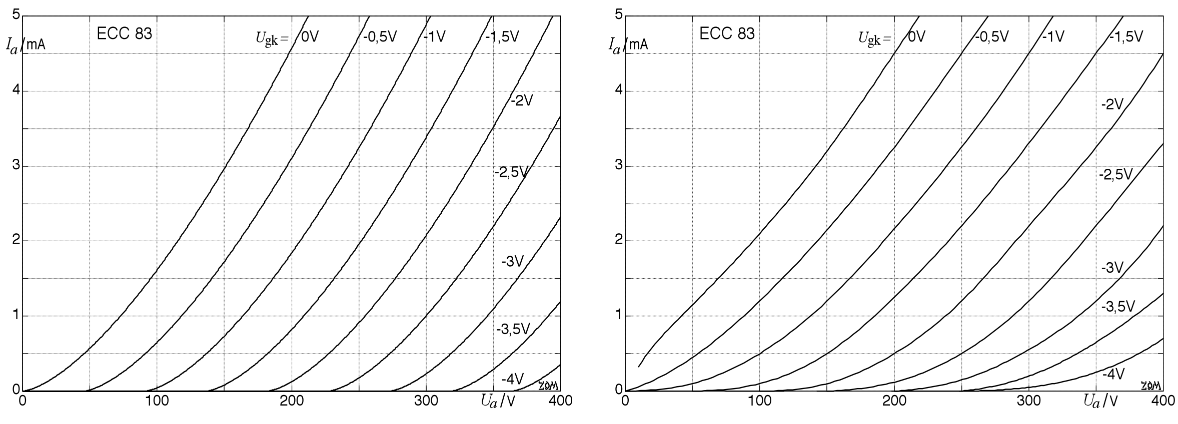

Abb. 10.1.6: Tube characteristics. Left: idealized according to Child-Langmuir. Right: data sheet info.

♣ D. Child: Phys. Rev., Vol. 32 (1911), p.498. I. Langmuir: Phys. Rev., Vol. 2 (1913), p.450.
♥ The drive-limit for the linear range may easily be exceeded by a factor of 30.

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The plate-current I_a depends on both the control voltage♣ U_{gk}, and on the plate-voltage U_a (Fig. 10.1.6). In a real amplifier circuit (Fig. 10.1.1), both of these values change, and consequently the transmission behavior may not be taken as such from Fig. 10.1.6. **Fig. 10.1.7** therefore directly indicates the mapping of the input voltage U_e onto the plate-voltage U_a. In the left part, the input voltage (multiplied by a factor of -58) is included, as well, in order to clearly show the effect of the non-linearity: both half-waves experience limiting. The latter is explained only from the overall transmission behavior, and not merely from the input characteristic. The right hand part of the figure shows the plate-voltage for input voltages of 1 V_{eff} and 4 V_{eff} respectively.

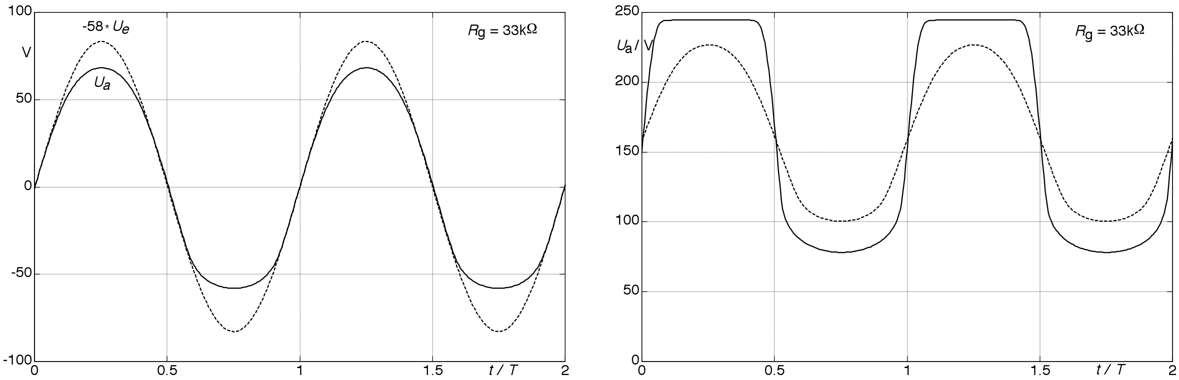

Fig. 10.1.7: ECC83: non-linear distortion of the plate-voltage; on the left with vertical offset. R_a = 100kΩ.

The figure shows how the negative half-wave is flattened first as the drive level increases; for strong overdrive, heavy **clipping** is introduced for the positive half-wave. The plate loading (5 MΩ-probe) is the reason why the plate-voltage does not fully reach the supply-voltage (250 V). That the minimum voltage is not closer to zero is due to the grid-resistor – it attenuates positive input voltages on their way to the grid (Fig. 10.1.4) and prohibits full drive of the tube. **Fig. 10.1.8** shows the influence of the grid-resistor: without R_g, larger plate-currents and smaller plate-voltages are possible – this kind of operation is, however, not typical for input stages of customary guitar amplifiers, and it will not be investigated further. What does require consideration is the **plate-load** that has, in the measurements so far, been very small (at 5 MΩ). In the classic tube amps (Fender, VOX, Marshall), the input tube often feeds the **tone-control stage** that exerts considerable loading onto the plate.

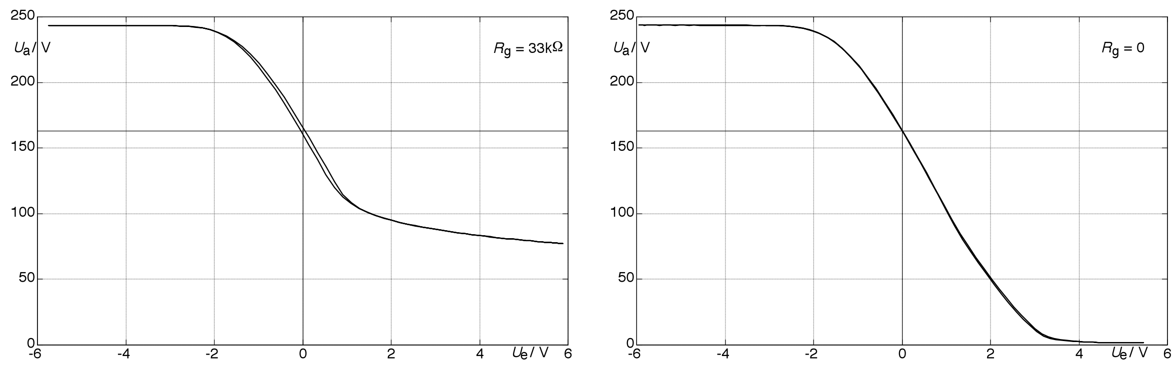

Fig. 10.1.8: transmission characteristic $U_e \rightarrow U_a$. For a 33-kΩ-grid-resistor (left); for shorted grid-resistor (right). R_a = 100 kΩ (plus 5 MΩ load).

♣ May take different meanings; in this case: grid/cathode-voltage.

The input impedance of the tone-control stage is complex, and therefore the analytical description now begins to become complicated (non-linear and frequency-dependent behavior). As a first approximation, however, we may replace the input impedance of a typical tone-control network by the series connection of a 100-kΩ-resistor and a 0,1-µF-capacitor – this enables us to describe the important effects already pretty well. More precise, amplifier-specific models would go beyond the scope of these basic considerations. The cutoff-frequency of the load-two-pole is low enough that the plate is loaded with 100 kΩ in the steady-state condition. Compared to the situation without load (as it has been looked at so far), the AC-plate-voltage is reduced by about a third (**Fig. 10.1.9**). The measured **small-signal-gain**, i.e. the gain for small drive levels (e.g. 0,1 V), amounts to -42. In theory, the small-signal-gain results from the multiplication of the transconductance S (data sheet: $S = 1,6$ mA/V) with the operational resistance. The latter consists of the parallel-connection of the internal impedance of the tube (data sheet: 63 kΩ), the plate-resistance (in the present example 100 kΩ), and the load resistance (again 100 kΩ). We calculate a small-signal-gain of -45, from this i.e. a reasonable correspondence. What needs to be borne in mind, though: the data-sheet information may be taken only as a guide number: swapping a tube for another can easily change the small-signal gain by 3 dB! The drive limits are specific to the respective tube specimen, as well.

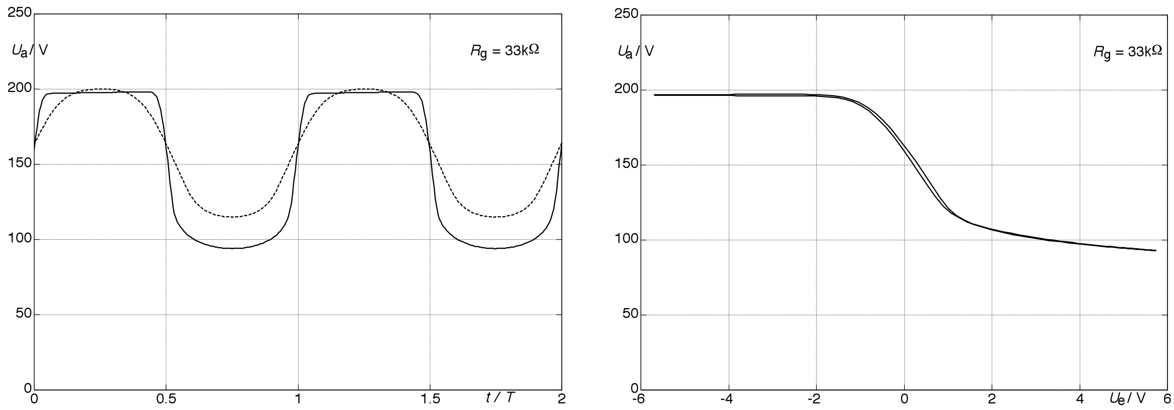

Fig. 10.1.9: Plate-voltage for input voltages of $1V_{eff}$ and $4V_{eff}$ (with R_g, and plate loading, left); transmission characteristic (with grid-resistor R_g and plate loading, right).

The comparison between Figs. 10.1.7 and 10.1.8 has already shown how important the internal impedance of the signal generator is. Whether the tube grid is driven from a low-impedance source ($R_g = 0$), or via a grid-resistor ($R_g = 33$ kΩ) makes a big difference. Of course, the serially connected impedance of the signal generator needs to be considered in addition. Active pickups (e.g. EMG) feature internal impedances similarly low as those of the generators used for the measurements; however, most guitars have high-impedance passive pickups. For an exact analysis, the operation with a 50-Ω-generator is therefore not indicative of the behavior when driven by an electric guitar. The latter may easily show an internal impedance of 100 kΩ in the range of the pickup resonance (2 – 5 kHz). Since the **internal impedance** of the electric guitar is frequency dependent (e.g. 6 kΩ at low frequencies and 100 – 200 kΩ at resonance), and since the input impedance of the tube is non-linear, complicated interactions between the different systems occur already in the input stage of a tube amp. Such an amp will make the guitar see an entirely different load compared to a “modeling amp”. In the latter, the guitar-signal will be normally fed - via a high-impedance OP-amp-stage - to the AD-converter, and all signal processing will be taken care of in the digital realm. However, which tube characteristics will in the end lead to audible differences can only be investigated via listening-experiments.

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For two different source impedances, the grid-voltage limiting is shown in **Fig. 10.1.10**. It is evident that even relatively small positive voltages are visibly reduced. The internal impedance of the generator is, however, purely ohmic in this measurement – which does not correspond to the situation for a connected electric guitar fitted with passive pickups. To better simulate this operational condition, a small **transmitter coil** was laid on top of the pickup of a Stratocaster (with original wiring). Driving this coil with a power amplifier generated a magnetic AC-field inducing a sinusoidal voltage into the pickup. The internal impedance of this arrangement therefore realistically corresponded to the actual operation.

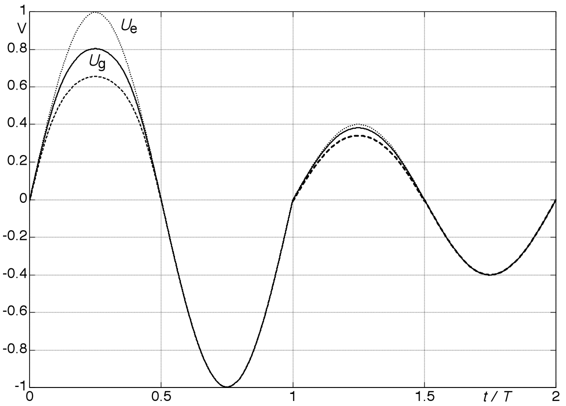

Fig. 10.1.10: Limiting of the generator voltage (U_e) in the grid circuit. Internal generator impedance = 0 (——), and 100 kΩ (----), plus grid-resistor R_g = 33 kΩ. Two periods with different voltage-amplitude shown (1V and 0,4V).

In its left-hand section, **Fig. 10.1.11** shows the corresponding measurement results. The dashed line relates to the source voltage of the guitar corresponding to the open-loop voltage generated by the unloaded guitar. With the load of the tube amplifier, the guitar voltage is bent out of shape; however, this does not happen such that the positive half-wave would simply be compressed (as it would be the case for an ohmic source impedance). Rather, the complex guitar impedance leads to phase shifts between the spectral distortion components (especially in the 1st and 2nd harmonic), and thus the voltage curve is also changed for the negative half-wave. The grid-voltage changes correspondingly (right hand section of the figure), and in the plate-voltage the duty factor is shifted (compare with Fig. 10.1.9). These measurements show that already the first interface between guitar and amplifier-tube has an effect on the signal. Precise observation indicates that the tube input is not of ideally high impedance but acts as a non-linear load-resistance already at moderate voltages. Whether the corresponding changes in the signal are audible compared to other non-linearities, is another question and can, however, be determined only for the individual case.

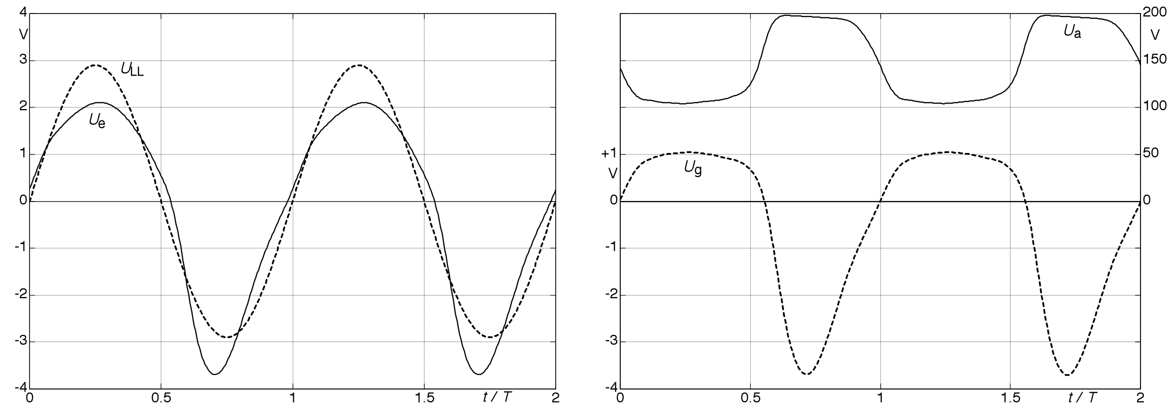

Fig. 10.1.11: Mapping of the guitar-source-voltage U_{LL} onto the terminal voltage of the guitar (left). Load resistance for the guitar is the input-circuit of the tube, R_g = 33 kΩ, f = 2kHz. On the right, the corresponding grid- and plate-voltages are shown; the plate is loaded as given in Fig. 10.1.9.

It is almost impossible to describe the transmission behavior of a guitar amplifier in its entirety by formulae and diagrams. This is not because the relations and connections would be unknown, but rather because too many dependencies would have to be defined. While the small-signal behavior can easily by specified via the frequency-response, there is – strictly speaking – not even a transfer-function for the large-signal operation because this function is only defined via the LTI-(linear time-invariant)-model. Mixtures of small-signal frequency-response and harmonic-distortion characteristic are either incomplete or too extensive. Non-linear distortion is dependent on frequency and on level, and thus is a bi-variant quantity. There are, in fact, *many* bi-variant quantities: 2nd-order (k_2) and 3rd-order (k_3) harmonic distortion, as well as 2nd-order and 3rd-order difference-tone-distortion, just to name the most important ones. For the – frequently occurring – strong-overdrive condition it is not adequate in the least to assess distortion up to merely the 3rd-order; rather, it would be necessary to determine a multitude of individual harmonic-distortion- and difference-tone-factors, and represent this as a function of two variables. And even if we would make such an effort: the result would be all but impossible to interpret. For example, how would we evaluate a circuit change that results in a reduction of the 3rd-order harmonic distortion at 0,5 V and 1 kHz, while the 2nd-order harmonic distortion increases at 0,8 V and 2 kHz? While at the same time the 4th-order harmonic distortion at 0,8 V (2 kHz) drops strongly but the 2nd-order difference-tone-distortion generally grows stronger? Is this desirable or counter-productive? General judgments such as: *for tubes, 3rd-order distortion (= good) dominates, for transistors 2nd-order (=bad) does* are far too unsophisticated, but unfortunately they keep getting copied again and again from textbook to textbook. Listening experiments remain indispensable. Still, a few fundamental relations can be taken from the theoretical models, after all – even if it the result is not much more than the insight that the circuit layout (than cannot be derived from the schematic) can be highly important, or that tube data have a considerable scatter range. In the following analyses, we will give some data on harmonic distortion for a tube driven via an ohmic source impedance, all the while remaining fully aware that only part of the topic can be covered this way, and that additional research would be highly desirable.

10.1.4 Non-linearity, harmonic-distortion factor

Here is a simple **example** regarding the topic of non-linearity: an amplifier generates – at an input voltage of 1 V – pure 2nd-order harmonic distortion with k_2 = 5%. Let's set its gain factor to v = 1. Now, a second amplifier (also with v = 1) also generating k_2 = 5% at 1 V is connected in series with the first one in a non-reactive fashion. How big would the harmonic distortion of the overall system be?

Would that be: k_2 = 10%, or 7%, or an unchanged 5%?

It is not even possible to answer this question without supplementary data: we do not know the phase of the distortion. In case the 2nd-order distortion is generated in-phase in both amplifiers, k_2 is doubled, but if it happens to be in the opposite phase, the 2nd-order distortions all but cancel themselves out. In both cases an additional 3rd-order distortion appears at k_3 = 0,5%. If there is a random phase-shift between the two amps, k_2 can assume any value between 0 and 10%. Already this simple example shows that it is very difficult to derive any statements about the distortion of the overall system from the non-linear behavior of the single amplifier stages.

So, are you having fun yet, dear audio-engineers? O.K. then – let's go for a **second example**: now both amplifier stages feature pure 3rd-order distortion at k_3 = 5%. Right … use the above: the series connection results in k_3 = 10% for the in-phase condition, and for the out-of-phase condition in k_3 = 0%; plus additionally k_4. Hm … are you sure? Then do turn the page!