10.1.5 Frequency limits

The frequency limits of the spectrum of an electric guitar are located at about 82 Hz and about 5 kHz; a guitar amplifier does not have to reproduce any lower or higher frequencies. That is a common and not entirely wrong opinion. The open low E-string vibrates with a fundamental frequency of 82.4 Hz, and the spectrum is limited towards the higher frequencies by the pickup-resonance often located between 2 and 5 kHz. However: bandwidth of the electric guitar and bandwidth of the amplifier are two different things. It is not necessary in a first modeling step to look into the issue that a guitar generates time-limited sounds and therefore the associated spectrum cannot even become zero below 82 Hz. What does require in-depth consideration is the fact that an amplifier with a non-linear distortion-characteristic will generate difference-tones of a frequency far below 82 Hz. Using operational amplifiers (OPs) it would be possible to DC-couple the output of each amplifier stage with the input of the following stage and thus transmit any desired low frequency (down to 0 Hz if we wait long enough …). Such an arrangement is sometimes seen as ideal for recording studio technology because there will be neither phase- nor amplitude errors in the low-frequency region. However, as already previously mentioned: the guitar amplifier is a part of the instrument, it *is supposed* to generate lots of errors. "Errors" from the point of view of classic circuit design, that is, which in the present context are better termed with "signal alterations". The latter should be of the right kind, i.e. those that sound good – and only those. What sounds good or bad is of course a matter of subjective judgment. If a guitarist wants to hear low-frequency difference-tones, amp and speaker need to reproduce these. This feature is, however, not the norm, because the resulting sound will be assessed by many players as "undifferentiated" and "mushy". In your typical guitar rig, we therefore see even whole bunch of high-pass filters taking care of effectively attenuating the very low frequencies: several RC-high-passes, the output transformer, and the loudspeaker. An extreme case was already mentioned in Chapter 10.1.4: in some Fender amps, you will find an RC-cutoff as low as 3 Hz. But then there is the other extreme: the 600-Hz-high-pass in the VOX AC-30.

Low frequencies may be attenuated not only in the plate-circuit where the RC-coupling works as a high-pass, but also in the cathode circuit. To obtain the highest possible gain, the cathode-resistor is often bridged by a capacitor. This **cathode-capacitor** will, however, only have an effect as long as its impedance is not significantly higher than the value of the resistor it bridges. Since it is not possible to make this capacitor indefinitely large, two cutoff-frequencies appear: below the lower cutoff-frequency, the capacitor is almost without any effect and the gain here is v_T, while above the upper cutoff-frequency, the gain is v_H, with a monotonous increase in between (**Fig. 10.1.25**).

For the small-signal model, the tube is replaced by an AC-voltage-source of the voltage $U_0 = \mu \cdot U_{gk}$. Here, μ is the open-loop gain of the tube – a theoretical parameter amounting to about 100 for the ECC83. The internal impedance R_i of the tube is connected in series to this source (internally within the tube); for the ECC83, its value is about 50 – 100 kΩ. If we postulate that there is no current though the grid, the plate-current equals the cathode current and is calculated as $I_k = U_0 / (R_k + R_i + R_a)$. I_k generates a negative feedback voltage across the cathode-resistor. The input voltage U_e decreases by the amount of this feedback voltage $U_{gk} = U_e - I_k \cdot R_k$. This enables us to calculate the plate-voltage $U_a = R_a \cdot I_k$:

$$U_a = U_e \cdot \frac{-\mu}{1 + \left(R_i + R_k \cdot (1+\mu)\right) / R_a} = U_e \cdot v_U \qquad \text{Voltage gain (without load)}$$

$\mu = 100$, $R_i = 72$ kΩ, $R_a = 100$ kΩ, $R_k = 1{,}5$ kΩ, yields $v_U = v_T = -30{,}9 \mathrel{\hat{=}} 29{,}8\,\mathrm{dB}$.

Including the cathode-capacitor (which is taken as a short in the high-frequency region) sets the voltage gain to $v_H = -58{,}1 \mathrel{\hat{=}} 35{,}3\,\mathrm{dB}$; this again is for the unloaded tube. A load resistor is simply connected in parallel to the plate-resistance: with a load of e.g. 100 kΩ, R_a is reduced to 50 kΩ and the voltage gain drops to, $v_T = -18{,}3 \mathrel{\hat{=}} 25{,}2\,\mathrm{dB}$ and $v_H = -41{,}0 \mathrel{\hat{=}} 32{,}3\,\mathrm{dB}$, respectively. For the tube without load, the cathode-capacitor will generate a treble-boost of 5,5 dB, and for the tube loaded with 100 kΩ, the boost will be 7,1 dB (**Fig. 10.1.25**). The capacitance of the cathode-capacitor – in conjunction with the remainder of the circuitry – determines in which frequency-range the transition from v_T to v_H happens. We could surmise that, besides C_k, it is only R_k that sets the treble-boost because this is the resistor that C_k bridges. However, in fact the cathode needs to be considered as load of this two-pole, as well. The relative treble-boost is:

$$v_H / v_T = 1 + R_k \cdot (1+\mu) / (R_a + R_i) \qquad \text{Relative treble-boost}$$

The center-frequency f_Z (marked with a small circle in the figure) computes to:

$$f_Z = \sqrt{1 + R_k \cdot (1+\mu)/(R_a + R_i)} \Big/ (2\pi \cdot R_k C_k) \qquad \text{Center-frequency}$$

If the cathode-resistor is bridged with a “large electrolytic cap” of e.g. 25 µF or more, the center-frequency is located so low (e.g. 5 Hz) that the gain receives a broadband increase – this being the normal approach for Fender amplifiers. Typical examples for small capacitor values (e.g. 0,68 µF) are found in some Marshall amps (f_Z = 150Hz, ΔG = 8dB).

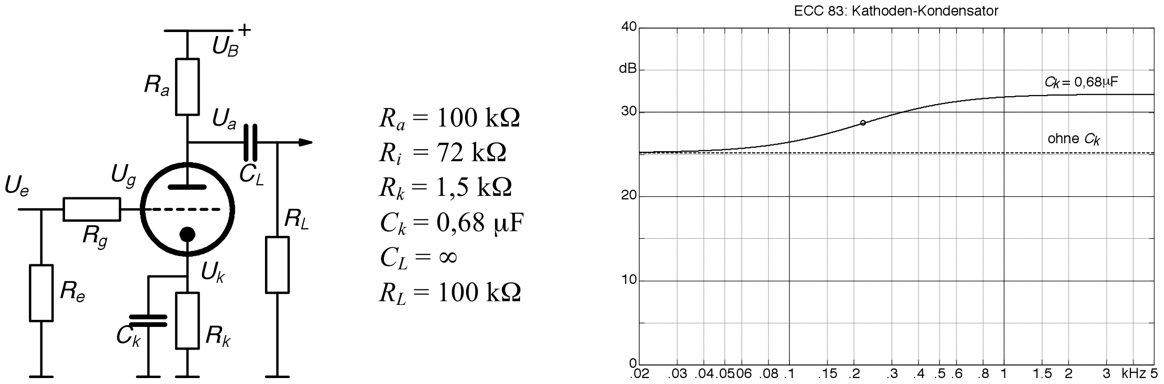


Fig. 10.1.25: Input-circuit of a tube amplifier (left), effect of the cathode-capacitor (right).

In the circuit according to Fig. 10.1.25, the coupling capacitor C_L is taken to be of infinite capacitance in order to be able to show the effect of the cathode-capacitor by itself. In guitar amps, C_L often has a value of 22 nF, but larger values (0,1 µF) are used, as well, as are smaller capacitances (500 pF). When calculating the resulting high-pass cutoff frequencies, it should be considered that the internal impedance of the tube circuit is not zero but is given by the R_a and R_i connected in parallel.

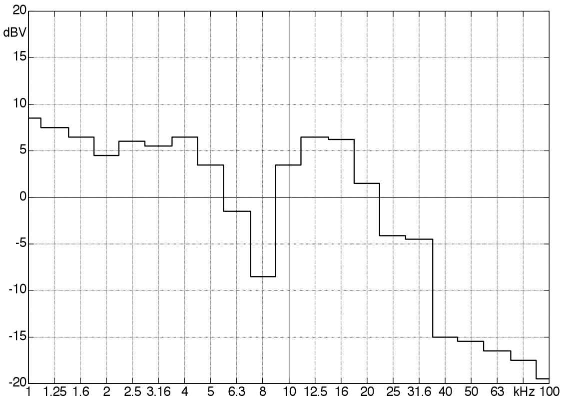
The classic guitar amplifier contains 4 tube-stages and thus has 3 coupling capacitors – the output of power stage is not picked up via a capacitor but via the output transformer. While it is easy to calculate the effect of the coupling caps on the low-frequency-response, the output transformer constitutes a complex system the data of which cannot be seen in the circuit diagram. The upper cutoff-frequency is not apparent, either.

We could assume that the upper cutoff-frequency is always sufficiently high to reproduce the guitar signal (which is low-pass-limited by the pickups), and that therefore it would be not necessary to specially consider it. This assumption is, however, only admissible as long as the amplifier is considered as a linear system. If an overdrive situation occurs, we get signal components in the **ultrasonic range**. These would be inaudible by themselves – however, as ultrasonic signals hit a non-linear amplifier stage, difference tones may be formed that may be audible, after all. A tone-pair constituted of two ultrasound signals (e.g. 24 kHz and 25 kHz) is inaudible at normal levels. Feeding the tone-pair to a 2^{nd}-order distortion-characteristic will generate (among other components) a 1 kHz 2^{nd}-order difference tone that may be audible. This effect should not be seen as all that dramatic, but it should not be entirely disregarded, either. Whether the 1-kHz-tone is in fact audible depends on its level and the levels of further neighboring tones which may have a masking effect. After all, the two ultrasound-tones are not generated in isolation, but are part of a spectrum generated by preceding amplifier stages, and they will not have very large levels. However, since guitar amps may include a very strong emphasis in the high frequency register, a bit of out-of-the-box thinking is advised. We have distortion, treble-boost and subsequently more distortion: there is potential for audible sound differences the reason for which *may* lie in the ultrasonic region.

Why do we not find any **upper cutoff-frequency** in the data-sheets of tubes? Some manuals will give 300 MHz for triodes, or – depending on the type – 1 GHz; however, specifically for the ECC83 this field is usually left empty. The reason is actually rather trivial: the upper cutoff-frequency is determined by the circuitry around the tube. Let's speculate a bit how all this started: the first guitar amps had to be economical regarding the use of power – that made (after the octal-socket-era had passed) the 12AX7 with only 1 mA plate-current highly welcome. As a result, the circuitry had to be of rather high impedance, with 1-MΩ-potentiometers (Fender, Marshall) necessary so as not to load the plate circuits too much. With the center-tap of such a potentiometer set in the middle of the range, its internal impedance is about 250 kΩ[♣]. Connecting this center-tap to the next high-gain triode with an input capacitance (enlarged by the Miller-effect) of about 150 pF, we get a low-pass with a cutoff frequency at about 4,2 kHz. That is kilohertz, not megahertz! You would not want to include such a low upper cutoff frequency in a data-book – it would look quite bad. The relatively high input capacitance is generated by the capacitance between grid and plate (12AX7: $C_{ga} = 1{,}6$ pF) that is enlarged by a factor given by the voltage gain. With $v_U = 50$, this already amounts to 80 pF, and since the wiring leading up to the tube also has a capacitance, 150 pF are easily reached – or even more. The low-pass mentioned above is not always there, though: if the center-tap of the 1-MΩ-pot feeds a cathode-follower (common-plate circuit) the cutoff-frequency will be much higher. In Fenders "Twin-Reverb" (just to name one example), however, the center-tap of the potentiometer directly connects to a common-cathode circuit the input capacitance of which is relatively high. In many Marshall amplifiers there is even a 470-kΩ-resistor in series to the center-tap (summation-stage, total series resistance = 320 kΩ). At this location in the circuit there was also an opportunity to include a low-cost supplement increasing the treble response: a fixed capacitor (Marshall) or a switchable capacitor ("Bright"-switch, Fender). The overall actual cutoff frequency resulting from this hodge-podge of frequency-boosts and frequency-cuts can be calculated via complicated models but depends on many parameters – not least on the layout. The distance between lines leading to grid and plate does influence, via the Miller effect, the input capacity and the upper cutoff-frequency.

[♣] The internal impedance of the tube will also make a small contribution.

Fig. 10.1.26 shows a section of the layout of a Fender amplifier (Super-Amp). Resistors and capacitors are soldered to eyelets on a carrier board, and wires lead from the long side of the board to other sub-assemblies (connectors, potentiometers, tubes, transformers). Some wires are laid out below the board at only a small distance to the components above. For example, the wire connecting the grid of a tube is located directly below the coupling capacitor connected to the plate of the same tube – this certainly is not the best possible decoupling approach. Even more extreme is the situation with three wires coming out of an access-hole (in the top section of the picture): two of these are connected to the input jacks, the third carries the plate-AC-voltage of the corresponding input tube. The resulting capacitive coupling is not particularly strong but we need to consider that the grid-plate-circuit is especially sensitive, and that such coupling has the effect of a low-pass. It cannot be excluded that such a low-pass is in fact intended, but comparisons with many other Fender layouts do not really support this assumption. The various wires seem to too arbitrarily keep or change their positions over the years.



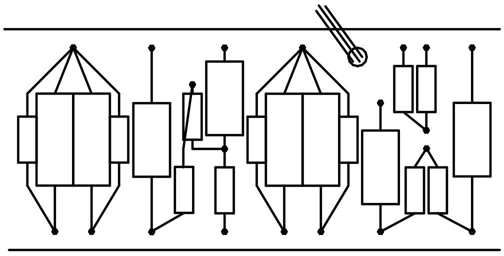


Fig. 10.1.26: Fender component board (excerpt, above).
Third-octave spectrum of the power tube-g_1-voltage (Fender Deluxe). Stratocaster, Stratocaster (left).

As distortion occurs, frequencies above 5 kHz result. The above third-octave-diagram shows this – it is taken from the grid of a power-tube; similar situations can be present at other tubes, as well. The effect of the input capacitance of a tube is shown in **Fig. 10.1.27** using the example of volume-pot: as it is turned down we obtain a low-pass-effect. The cutoff-frequency is lowest at an attenuation of about 6 dB. A "Bright"-capacitor bridging the pot (from the anode to the grid of the subsequent tube) compensates this treble loss but as the control is turned down further, an over-emphasis of the treble occurs. The individual characteristics are strongly dependent on stray-capacitances and on the gain of the individual tubes.

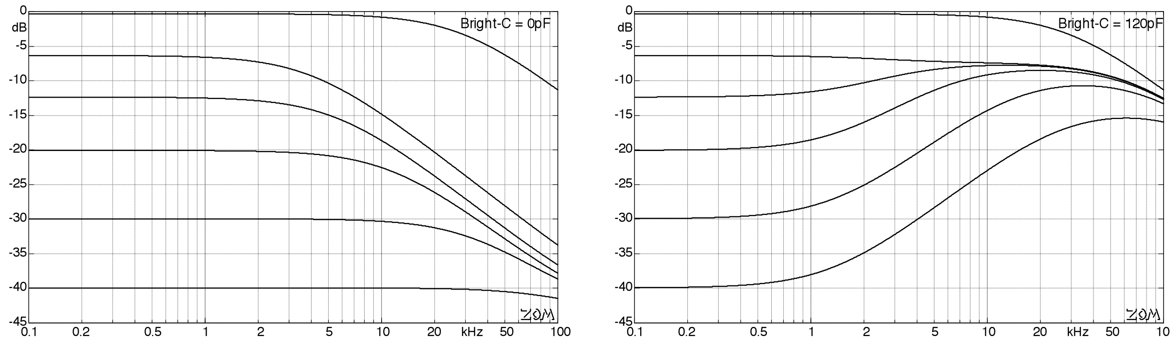


Fig. 10.1.27: Transfer-function of a volume-pot loaded by a capacitance (1 MΩ); tube-input-capacitance 150 pF.