10.1.8 Noise processes

Noise belongs to the stochastic signals – it is not possible to predict its exact course. The simplest quantitative description involves RMS-value, bandwidth, and spectral-envelope characteristic (dP/df = const. or $1/f$). Supplementary specification regarding the time function is given by distribution (= probability-density-function) and cumulation (= probability-density-distribution), further information regarding the spectral distribution results from DFT- and 1/3rd-octave-sprectra. The literature listed at the end of this chapter may serve as guide to the theoretical principles for the description of random signals – the following listing in short introduces the most important noise processes.

a) Thermal noise (white noise i.e. dP/df = const.)

The temperature-dependent random-movements of free charge-carriers in a conductor (or in a resistor) lead to a thermal open-loop voltage at the connecting terminals (without any load); the RMS-value of this voltage is computed to:

$$\boxed{\tilde{U}_n = \sqrt{4kT \cdot \Delta f \cdot R}, \quad e_n = \sqrt{4kT \cdot R}} \qquad 4kT = 1.70 \cdot 10^{-20}\ \text{Ws}, \quad T = 308\text{K}$$

Open-loop noise-voltage density e_n and RMS-value of open-loop noise-voltage $\tilde{U}_n$ for Δf = 10 kHz at resistor R:

R =	58.8	100	200	1k	10k	100k	1M	Ω
e_n =	1.00	1.30	1.8	4.1	13.0	41.2	130	nV / $\sqrt{\text{Hz}}$
$\tilde{U}_n$ =	0.1	0.13	0.18	0.41	1.3	4.12	13	μV

b) Shot noise (white, i.e. dP/df = const.)

Shot noise occurs in semiconductors and amplifier tubes. It is caused by statistic fluctuations of the current-flow through an interface layer between potentials. As an example, the electron-emission at an amplifier cathode may be modeled by a Poisson-distribution, with the current not continuously flowing but having statistic fluctuations. The real tube-noise is (given the space-charge conditions) slightly less than the theoretical maximum value calculated below for saturation [Meinke/Gundlach]:

$$\boxed{\tilde{I}_S = \sqrt{2e \cdot \Delta f \cdot I_0}, \quad i_S = \sqrt{2e \cdot I_0}, \quad \tilde{U}_S = \tilde{I}_S \cdot R} \qquad 2e = 3.204 \cdot 10^{-19}\ \text{As}$$

Noise-current density i_S, (RMS) noise-voltage $\tilde{U}_S$ across a 10-kΩ-resistor for 10 kHz bandwidth, generated by DC I_0: [f = Femto = 10^{-15}, p = Pico = 10^{-12}]

I_0 =	10 n	100 n	1 μ	10 μ	100 μ	1 m	10 m	A
i_S =	56,6 f	179 f	566 f	1,79 p	5,66 p	17,9 p	56,6 p	A / $\sqrt{\text{Hz}}$
$\tilde{U}_S$ =	56,6 n	179 n	566 n	1,79 μ	5,66 μ	17,9 μ	56,6 μ	V

The relation between shot-noise voltage $\tilde{U}_S$ and thermal noise-voltage $\tilde{U}_n$ depends on the DC voltage across the resistor and on the temperature voltage:

$$\boxed{\tilde{U}_S / \tilde{U}_n = \sqrt{U_0 / 2U_T}}$$

U_0 is the DC-voltage across resistor R; $2U_T$ = 2·26 mV = 52 mV.

c) Flicker noise (approximately pink, i.e. $dP/df \sim 1/f$)

This is low-frequency $1/f$-noise caused by inhomogeneities in the material, deficiencies from manufacture, contaminations, and charge-fluctuations at surfaces. The designation stems from the burn spots jumping around (flickering) on the cathode of an amplifier tube. Simplified, the power-density decreases towards high frequencies with $1/f$ (pink noise). However, also observed were noise processes the spectral density of which does not correspond exactly to the $1/f$-hyperbola. Flicker noise is only relevant in the low-frequency range.

The $1/f$-noise caused in **resistors** carrying DC is characterized by the **Noise-Index** *NI*. Metal-film resistors (homogeneous crystal lattice structure) feature a small *NI*, while carbon composition resistors have large *NI*-values. In general, resistors with a high power-handling capacity (and requiring a larger volume) generate less noise than their low-power cousins of the same basic build.

$$\boxed{NI = 20 \cdot \lg \frac{U_{10} / \mu\text{V}}{U_0 / \text{V}} \text{dB}} \qquad U_{10} = U_0 \cdot 10^{-6} \cdot 10^{NI/20\text{dB}}$$

U_0 represents the DC-voltage across the resistor, U_{10} is the resulting $1/f$-noise-voltage (RMS value) per frequency-decade; $NI = 0$ dB $\Rightarrow$ 1 μV/V.

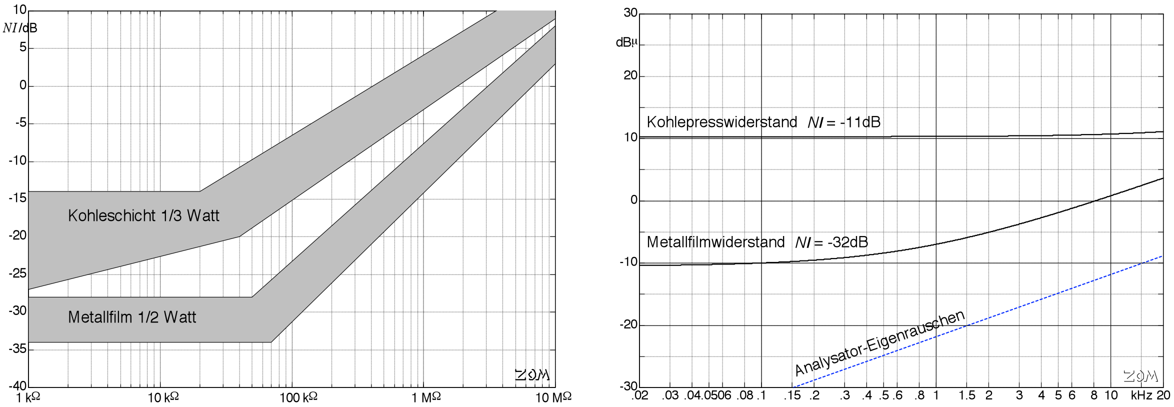

Abb. 10.1.30: left: noise-index *NI* for two different resistor types (Kohleschicht = carbon layer, Metallfilm = metal film). The grey areas show the scatter range between typical average values and typical maximum values. Right: measured 1/3$^{\text{rd}}$-octave noise-; dashed: intrinsic noise of analyzer. Pink noise results in frequency-independent 1/3$^{\text{rd}}$-octave-level voltage levels; for white noise the 1/3$^{\text{rd}}$-octave-levels rise with 10 dB/decade. Kohlepresswiderstand = carbon-composite resistor; Metallfilmwiderstand = metal film resistor; Analysator-Eigenrauschen = intrinsic noise of analyzer.

In **Fig. 10.1.30**, *NI* is listed for different resistor types. The areas marked in grey can only give very approximate orientation-values since the individual build has significant influence on the *NI*. In the right-hand section of the figure, we see measurements taken with two serially connected 68-kΩ-resistors carrying a DC of 1 mA. The two incoherent noise currents of the two resistors need to be added via a Pythagorean summation, and the mutual loading plus the loading via the analyzer (100 kΩ) has to be considered, as well. The metal-film resistors show a thermal white noise in the high-frequency region, and a current-dependent pink noise at low frequencies. In the carbon-composite resistors, current-dependent pink noise dominates throughout practically the whole frequency-range. These measurements give a noise index of the carbon-composite resistors of -11 dB, and an *NI* for metal film resistors of -32 dB. At low frequencies, the noise power densities of these two resistor-types therefore differ by **a factor of 126**. This factor is current-dependent; 1 mA is typical for plate-currents in preamplifiers.

Despite this considerable current-noise, carbon-composite resistors are listed as “**absolute high-end**” in the catalog of a retailer; one is very temped to interpret this as “absolute upper range of the resistor noise”. The high-end fan must furthermore not be irritated by the fact the carbon-composite resistors have also considerably larger tolerances (compared to metal film resistors): maximum ±10% (carbon) vs. maximum ±1% (metal). Measurements confirm this: + 7% (carbon) vs. -0,3% (metal). What about the price-difference? As expected, carbon-composite resistors are about 10 times as expensive as metal film resistors. Say no more: it’s about more noise – more tolerance to resistance – more money …

What remains is the question whether differences in the current-noise play any role at all compared to the **shot-noise** generated in the tube. For an **ECC83** (12AX7), the equivalent input-noise voltage-density may be set to about 5 $\mathrm{nV}/\sqrt{\mathrm{Hz}}$ as a good approximation. With a voltage gain of 34 dB, this is equal to 250 $\mathrm{nV}/\sqrt{\mathrm{Hz}}$ at the plate, corresponding to a third-octave-level of 11.6 dBμ at 1 kHz (bandwidth 232 Hz). In comparison, the thermal noise from the **grid-resistors** (68 kΩ // 68 kΩ = 34 kΩ) typically found in the input-stages of guitar amps is five times as much (Chapter 10.1.7), reaching some ample **26 dBμ** in the third-octave band around 1 kHz. And how are we doing regarding the resistor-noise created by the plate-current? Given a 100-V-voltage-drop across the plate-resistor, and including a noise-index of NI = -11 dB, we would be confronted with a 1-kHz-third-octave-level (open loop) of no more than 19 dBμ. With the loading by the internal impedance of the tube, this would decrease to about **11 dBμ**. Consequently, the current-noise of a carbon plate-resistor (NI = -11 dB) at 1 kHz is lower than the noise of the preamplifier by 15 dB. For higher frequencies, this difference will grow even bigger, and only below 31 Hz, the current-noise would become dominant for the present model.

So: The current-noise of customary carbon resistors is inaudible in the investigated circuits
But: Supposedly there are carbon-composite resistors with NI not at -11 dB, but at 0 dB,
Or even higher – that could then just become audible.
Question: Is that worth 10 times the price? *Answer:* sure, the retailers are happy.

Two advantages are often highlighted to scientifically support the apparent superiority of the carbon-composite resistors: high power capacity with impulses, and small inductance. There may be scenarios in which the relative long thermal time-constant of the carbon-composites helps to avoid overheating, but pre-amp stages in guitar amps are not even remotely in the playing filed here. O.K. then: the reported low **inductance** of composite resistors will be crucial, won’t it? No, sorry, that aspect is utterly insignificant in the relevant frequency-range! The impedance of a 100-kΩ-resistor will increase by 0,000000002% at 100 kHz (with a inductance of 1 μH as a baseline). This increase should be seen relative to the manufacturing tolerances in carbon-composites: 10% according to data sheets. Plus: do not forget that 1 μH is already a high value; in data sheets we often find the entry “a few nano-henry”. BTW, our metal-o-phobic friends prefer not to mention capacitive reactive values, although these exist in carbon resistors, as well. Do you need to consider those? Course you do … if you want to look beyond 1 MHz, where the reactive currents start to achieve some significance.

The never ending Internet saga of Carbon Comps: “Smooth, creamy sound…Are unstable, should not be used…Very clean and natural sound…Should be avoided…Taut and 3-dimensional sound…Make the working point drift away…Are the only choice for guitar amps…Never heard any difference in sound…Light-years ahead.” More examples are available …

Literature: Motchenbacher/Connelly: Low-Noise Electronic System Design, Wiley 1993. **Connor**: Rauschen, Vieweg 1987. **Hänsler**: Statistische Signale, Springer 1991. **Bendat/Piersol**: Random Data, Wiley 1986.