10.10.5 At which strength is harmonic distortion audible?

This is a difficult topic because there are so many details influencing it that a single number is not even close to doing the job. We can merely state: ”somewhere between 0,03% and 10%.” For synthetic test signals it will be more towards the lower value while for guitar sounds, it will be more towards the upper.

Nonlinear distortion of a **sine-tone** can be detected only at strong distortion levels because the new (higher-frequency) partials generated by the nonlinearity are masked to a large extent [12]. **Two-tone signals** are more critical since their nonlinear transmission generates (on top of the masked summation tones) low-frequency difference tones, as well – and these can relatively easily be detected. Webers writes in his book "Tonstudiotechnik" (recording studio technology) that tones of flutes are seen as particularly problematic. He notes a threshold of detection of k_2 = 1% for 2nd order distortion and of k_3 = 0.3% for 3rd order distortion. Rossi lists even smaller limits of audible distortion but feels that 1% intermodulation-distortion is acceptable. Our guitar amps? No, they do not fit at all into this system of (mostly purposeful) rigid values of audibility thresholds found in recoding studio technology. Still, it would be helpful to have an understanding of the distortion levels at which *clean* becomes *crunch*, of the characterization of *strong* and *ultra* distortion, respectively, and of what *even* and *odd* distortions are.

Nonlinear distortions happen at curved transmission characteristics, i.e. predominantly in tubes and semi-conductors. Curved characteristics may be developed into mathematical series expansions, and if these expansions include odd powers only (x, x^3, x^5, ...), they generate odd distortions. If only even powers occur (x^2, x^4, ...) on top of the linear term (x), even distortions result [Taylor/MacLaurin, Fourier-series, communication technology]. To start with a simple signal (even if it barely shows any similarities to a guitar tone): in **Fig. 10.10.23** a sine-tone receives nonlinear distortion via the characteristic $y = x - x^3$.

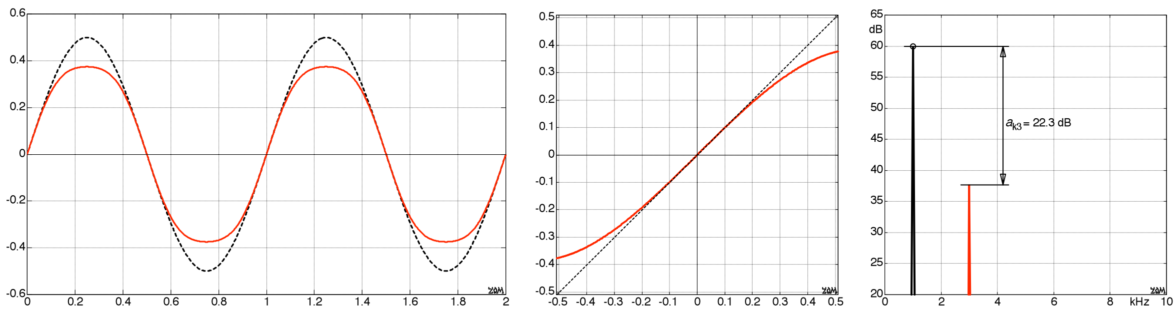

Fig. 10.10.23: Nonlinear (3rd order) distortion of a sine-tone; time function, transfer characteristic, spectrum.

Inserting for $x = \sin(\omega t)$ into the characteristic and calculating the equation immediately shows the result as seen in the spectrum: we obtain a new spectral line at three times the fundamental frequency with a level-distance of 22 dB re the level of the fundamental. In the following formula, the index i stands for the order of the partial tones (i = 1 marks the fundamental), u is the distorted voltage, and u_i is the voltage of the i-th partial (all voltages are RMS-values). Consequently, k_3 is the 3rd order “Harmonic-Distortion”-factor (HD), and a_{k3} is the difference level between the fundamental and the distortion products. This approximation works the better, the smaller the HD is.

$$k_i = \frac{u_i}{\sqrt{u_1^2 + u_2^2 + u_3^2 + \ldots}} = \frac{u_i}{u} \approx \frac{u_i}{u_1} \qquad a_{ki} = 20 \cdot \log(1/k_i)\,\mathrm{dB}$$

Harmonic Distortion

Fig. 10.10.24 shows the corresponding results for purely 2nd order distortion; the chosen characteristic was $y = x - 0.3x^2$. The new partials generated are now at twice the fundamental frequency and at 0 Hz. The DC-component is usually blocked using coupling capacitors because it may disadvantageously shift the operating point depending on the signal.

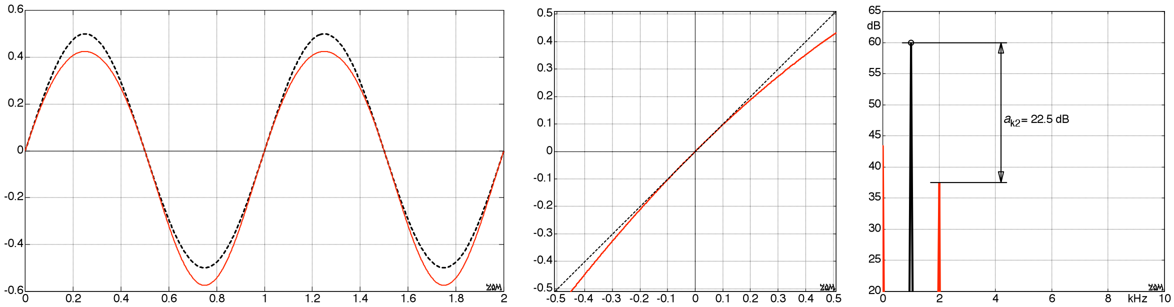

Fig. 10.10.24: Nonlinear (2nd order) distortion of a sine-tone; time function, transfer characteristic, spectrum.

Using **two-tone signals** we achieve a step towards more natural signals, but we also increase the number of degrees of freedom: we may now choose the frequency relation between the two primary tomes, the difference in their level and the difference in their phase. For **Fig. 10.10.25**, a frequency relationship of 6/5 is chosen, with the levels of the primaries being equal. For **3rd order distortion**, new lines are generated at the frequencies $2f_1 - f_2$, $2f_2 - f_1$, $3f_1$, $2f_1 + f_2$, $2f_2 + f_1$, $3f_2$. At $2f_1 - f_2$ we find the **3rd order difference tone**.

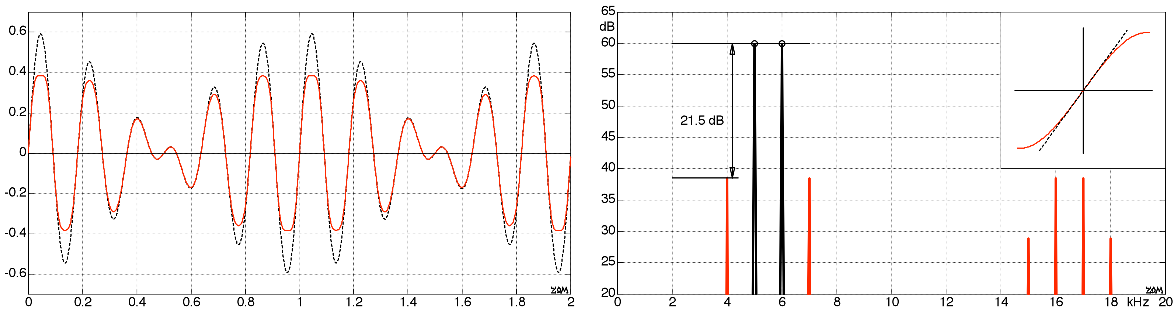

Fig. 10.10.25: Nonlinear (3rd order) distortion of a two-tone signal; time function, transfer function, spectrum.

With **2nd order** distortion (**Fig. 10.10.26**), a DC-component results, as well as new lines at $f_2 - f_1$, $2f_1$, $f_1 + f_2$, $2f_2$. At $f_2 - f_1$ we find the **2nd order difference tone**.

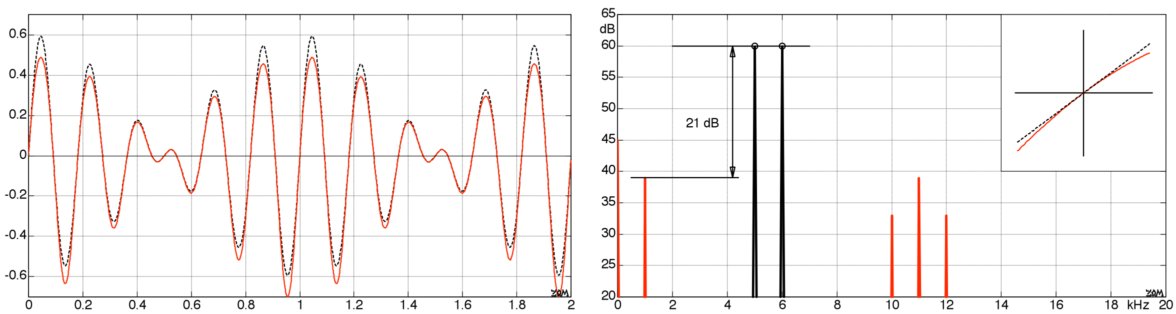

Fig. 10.10.26: Nonlinear (2nd order) distortion of a tow-tone signal; time function, transfer function, spectrum.

The distortion does not only generate lines at new frequencies but also at the frequency of the primary tones. The level and phase of the latter is correspondingly changed.

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A tone from a guitar is much more complex than the signals just looked at, and therefore the multitude of parameters explodes. The HD is not a fixed value but dependent on the drive level. Doubling the input signal makes the 2nd order HD grow by a factor two and the 3rd order HD by a factor of four; k_2 is proportional to the drive level while k_3 is proportional to the square of the drive level. Changing the phases of the partials changes the crest-factor (peak-value/RMS-value) and thus the HD even if the drive level remains constant. For a guitar signal, this drive level is of course not constant but decreases quickly after a strong attack. So, what should we reference the HD to? To the maximum value that lasts only a few milliseconds? Or some kind of average value defined one way or another? For sine-shape drive signals it is easy to specify the HD but driving a system with a guitar signal creates a problem.

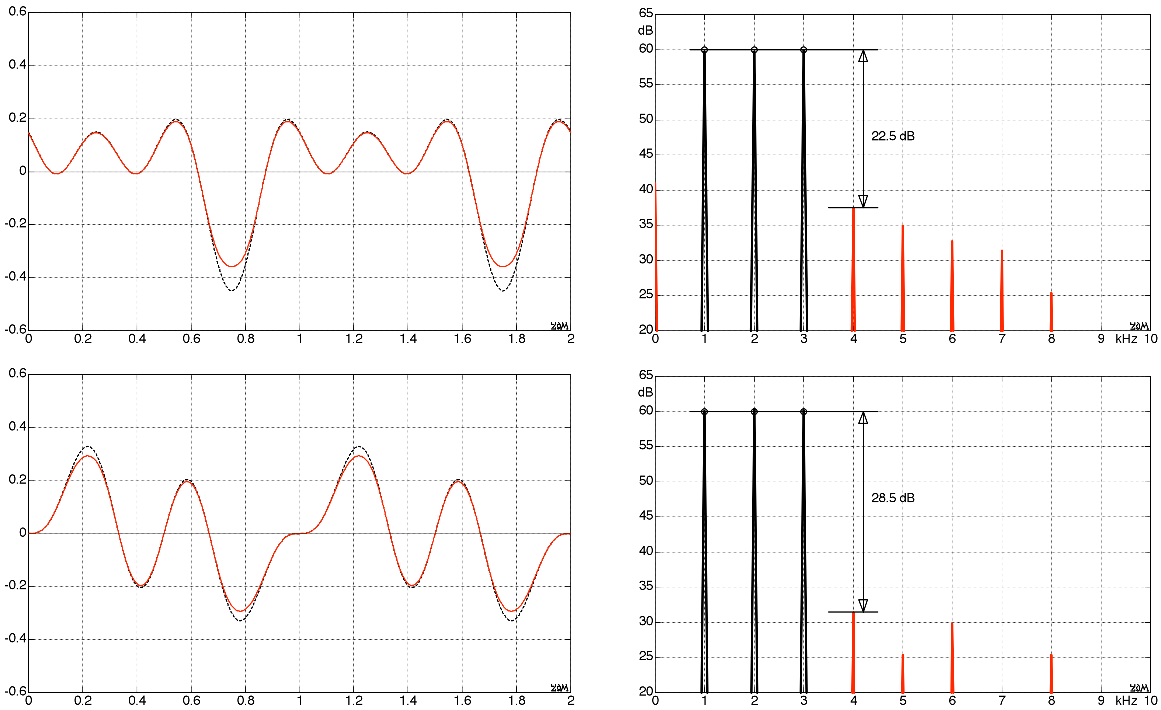

Fig. 10.10.27: Changes in the 3rd order distortion spectrum as the phase of the partials in a three-tone signal is changed. The RMS-value, and thus the level of the primary signal is identical for both cases.

For **Fig. 10.10.27**, a signal consisting of 3 partial tones is distorted. Changes of the phases of the partials do change the level of the strongest distortion product by no less than 6 dB. This does not mean that measuring of (T)HD (or intermodulation- or difference-tone-distortion) is not purposeful – in fact these measurements are highly suitable to describe the nonlinear behavior of a system. An approximate estimation of how strongly a specific signal is distorted by this system is possible, but does not really indicate how the resulting distortion in fact sounds.

After this introduction we will now look at real **guitar signals**, using the pickup voltage of a Telecaster. As a string is plucked with little force, the levels of the partials decay approximately linearly over time, as it has been shown in Chapter 7.7. For strong plucking (with the string hitting the frets - Chapter 7.12.2) we find a strong level-decay of up to 10 dB during the first 20 – 50 ms, and a slow decay afterwards, similar to weak plucking. In a simple model generating merely 3rd order distortion, the HD would change by a factor of 10 during the first 50 ms. For such time-variant signal a single HD-limit is not very purposeful.

In **Fig. 10.10.28** we see the time functions of two pickup voltages. A non-linear amplitude limiting to e.g. ±0.5 V would have very different implications for the two signals. This example clarifies that for HD-limit-values not only amplitude limits are significant, but durations in time, as well.

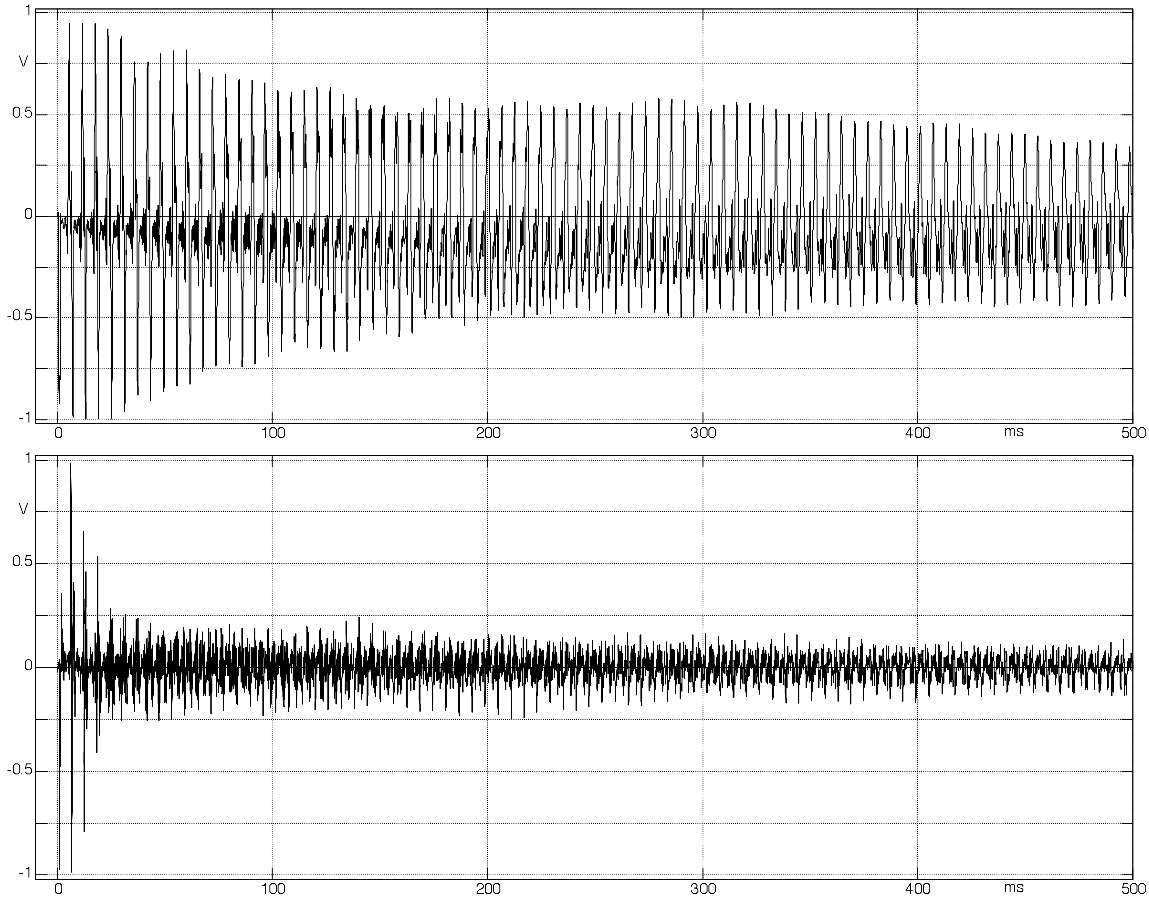

Fig. 10.10.28: Two different pickup voltages normalized to the same maximum value. In the upper section the string was weakly plucked, and strongly on the lower section. Telecaster, bridge pickup, E3 on D-string.

Before we subject these pickup voltages to distortion, we first return to the series expansion of the characteristic curve. For small HD it is purposeful to study the behavior of purely 2nd order and purely 3rd order distortion. In guitar amplifiers, however, strong distortion occurs, as well, and therefore the model using purely 2nd order and purely 3rd order distortion is incomplete. Tubes (as well as semi-conductors) **limit** on **both sides** for strong drive levels – this is the domain of **odd** distortions. A straight, symmetric characteristic (such as $y = x^2$) cannot generate limiting to both sides. A 3rd order characteristic can do this – however only within a small range, as shown in **Fig. 10.10.29.** The blue line approximates the characteristic of a tube close to the origin, but it turns off in the opposite direction as it moves away. And it continues to grow without any limiting.

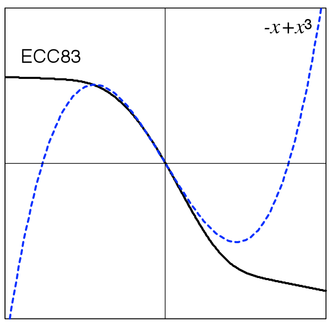

Fig. 10.10.29: Tube characteristic (ECC83); third-order parabola.

To better adhere to the tube characteristic depicted in Fig. 10.20.29, the approximation-polynomial would require further odd-order members in the series (x^5, x^7, ...), and in addition series-members of even order would be necessary, because the amounts of the limit-values differ (tube-characteristics are not exactly point-symmetrical). Therefore, the distortion in the following is done not by a polynomial characteristic but by a real tube characteristic (ECC83).

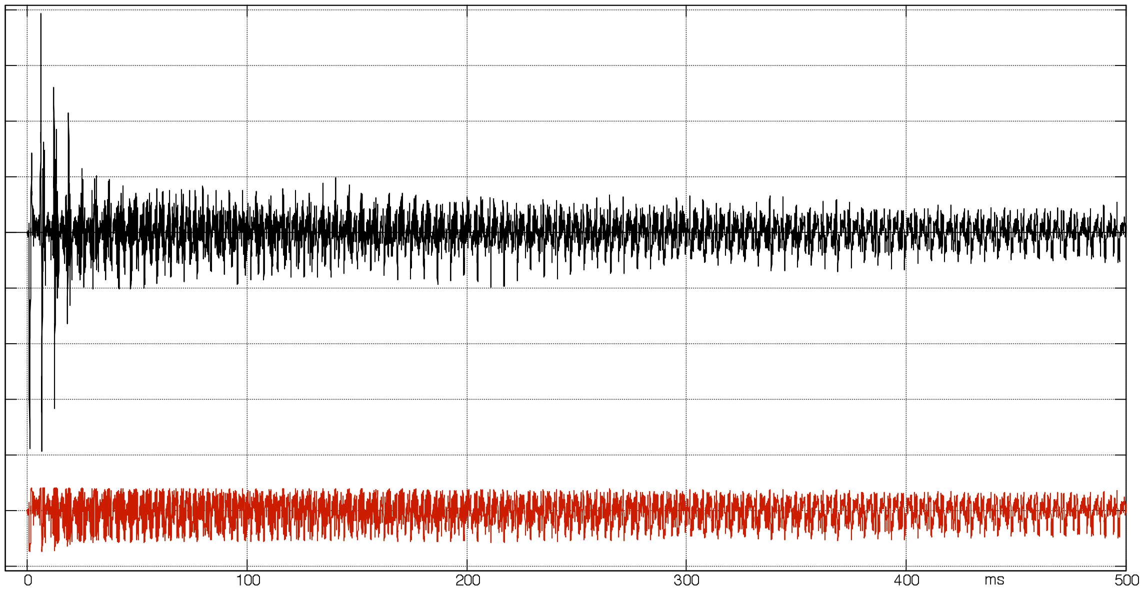

Fig.10.10.30: Pickup voltage, **without** (top) and **with** non-linear tube-distortion (bottom). String strongly plucked, Telecaster, E3 on D-string, bridge pickup.

In **Fig. 10.10.30** we again see the signal from Fig. 10.10.28, with and without tube-distortion. It may be hard to believe, but these two guitar signals do not in fact sound that different. One does hear differences but not in terms of "undistorted/distorted". The attack is louder for the undistorted signal, but afterwards there is no audible difference. This may be due to post-masking [12], and/or due to the fact that any limiting in the subsequent development affects merely very short signal peaks. Another reason: for a **strongly plucked string**, contacts between string and frets occur for a relatively long time period, and these sound similar to slight overdrive and hamper the recognition of actual tube distortion. A value for the HD in the signal shown in Fig. 10.10.30 cannot be established since there is no definition of a HD for such a multi-tone-signal. It is however possible to create a sine-tone with the same envelope, and to distort it in the same way (i.e. feed it through the same tube characteristic). The result is that at first 3rd order distortion dominates with k_3 reaching 28%. From 50 ms the 2nd order distortion starts to dominate with $\boldsymbol{k_2 \approx 5\%}$. It is noted again, however, that despite these large HD's the guitar does not actually sound distorted but is limited in its dynamic range. The "thud" at the beginning is softer – and that's it.

We obtain an entirely different result as the string is merely **lightly plucked**. Without distortion, it sounds weaker in the treble range than the strongly plucked string. Therefore, and also because the level does not decrease as fast, distortion can be heard clearly as the signal is fed to the same characteristic as the strongly plucked string (with both signals normalized – pre-distortion – to the same maximum drive.

For the lightly plucked string, **Fig. 10.10.31** shows the time function of the undistorted and distorted pickup voltage. Despite the same maximum drive level and the same distortion characteristic, the subjective degree of distortion is different. This is because the hearing system does not exclusively evaluate the attack, after all. It is well known from experiments on loudness scaling [12] that the loudness of short bursts decreases: the hearing integrates over 100 ms.

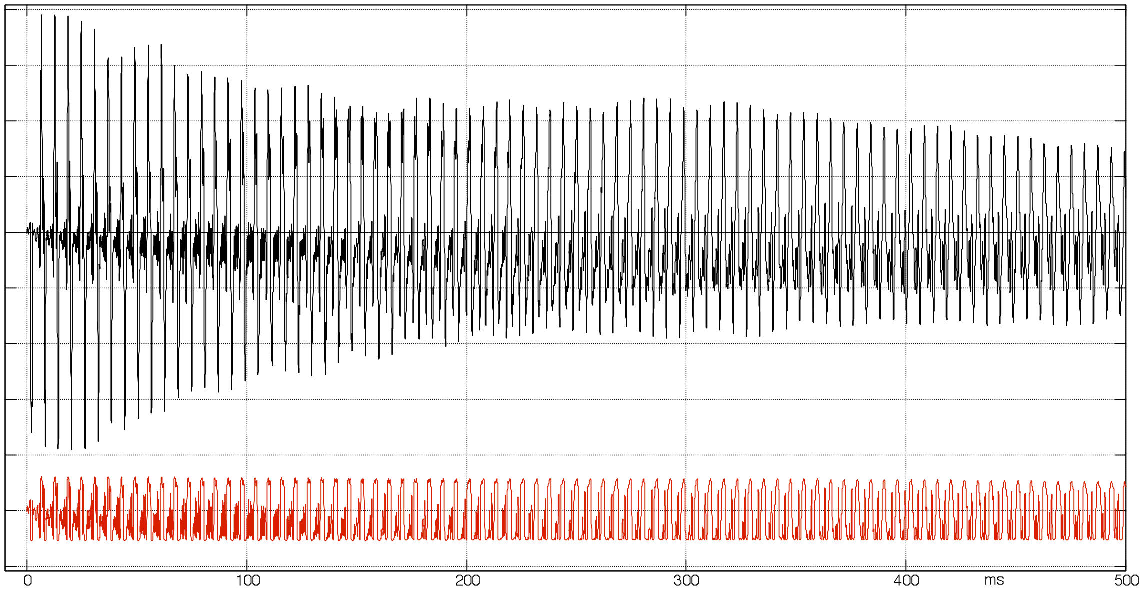

Fig.10.10.31: Pickup voltage, **without** (top) and **with** non-linear tube-distortion (bottom). String lightly plucked, Telecaster, E3 on D-string, bridge pickup.

It has already been elaborated repeatedly that the inharmonicity of the guitar signal plays a role, as well (Chapter 1.3, 8.2.5, 10.8.5). For a strictly harmonic sound, all spectral lines generated by the distortion fall onto already existing lines, and it is merely level and phase of the frequency component that changes. However, for an inharmonic sound the non-linearity will cause new spectral lines at frequencies where no partial was present in the undistorted signal. The subjectively perceived sound may change considerably due to this, depending on the circumstances. It will obtain a more stochastic character and sound as if noise had been added (Fig. 10.8.23). Because the inharmonicity depends on the type of string, on characteristic of the circuitry, and on the individual tubes, and on the guitarist, it is not possible to give a single threshold value for the audible HD. It may be noted as an orientation, though, that we are not talking about values in the range of or even below 0,1% here. There are investigations comparing capacitors with a THD of below 0,0001%. This is extremely sophisticated metrology but entirely without meaning for auditory acoustics.

Well then – despite all constraints, the reader will expect a number here, and now. And so, to the best of our knowledge: ***k* = 3%**. This would be the orientation value – and surely a basis for splendid discussions. Guitar-distortion becomes just audible as a sine-tone of the same level distorts with a THD of 3%. “The same level” should be interpreted such that not the level at the attack of the guitar tone is measured, but the level of a purposeful section of tone following the attack region. This puts the responsibility back to the esteemed reader and hopefully helps to avoid a discussion in internet fora (e.g. dedicated to the question of whether the threshold of audibility is not at 2,6%, after all).

 Translation into English by Tilmann Zwicker