4.10.3 Permeability

In chapter 4.3 the permeability was defined as the *specific magnetic conductivity*. It is a material property like, e.g. the electrical conductivity in a current circuit. Air has a relatively poor magnetic conductivity (μ_0 = 1,257 µH/m) and the magnetic resistance of air is relatively high. The permeability of magnetic materials is often depicted in relation to the permeability of air♣ as the **relative material permeability**: $\mu = \mu_r \cdot \mu_0$. Here, μ_r is the relative permeability, also called **permeability number**.

The permeability number of air equals one, to very good approximation The difference to vacuum is not significant. In contrast to air, in which the relationship between the magnetic field strength H and the magnetic flux density B in air is proportional: $B = \mu \cdot H$, in ferromagnetic materials (Alnico, steel, iron, nickel) the permeability number is larger than one and dependent on the magnetic field strength. The literature for iron claims maximum permeability numbers of approx. 5000, but also up to 250000, if one deals with purest iron heat treated in hydrogen. If one would multiply the field strength in the close vicinity of a pickup of 40 kA/m with $5000 \cdot \mu_0$ this would yield, in purely mathematical terms, a flux density of 251T. But this value has of no practical meaning because, on the one hand, generally the magnetic field strength changes when materials with magnetic conductivity are inserted into the field, on the other hand the saturation value for the magnetic flux density of iron is approx. 1 T. **Saturation** means that the field enhancement in iron reaches a limit that cannot be exceeded. The origin of a magnetic field is supposed to be a moving charge and, hence, an electric current. In all magnetically non-neutral materials internal circular currents (elemental currents) align themselves with respect to external magnetic fields and form an internal magnetic field, which is superimposed to the external one. The flux density B generated in the iron can be considered as being the sum of an externally generated component $B_0 = H \cdot \mu_0$ and an internal field enhancing **polarization** J. In other words, the component B_0 forms principally and everywhere in the magnetic field. In magnetic materials the polarization J is added to it.

$$B = B_0 + J\,; \quad J = \chi_m \cdot \mu_0 \cdot H\,; \qquad \chi_m = \mu_r - 1 = \text{relative susceptibility}$$

The relationship between B_0 and H is proportional; it is not subjected to saturation. However, the polarization depends decreasingly on H and tends to a non-exceedable limit = saturation polarization J_{sat} (**Fig. 4.37**).

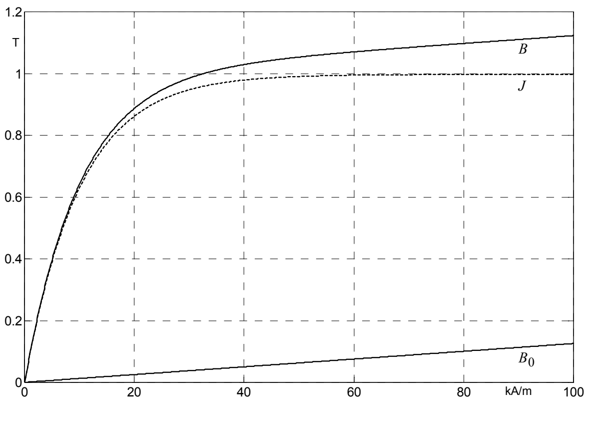


Fig. 4.37: Relationship between field strength H, flux density B and polarization J. In this example the saturation polarization is 1 T.

♣ Actually, vacuum is the reference: $\mu_{\text{air}} = 1.0000004 \times \mu_{\text{vacuum}}$.

In the pickup an electrical voltage can only be generated (induced) if the magnetic flux density B changes as a function of time t and, as B depends on H through μ, the magnetic field strength H also changes. The calculation of such field changes is highly complicated and only possible as a rough approximation. The string and the pickup-magnet are ferromagnetic in different ways and, in addition, the screws, pole pieces and plates may possibly influence the field. In every differential metal volume a different B and, hence, a different μ can dominate. Adding to the difficulty, small changes of the field ΔH lead to small changes in the flux density ΔB which cannot be deduced from the slope of the magnetization curves (hysteresis loops). In **Fig.4.38** in the right picture we have depicted a section of the magnetization curve. If the field strength increases from the working point H_A by a small amount ΔH, the corresponding flux density will adjust to $B_A + \Delta B$, which is not located on the large continuous curve; this one can only be traversed in the direction of the arrow. The quotient from $\Delta B / \Delta H = \mu_{rev}$ is called the **reversible permeability**. It is smaller than the **differential permeability**, which can be viewed the as differential quotient or slope of the hysteresis (dashed line in the figure). According to [7] the reversible permeability not only depends on flux density B, according to Gans [Phys. Z., 12/1911], but also on on the polarization J. It also has – and this complicates the numerical calculations – a tensor character for isotropic media!

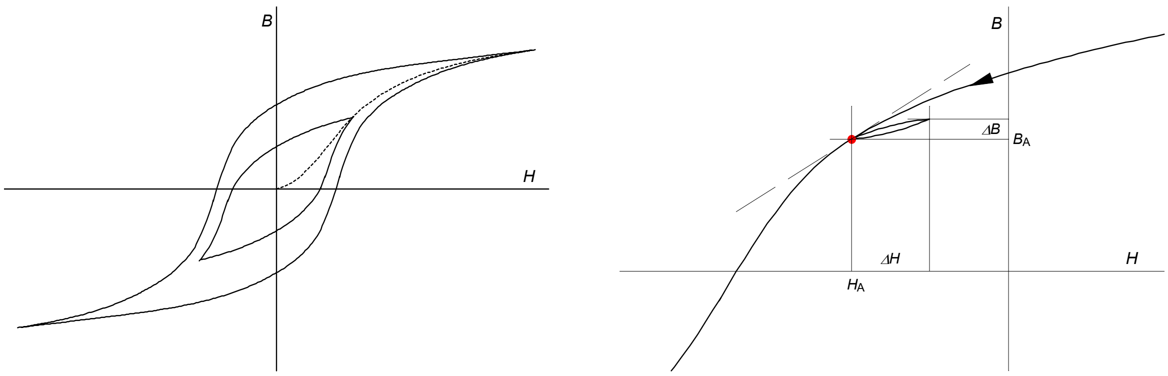


Fig. 4.38: Difference between differential and reversible permeability. ΔH and ΔB represent small changes; in the picture they are considerably exaggerated (cf. Fig. 4.6).

The reversible permeability μ_{rev} is maximum for $B = 0$ and decreases monotonically towards μ_0 with increasing value of the flux density. Consequently, ferromagnetics conduct alternating magnetic fields much more poorly the higher the static percolating magnetic flux is. Thus, it is incorrect to simply assign a better magnetic conductivity to a **steel-string** because steel is depicted as ferromagnetic material in tables ($\mu >> 1$). Only steel strings without static pre-magnetization may exhibit permeability numbers which exceed 50. However, as soon as a string is in the vicinity of a pickup magnet, a considerable static magnetic flux will flow through it, and the reversible permeability will drop to values only slightly higher than that of air (**Fig. 4.39**)

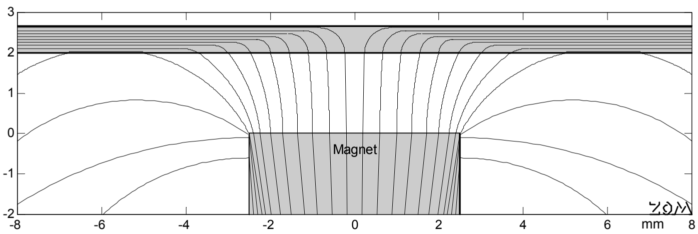


Fig. 4.39: Approximate progression of the static magnetic flux for magnet, air gap and string (→ Fig. 5.4.8).

In **Fig. 4.39** one can recognize, how the static magnetic flux density (depicted as line density) increases within the string along the string axis. Directly above the magnet axis the axial string flux density is zero, but already after approx. 6 mm a maximum is reached which practically means full saturation (1.7 Tesla, steel string with Alnico-5-magnet, chapter 5.4.2). If now the string loses its good conductivity for alternating magnetic fields already after some millimeters, they will have no reason to follow the string and, consequently, leave it. Thus, vibrations of the string only influence the magnetic field in a relatively small volume and the **magnetic window** (the magnetic aperture) is relatively short (chapter 5.4.4). The closer the magnet is located to the string and the stronger it is, the shorter is the magnetic aperture (the string sampling is more selective) and the corresponding damping of the treble frequencies is lower.

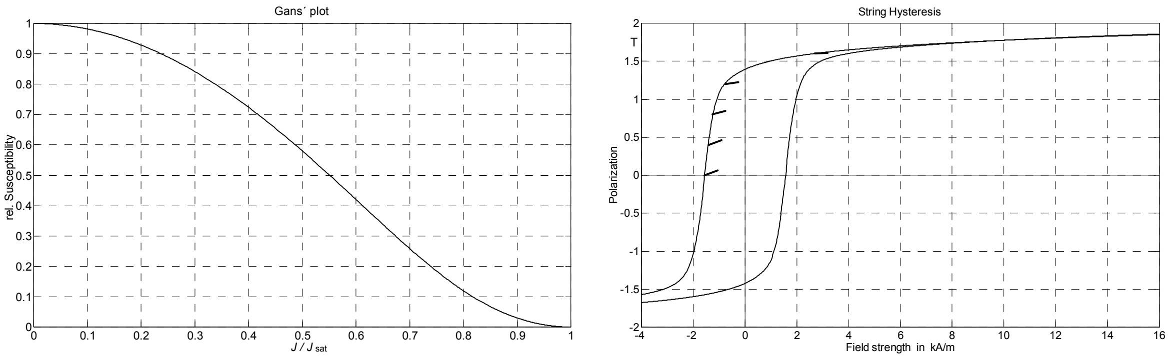


Fig. 4.40: Gans curve for the reversible susceptibility (left) and string-hysteresis (right).

In **Fig. 4.40** we have depicted the **"Gans curve"** in the left picture. It describes the relationship between the normalized polarization up to the saturation polarization J_{sat} and the normalized reversible susceptibility up to the starting susceptibility χ_A. Their formula [21] reads, in parameter representation:

$$\frac{\chi_{rev}}{\chi_A} = 3 \cdot \left(\frac{1}{x^2} - \frac{1}{\sinh^2(x)} \right); \qquad \frac{J}{J_{sat}} = \coth(x) - \frac{1}{x}; \qquad \text{Gans curve}$$

In this parameter representation x stands for the parameter (0 … 100), J for the polarization and χ_{rev} for the reversible magnetic susceptibility. The starting susceptibility acts at the beginning, i.e. at the origin of the new (first) curve. In the right picture of Fig. 4.40 the hysteresis curve of a steel string is shown, together with little lines indicating the slope of small signal changes. **Fig. 4.41** shows the measured results of a typical steel string.

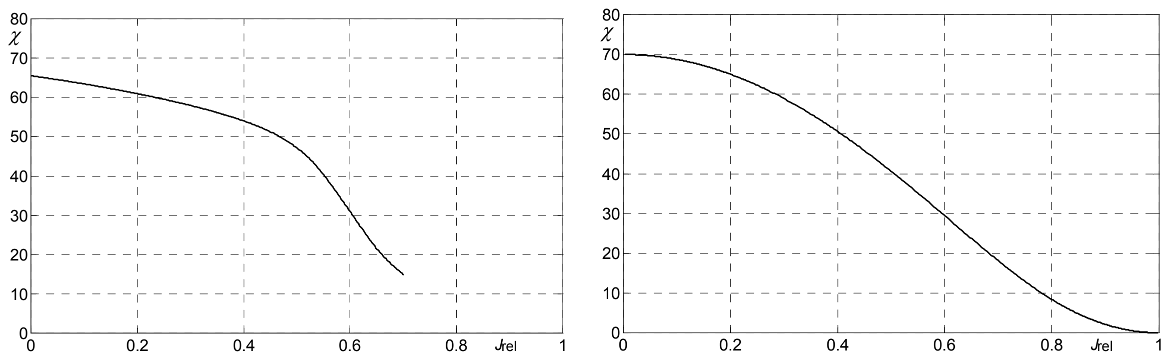


Abb. 4.41: Measured reversible susceptibility (left) and theoretical simplification of the Gans curve (right).

4.10.4 Magnetic Losses, magnetic skin-effect

The field enhancing effect of ferromagnetic materials is caused, on the one side by the internal shift of domain walls (Bloch walls, chapter 4.4.1), and on the other side by initially randomly oriented elemental magnets that are turned into a common direction by the external field. A small part of the energy that is necessary for shifting and/or rotating is irreversibly transferred into heat. The thermal energy that is produced by a kind of micro-friction is "lost" from the electromagnetic field and this is the reason why one talks about **loss of electromagnetic field energy**, or in short about magnetic losses; the designation **iron losses** is also common. Losses will decrease the voltage generated in the pickup – an effect which may mainly affect higher frequencies as brilliance loss.

The two most important loss mechanisms are eddy current losses and hysteresis losses. The field-energy per volume w can be derived from the relationship between the magnetic field strength H and magnetic flux density B, as given by the hysteresis curve:

$$w = \int_{B_1}^{B_2} H dB$$ Volume-specific magnetic field energy

If the hysteresis curve is a (curved) line, the magnetic energy would be increased by elevating the flux density from B_1 to B_2 and likewise would be diminished by the same value by decreasing from B_2 to B_1 – the process would be reversible. However, as each hysteresis loop consists of two different branches, a complete circuit leading back to the origin does not yield $w = 0$ but an energy density which is proportional to the enclosed area and which represents a measure for the energy loss. For guitar strings, the specific **energy loss** for a *boundary loop* circuit is about 10 µWs/mm³. If one multiplies this value with a 2 cm long 0.7 mm string one ends up with an energy loss of approximately 77 µWs for the total hysteresis circuit. However, the working point of a vibrating string does not follow the boundary hysteresis (from negative saturation to positive saturation and back) but only a small fraction of it. The fraction of it heavily depends on the distance of the string to the magnet and on the amplitude of the string deflection. The steady flux is also high in the regions of high alternating flux and – conservatively estimated – the alternating flux may reach about one tenth of the steady flux. In addition, if one considers that the small signal changes yield relatively small areas, lancet-shaped hysteresis loops (also called **Rayleigh-Loops**), it becomes clear, that the energy losses caused by the string are of only marginal significance. As an order of magnitude one can estimate 10 mWs for the string energy and 1 µWs for the iron losses per cycle. If the string vibrates with 150 Hz with this assumption it will lose 1.5% of its vibration energy, which would be negligible. A more precise computation of the iron losses would be laborious, because one has to deal with a three-dimensional inhomogeneous field, for which material tensor parameters would have to be known. In addition, measurements are difficult because one has to discriminate from other damping mechanisms. But, even for the case that the above approximation would be unrealistic and the string energy loss per second would be 26% instead of 1.5% this would be equal to a level decrease of 1 dB/s – insignificant against other damping mechanisms. The bottom line of these approximations is, therefore, (without proof): the **hysteresis losses** (magnetization-change losses) **emerging within a string are negligible**.

Other than in the string, hysteresis losses are also possible in the magnet or in nearby ferromagnetics. Although these losses are not generated within the string, nevertheless the energy necessary for changing the magnetization of these ferromagnetics has to be delivered by the vibrating string. The magnetic volume affected by relevant alternating fluxes for single coils is larger, by more than one order of magnitude, compared to the above considered volume of the string. However, the relative change in flux density inside the magnet is also one order of magnitude smaller than inside the string so, on the whole, again an effect of marginal importance. As long as one does not move a very strong magnet close to the string (which is in contradiction to a large string displacement) **the conclusion is: hysteresis losses are negligible**. This statement is indeed speculative but is supported by measurements which yield, without doubt, that string vibrations are damped more intensely by the fretting hand of the guitarist than by the magnetic field of the pickup (chapter 4.11).

A second source for losses are **eddy current losses**. The induction law discussed in chapter 4.10.1 generates a voltage and a current in the pickup coil but also in every conductor that is in close vicinity to the pickup. As metals represent electrical resistances, an effective electrical energy or thermal energy is produced which weakens the magnetic field or the string vibration. The electrical voltage that causes the eddy current is dependent on the *change* of the magnetic field and, therefore, eddy currents do not play any role at low frequencies. With increasing frequency they become more and more important, however, the **skin effect** has also to be taken into account as reverse effect (chapter 5.9.2.2): the magnetic counter field induced by the current flow forces the current more and more into the boundary regions and consequently increases the eddy current resistance (chapter 3.3.2)

Eddy current losses cannot generally be neglected, but indeed deteriorate the treble reproduction of every pickup; not only marginal but possibly by 5 dB and more, if thick low-ohmic metal plates are employed. One could interpret this fact as a sound characteristic which is deliberately chosen by the developer, but one should take into account that very dominant treble can be reduced easily by a potentiometer in parallel to the pickup, which is not possible the other way round. A pickup with less eddy currents may sound brilliant as well as dull; a pickup damped by eddy currents may only sound dull[♣]. Pickups that exhibit small eddy current losses are the ones with 6 Alnico magnets as sole metal pieces (USA-type Stratocaster). Soft-magnet pole-pieces with underlying bar magnets increase the eddy currents, as do tin covers. If one wishes to have a shielding case with small eddy current losses, thin-walled German silver cases are recommended. One pickup that sounds brilliant, despite having a metal case, is the Gretsch humbucker.

Eddy currents are not only present in magnets, pole-pieces and shielding cases but are also possibly in metal support plates and shielding foils. When replacing a plastic by an aluminum pick-guard one experiences a small treble loss. However, the loss can mostly be avoided by a small slit, which suppresses the circling eddy currents.

[♣] In general, sound filters built into guitar amplifiers cannot compensate for eddy current losses.

In order to obtain quantitative data for **eddy current losses**, a thin walled measuring coil was fabricated, into which cylinder-shaped ferromagnetics (∅ = 5mm) could be inserted. The 14 mm wide coil form was wound with 5500 turns of an 80-µ-CuL enameled copper wire (**Fig. 4.42**). In this representation the logarithmic impedance unit is depicted over the logarithmic frequency – unusual but convenient. 0 dB = 1 kΩ.

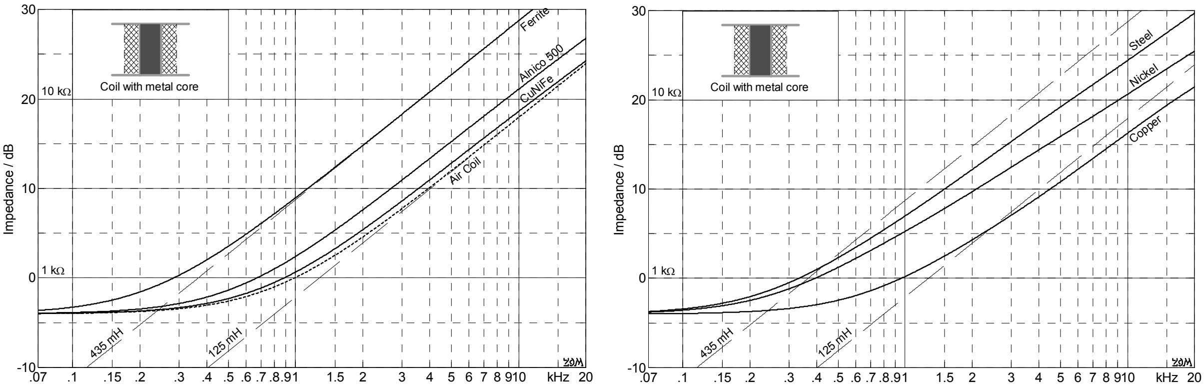


Fig. 4.42: Logarithmic impedance of a measuring coil ($N = 5500$); different core materials.

The wire resistance (630 Ω) is measured without a core ("air coil") at low frequencies and at high frequencies the impedance increase proportional to the frequency, the inductance (125 mH). The two terminal device is, thus, perfectly described by an *RL*-series connection in this frequency range. Insertion of an **Alnico**-500-magnet (5x14 mm) increases the inductance by 46%, insertion of a respective **ferrite** cylinder increases the inductance by a factor of 3.5. In both cases a frequency-proportional inductance increase happens at high frequencies, so that only one resistance is necessary in the equivalent circuit: the **wire resistance**♥. The inductance increase, however, does not mean that the relative permittivity of ferrite is only 3.5 (or for Alnico is only 1.46). These materials cover only a part of the field space, their effectiveness is, thus, substantially diminished. As an analogy one might think of two resistors in series, e.g. 1000 Ω and 10 Ω. The total resistance in this example is 1010 Ω. It decreases to 1001 Ω if the second resistor has only 1 Ω. At a 1V-source a current of approx. 1 mA will flow even if one will decrease the second resistor even further. This is similar for the magnetic circuit: the magnetomotive force is dominated by the low-conductive air field. With a little peculiarity: A change in the magnetic resistance of the core will also affect the shape of the field lines and, thus, the resistance of the air field.

The reason, why the impedance of Alnico and ferrite can be represented by an ordinary *RL*-two terminal device, is quite simple: in addition to the wire resistance no additional loss resistance has to be taken into account: eddy currents do not yet play a role♣. **Ferrites** are sintered out of oxide powder; they have a high electric resistance that prevents eddy currents. **Alnico**-alloys are, in comparison to ferrites, already quite good conductors. The fact, that they exhibit nearly no eddy current losses in the relevant frequency range, arises from their relatively small permeability (2 – 5). Good conductors with high permeability should, thus, produce enormous eddy current losses – and this is what they do, to be confirmed by the following measurements. To achieve this, we have inserted cylindrical cores made of different materials into the above mentioned coil: steel, nickel, copper (**Fig. 4.42** right)

♥ The denomination *copper-resistance* is disadvantageous here, because copper is also used for the coil core.
♣ The (nonlinear) remagnetization losses are also insignificant.

Copper is diamagnetic, its permeability differs only marginal from μ_0. **Steel** and **nickel** are ferromagnetic, their permeability is considerably higher than μ_0. Copper is a very good electrical conductor, Nickel has higher resistivity by a factor of 4, steel by a factor of 10 – 20. As can be seen clearly by the measured curves (Fig. 4.42, right) the high-frequency impedance increase with these metal cores is shallower than in air or in ferrites. The origin of this behavior is the **eddy currents**, which increasingly force the field lines out of the core with increasing frequency and, thus, decrease the inductance. **Fig.4.43** shows the internal field distribution for a steel cylinder (∅ = 5 mm, length = 14 mm) for three frequencies as well as the frequency-dependence of the magnetic impedance, which, as complex unit, consists of a real part (magnetic resistance) and an imaginary part (eddy current losses). The losses have to be imaginary because the magnetic resistance is generally defined to be real – different from *electrical* networks, where loss-resistances usually are defined to be real. However, these are only conventions, finally only orthogonality between effective and reactive power is necessary.

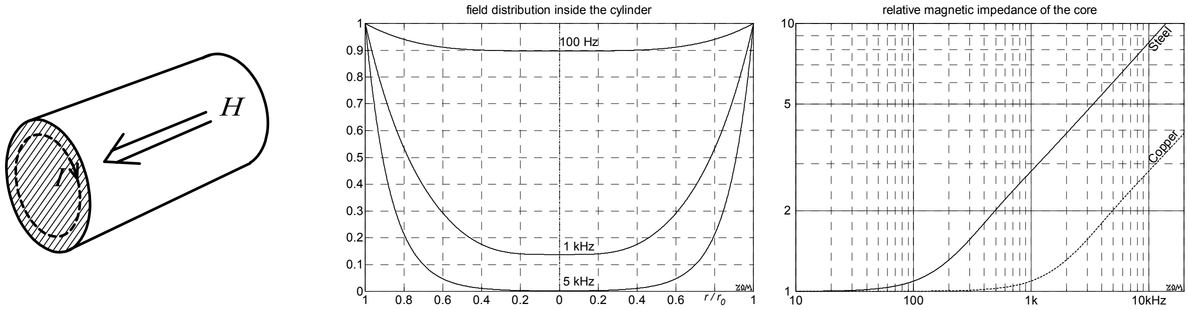


Fig. 4.43: An axial magnetic field *H* growing with time generates the eddy current *I* in the metal cylinder. This current will produce a circular magnetic field around itself, which is in opposite direction to the generating field and forces it out of the cylinder. The picture in the middle shows the radial distribution of the axial magnetic field in a steel cylinder (r_0=5mm), on the right the magnetic impedance normalized to low frequencies is depicted.

The basis of the calculations is **Maxwell's** Laws in their differential form under the simplifying assumption that the electrical conductivity σ and the permeability μ are constant. For the conductivity this assumption is true, for the permeability actually not: the space and time-dependent flux-distribution leads to a space and time-dependent μ. The exact calculation in an anisotropic non-linear medium is, however, so complicated that simplification is necessary. Both Maxwell Laws now read as:

$$rot\,\vec{H} = \sigma \cdot \vec{E} \quad \text{and} \quad rot\,\vec{E} = -\mu \cdot \partial \vec{H} / \partial t$$ Differential form of Maxwell's Law

In cylinder coordinates *H* only exists in the axial direction, the field strength *E* exists only in the circular (azimuthal) direction and the rotation *rot* can, therefore, be simplified to:

$$\sigma \cdot E = -\partial H / \partial r \quad \text{and} \quad -\mu \cdot \partial H / \partial t = E / r + \partial E / \partial r$$ In cylindrical coordinates

Both formulas combined will yield **Bessel's** Differential Equation, which can be solved for harmonic signals with complex units:

$$\frac{\partial^2 H}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial H}{\partial r} = \mu\sigma \cdot \frac{\partial H}{\partial t} \quad \rightarrow \quad \frac{\partial^2 \underline{H}}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \underline{H}}{\partial r} = j\omega\mu\sigma \cdot \underline{H}$$ Bessels Diff. Equation

The time differential operator $\partial/\partial t$ has been replaced by $j\omega$ (see system theory).

The **solution of Bessel's Differential Equation** for the radial distribution of the axially directed magnetic flux density $\underline{B}(r) = \mu\underline{H}(r)$ is:

$$\underline{B}(r) = \mu c \cdot J_0(kr) \qquad \text{with} \qquad k = (1-j)\cdot\sqrt{\pi f \mu \sigma} \qquad \text{or} \qquad k^2 = -j\omega\sigma\mu$$

Here c is an integration constant and J_0 is zero order Bessel's function of the first degree. The total magnetic flux that axially passes through the cylinder is given by the area integral over the cross section with r_0 = cylinder radius:

$$\underline{\Phi} = \int_0^{r_0} \underline{B} \cdot 2\pi r \cdot dr = 2\pi\mu c \cdot \int_0^{r_0} r \cdot J_0(kr)\cdot dr = \frac{2\pi\mu c}{k^2} \cdot \int_0^{kr_0} kr \cdot J_0(kr) \cdot dkr \qquad \text{Total flux}$$

The integration of Bessel's Function is carried out with $\int x \cdot J_0(x) \cdot dx = x \cdot J_1(x) + C$, where J_1 is a first order Bessel's Function of first degree. For the magnetic flux this yields:

$$\underline{\Phi} = \frac{2\pi c}{-j\omega\sigma}\left[kr \cdot J_1(kr)\right]_0^{kr_0} = j\frac{2\pi c k r_0}{\omega\sigma} \cdot J_1(kr_0) \qquad \text{Total flux}$$

The magnetic resistance is defined as quotient out of magnetomotive force and flux, the **length-specific** magnetic resistance R'_m is the quotient out of field strength and flux:

$$R_m = V_m/\Phi; \qquad R'_m = R_m/l = H/\Phi; \qquad \text{Magnetic resistance}$$

The length-specific magnetic resistance is calculated by dividing the field strength $H(r_0)$ by the flux Φ along the cylinder barrel; the result is complex and is, therefore, called the **length-specific magnetic impedance**:

$$\underline{Z}'_m = \underline{H}(r_0)/\underline{\Phi} = \frac{-j\omega c\sigma}{2\pi c k r_0} \cdot \frac{J_0(kr_0)}{J_1(kr_0)} = \frac{k}{2\pi r_0 \mu} \cdot \frac{J_0(kr_0)}{J_1(kr_0)} \qquad \text{Length-specific impedance}$$

For very low frequencies k tends to zero and, using a series expansion of Bessel's Function, one will get as a (real) limit value $\underline{Z}'_m \to 1/r_0^2\pi\mu$, or the inverse of the cylinder cross-section and the magnetic conductivity. This means, that for low frequencies, there is no field displacement at all, the flux density is <u>in</u>dependent of position for the entire cross-section. However, with increasing frequency the magnetic flux is forced from the center to the boundary area (casing vicinity), the magnetic resistance (impedance) increases and the cylinder will become 'less magnetic' (see also chapter 5.9.2.4).

The field displacement calculated by Bessel's functions can qualitatively explain the impedance/frequency relation shown in Fig. 4.42. However, precise quantitative data are not possible because the (possibly tensor) magnet data are not known precisely and the metal cylinder is not percolated exactly axially. In contrast to the pickup calculations for metal cylinders, a finite elements (FEM) computation would be possible but, also for this case, the problem of insufficient material data remains.

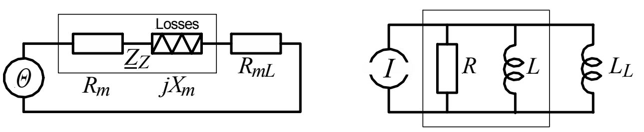
The (magnetic) field lines of the coil shown in Fig. 4.42 run partly in metal and partly in air. As already shown in chapter 4.6, one may visualize these magnetic networks by block diagrams, in **analogy** to electrical networks, in which resistors are displayed by rectangles. For the depiction of magnetic *loss* resistors there are no commonly defined symbols; in the following they are represented by rectangles enclosing a zigzag line (**Fig. 4.44**). The **magnetic impedance** Z_m (the inverse of which is the magnetic admittance Y_m) consists of real and imaginary parts: $\underline{Z}_m = R_m + jX_m$. It has to be kept in mind that the depicted loss resistors are imaginary – different from an electrical network. In order to project the networks onto one another using this analogy one has to define the flux quantity and the potential (difference) quantity [3]. The **flux quantity**♣ in electrical networks is the current, in magnetic networks it is the magnetic flux. The **potential quantity** is the electric voltage and the magnetomotive force, respectively. The flux quantity divides at nodes, where Kirchhoff's first law (or Maxwell I, respectively) is valid; in analogy, Kirchhoff's second law is valid for the potential quantity (or Maxwell II. respectively). Analogies that project flux quantities onto flux quantities create an **isomorphic** (equal structure) network; the projection of a flux quantity onto a potential quantity generates a **dual** network. Which is valid for **electromagnetic analogies**? Using the transformation mechanisms predominant for pickups (chapter 5) as orientation, one may find a projection of the magnetic flux to the voltage and of the current to the magnetic field strength – or **duality**. Written as equations:

$$U = N \cdot d\Phi/dt \qquad \text{and} \qquad N \cdot I = \oint H \cdot ds \qquad \text{Electromagnetic transfer formulas [3]}$$

The first formula represents the law of induction, the second the law of magnetomotive force (Ampere's law). Consequently, a magnetic series circuit will become a parallel circuit in the electrical block diagram. The differential occurring in the law of induction will be replaced by a multiplication with $j\omega$ for complex (sinusoidal) signals, which yields:

$$\underline{\boldsymbol{Z}}_{\mathbf{el}} = \frac{\underline{U}}{\underline{I}} = \frac{j\omega \cdot N \cdot \Phi}{\Theta / N} = j\omega \cdot N^2 \cdot \underline{\boldsymbol{Y}}_{\mathbf{m}} \qquad \Theta = \oint H \cdot ds = \text{Magnetic flux}$$

The magnetic and the electric impedance are thus reciprocal: The higher the permeability, the lower is the magnetic impedance and the higher the electric impedance. A real magnetic resistor will be projected into an imaginary electrical resistor (inductance, $\underline{Z} = j\omega L$), an imaginary magnetic (loss-) resistor will be projected into a real electrical resistor. The series connection of the magnetic real and imaginary part of the impedance $R_m + jX_m$ will become the parallel connection of the electrical resistance R and the inductance L; both are frequency-dependent. The magnetic in-series connection of the air resistance R_{mL} will become the parallel lying inductor L_L.



$$\underline{Y}_{el} = \underline{Z}_m / (j\omega N^2)$$
$$R = \omega N^2 / X_m$$
$$L = N^2 / R_m \,; \quad L_L = N^2 / R_{mL}$$

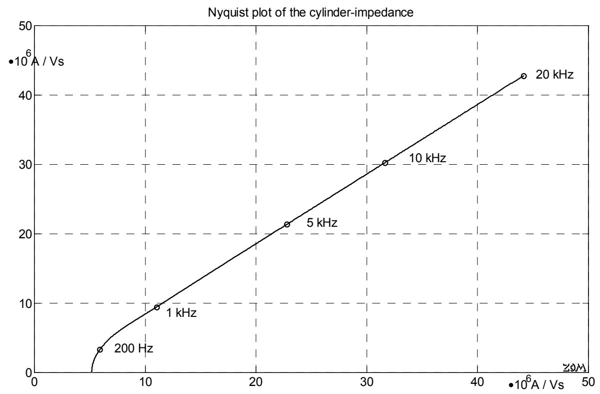
Fig. 4.44: Dual analogy between the magnetic (left) and the electric network (middle).
$\underline{Z}_Z$ = magnetic (metal-) cylinder-impedance, R_{mL} = magnetic air resistor.

♣ For the electromechanical FI-analogy [3] the electrical flux quantity "current" will be projected to the mechanical flux quantity "force" in equal structure; the FU-analogy projects dually.

The magnetic cylinder impedance $\underline{Z}_Z$ for a **metal core** with length l and radius r_0 located in the center of the coil is:

$$\underline{Z}_Z = \frac{k \cdot l}{2\pi r_0 \cdot \mu} \cdot \frac{J_0(kr_0)}{J_1(kr_0)} \quad \text{with} \quad k = (1-j)\cdot\sqrt{\pi\mu\sigma\cdot f} \qquad \text{Magnetic cylinder impedance}$$

Here, μ is the (absolute) permeability of the core and σ is the electrical conductivity. Both the argument (kr_0) as well as the resulting Bessel function are complex. **Fig. 4.45** depicts the frequency dependence of the real and the imaginary parts of the magnetic cylinder impedance. If the metal cylinder were the only magnetic resistor in the (closed) magnetic loop, one would obtain 5.9 H for the low frequency case, as shown in the Fig. 4.46 (left). However, as for the cylinder coil under consideration, the field lines close over a long air distance and an air resistor also has to be taken into account.



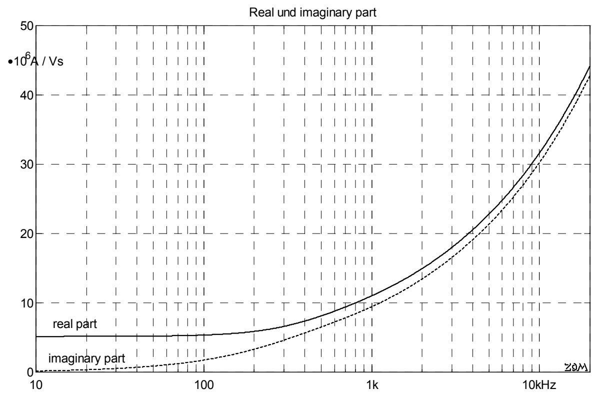
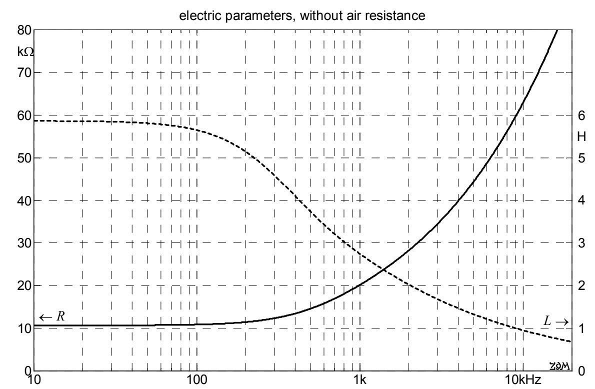


Fig. 4.45: Frequency-dependence of the complex magnetic cylinder impedance $\underline{Z}_Z$, $\mu_r = 110$, $\sigma = 5e6$ S/m.

If one considers, in a simple magnetic equivalent circuit, a series connection of core and air resistance (Fig. 4.44), this will reduce the absolute value of the inductance as well as its frequency dependence (**Fig. 4.46**). This simple model is well suited as long as the ferromagnetic metal core can sufficiently focus the field running through the coil. For small μ_r, however, a considerable part of the inner magnetic field flows within a kind of **hollow cylinder**, i.e. between core and average coil diameter. The magnetic resistance of this hollow cylinder is located parallel to $\underline{Z}_Z$ in the magnetic block circuit, hence, in the electrical equivalent circuit in series with the parallel connection of R and L (**Fig. 4.47**).



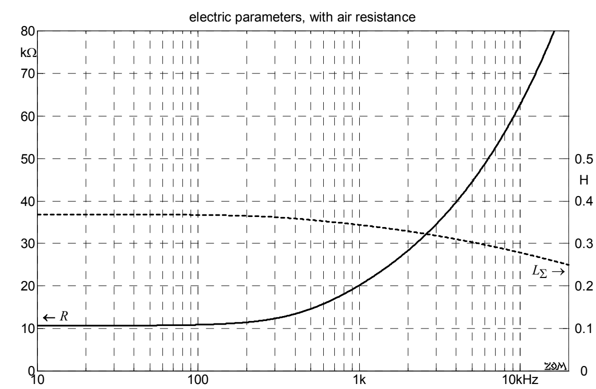


Fig. 4.46: Frequency dependence of R and L (left) as well as R und L // L_L (right) from Fig. 4.44 ($N = 5500$).

It is possible to explain every impedance/frequency curve in Fig. 4.42 with a good precision using this extended equivalent circuit diagram (**Fig. 4.47**). The magnetic resistance of this “hollow cylinder” is real and it is mapped onto the inductor L_{HZ} – by definition its *electrical* impedance is purely imaginary. The values of R and L are, as explained by Fig. 4.44, frequency-dependent.

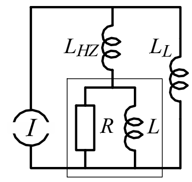


Fig. 4.47: ECD

For the **magnetic pickup** the magnetic eddy current losses have the following consequences: 1) The pickup resonance is damped not only by the wire resistance of the coil but also the ferromagnetic core inside the coil. 2) The inductance of the coil is frequency-dependent and decreases towards higher frequencies. A higher order *RL*-circuit can be employed in the block diagram as an alternative to the frequency-dependent inductance (see chapter 5.9.2.3). The different geometries and the diversity of the material parameters produce different damping and inductance frequency-dependencies. Using this, the pickup designer can purposely influence the frequency transfer characteristics.

Fig. 4.48 shows the impedance/frequency curves taken with a measuring coil (N = 5500, Fig. 4.44). The highest inductance is created by the ferrite rod, whose isolated elemental magnets do not allow eddy currents in this frequency range. The permeability of the humbucker screw, made of undefined steel, is practically the same for low frequencies (300 Hz), however, due to high eddy currents, its inductance decreases. The humbucker cylinder (“slug”) has a somewhat smaller inductance for lower frequencies but also smaller eddy-current losses. Alnico-magnets are practically free of eddy currents; the magnetically weaker Alnico 2 has a higher permeability as compared to Alnico 5 (Alnico 500), resulting in a lower pickup resonance (for otherwise equal parameters).

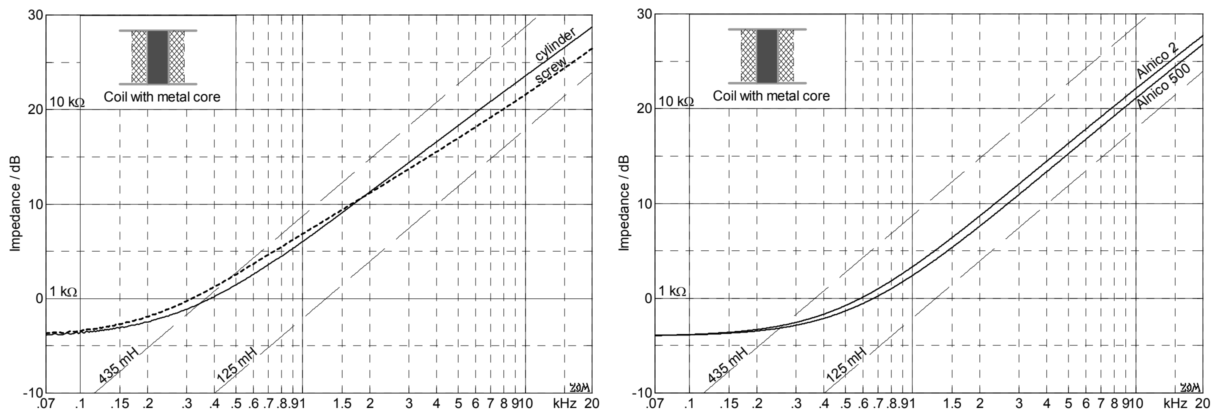


Fig. 4.48: Impedance/frequency curves, taken with the measuring coil; core dimensions 5x14 mm. “Cylinder” means the metal cylinders (= slugs) commonly used for humbuckers, “screw” depicts the humbucker screw (5.9.2.6).

Elaborate details for the construction of single-coil and humbucker pickups, as well as their technical data are summarized in chapter 5.

4.11 Magnetic Field Forces

Magnetic forces are the most obvious effect of the magnetic field: If one places a *ferro*magnetic material into the field of a permanent magnet, it will be drawn towards one of the magnetic poles. Magnetic forces also act for *para*- and *dia*magnetic materials, however, they are barely detectable. Only when the string is composed of a ferromagnetic material its vibration can be effectively detected by a magnetic pickup, because only then will the string significantly change the magnetic flux, so that a sufficiently high voltage is induced. At the same time, however, the magnetic forces will change the vibration mode of the string – the generation of a voltage in the pickup is, thus, not free of retroactive effects.

Theoretical physics does not view the electrical and magnetic fields as independent and self-contained conditions in space, but rather combines both phenomena into a unique field theory. Forces between stationary charges have to be treated differently from charges in motion: a relativistic approach is necessary even for small velocities. However, for the pragmatist says that *veritable is only what is appropriate for the act* and he gains the winning tender, in this case. The unique field theory is elegant but, for the present considerations, classical electrical engineering theory is sufficient and describes – as shown in the following – the force effects as independent phenomena.

4.11.1 Maxwell's Force

A ferromagnetic string brought into a magnetic field experiences a magnetic force. Here, it does not matter whether the string approaches the north or the south Pole; in both cases it will be attracted. The larger the field strength the larger the attractive force. The force, however, does not generally act in the direction of the of the field strength – and likewise also not generally in the opposite direction. Most simply one can interpret the magnetic force as a surface force that affects the entire surface of the string. Hereby it is understandable why an iron ball will stay at rest if brought into a *homogenous* magnetic field: the drag forces acting on both halves of the ball are balanced and the resulting sum of forces is zero. The fact that an iron ball in the field of a horseshoe magnet is nevertheless attracted by one of the poles is due to the inhomogeneity of the field. Only in a very theoretical middle position could an instable balanced condition be constructed; in every other position one of the two forces dominates and will accelerate the ball. This is completely different for the Coulomb force (4.11.6): a charged Styrofoam ball will also be accelerated in a *homogenous* electrical field.

The magnetic force effect may be very obvious; however, it still remains difficult to understand its underlying mode of action. Around the beginning of the 19th century magnet scientists still had the opinion that magnetized bodies would act on each other by a **long-distance effect**. This fact, that even an intermediate vacuum could not prevent this long-distance effect, lead to the conviction that the intermediate space was not involved and that the magnetic forces would directly act on the bodies without changing the space in between. The first person to define the **concept of the field** was Michael **Faraday** at around 1830, which changes the space between the bodies by force lines (**near-field theory**): the space itself will now become the medium and transmitter of the force.

James Clerk **Maxwell** (1831 – 1879) extended Faraday's ideas into a comprehensive electromagnetic field theory. A field is assigned to every point in space, which is defined by its **field quantity**. For the magnetic field these quantities are the field quantities H and B. The permanent magnet of the pickup generates an electromagnetic field that acts on other bodies (e.g. on a string) and produces forces there. However, the now magnetized string will also produce a field that acts on the permanent magnet. The generation and changes of these fields happen in the pickup practically without delay.

A mechanical stress state can be assigned to every point in the magnetic space within a very general force effect theory. The theory of elasticity distinguishes between **normal stress** (force perpendicular to the area) and **shear stress** (force runs within the area). For example, if a steel cylinder is stressed in the axial direction a tensile stress is generated which will elongate the cylinder. At the same time it will become a little bit thinner, because a compressive stress acts in the radial direction (lateral contraction). On the other hand, shear stresses are generated by shearing-off a whisker, which are also called **shear strains**.

A general quantity for the characterization of the mechanical state of stress is the **stress tensor**: it describes the mechanical stress load that a differential small volume of the string is exposed to. By integration over all these string volumes (mathematically formulated by Gauss's integral law) one will arrive at surface forces that act in a radial direction with respect to the string's surface. Since two magnetically very different materials converge at the string-air interface, one can obtain a very simple approximation for the normal force per area (*F/S*):

$$\frac{F}{S} = \frac{B^2}{2\mu_0} \qquad \mu_0 = 1{,}26 \cdot 10^{-6}\,\frac{\mathrm{Vs}}{\mathrm{Am}} \qquad 1\ \mathrm{VAs} = 1\ \mathrm{Nm}.$$

The area-specific force is proportional to the square of the flux density. For B one has to take the value that will result at the string surface and not the value which will be measured without the string. The value at a distance of approx. 2 mm in front of a pickup magnet without the string will be 20 – 50 mT, while it will be approx. 200 mT including the string. The string, as a consequence of its good magnetic conductivity, "sucks" the surrounding field lines as it were and, thus, increases the local flux density. As a rough approximation one will get 48 mN for the magnetic force for a string area of 3 mm² and 200 mT flux density. A precise calculation is difficult, because in this case the three-dimensional field distribution in two non-linear media would have to be determined. In contrast, **measurements** convey a sufficiently precise picture: for this a magnetic pickup was moved towards a steel wire (0.7 mm diameter) and the resulting magnetic force was measured (**Fig. 4.49**). Forces of 10 to 40 mN are detected for common separations – a good confirmation of the theoretical estimation. For a typical humbucker (e.g. Gibson '57-Classic) the forces are smaller.

In comparison to the string tension force (50 – 200 N) the magnetic forces are very small; the lateral string displacement caused by them is less than 0.1 mm. Nevertheless, the effect of the magnetic field must not be totally ignored, because its stiffness changes the frequency of the string. The nearer the string comes to the magnet, the more it is pulled. The differentiation of this force/distance relation will yield a distance-dependent stiffness of –1 ... –30 N/m; in contrary to common springs it is negative. The numbers are to be interpreted as guiding value; the measurement precision is only moderate.

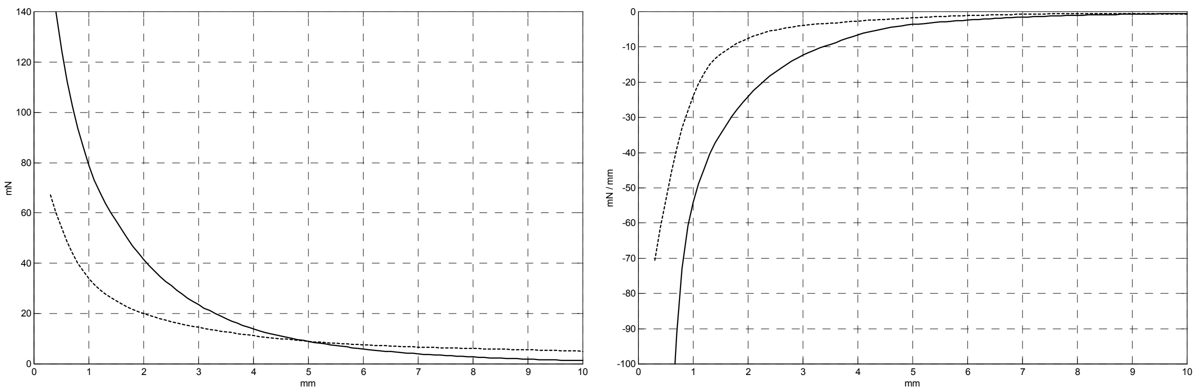


Fig. 4.49: Magnetic force as function of the clearance. Alnico-5, singlecoil (——), Gibson-humbucker (---). In the right picture the differential stiffness is shown as a function of the clearance.

However, the **negative magnetic field stiffness** not only affects the vibrations normal to the fretboard; for vibrations parallel to the fretboard the string/magnet distance is practically constant, the magnetic field stiffness is therefore negligible♣. For vibrations normal to the fretboard the negative field stiffness generates a decrease of the mechanical stiffness and, hence, a decrease of the partial tone frequency of the string. The effect is not dramatic, but audible for strong magnets: if the magnet is moved closer the tone frequency drops. However, every vibration of the string will occur as spatial wave, not as plain transversal wave. Even if a *plain* vibration is prevalent shortly after plucking, mode coupling in the supporting points and, last but not least, the magnetic field will cause a rotation of the original vibration plane. The rotation frequency is low (some Hertz) and beat-frequency-like **amplitude variations** will evolve in the pickup signal (4.11.3).

In this way the magnet does not just change the tone frequency but also the tone color of the vibrating string. Whether this is good or bad depends on subjective assessment criteria. Many guitarists have the opinion that the chorus-like beat frequencies of a Stratocaster belong to the typical sound of this guitar – as long as they are not too dominant. The assumption (or "certainty" of expert authors) expressed in several books that "the harmonics are slightly detuned in comparison with the fundamental*s*" is incorrect: the **fundamental** will be detuned the most. One can ask, since 2001, why it is suddenly referred to in the plural, and who feels that the announcement: "*further handbooks are in preparation*" is mere threat. However, this is just how they are, those string fundamentals.

4.11.2 Field-Induced Deviations of the Tone Frequency

When a magnetic pickup approaches a string, three effects can be anticipated: The **tone frequency** decreases, chorus-like **beat frequencies** evolve and the **amplitude** changes. The frequencies, especially the fundamental, will decrease due to the negative field stiffness, which can be audible for large values. The detuning between the fretboard-normal and fretboard-parallel vibrations induces beat frequencies; the altered frequency relationship between the partial tones in the subsequent non-linear systems causes additional partial tones, which can further increase the chorus impression. True damping, i.e. removal of vibrating energy, occurs only to a negligible extent. Firstly to the tone frequency:

♣ If the string is located substantially beyond the magnet axis, both vibrating planes are affected.

The resonance frequency of a vibratory mass-spring-system depends on the square root of the spring stiffness. The stiffness caused by the magnetic field is negative because approaching the magnet does not need a force in the direction of the movement (as it is for every common spring) but in the opposite direction: the magnet does not need to be pushed towards the string with force but, to the contrary, must be held back. The negative stiffness which is acting thereby decreases the total stiffness of the string and reduces the frequency. The frequency-dependence of this effect or which partial tones may be affected can be investigated with the **conduction-analogy**. Here, the mechanical system is described by an analogous electrical circuit with the analogies: force/current, velocity/voltage, spring/coil, mass/capacitor [3]. The principle effect can be investigated for an undamped plain transversal wave which is ideally reflected at a solid mounting. The string corresponds to an electrical conductor shorted at the end and whose length is large in comparison with the wavelength [e.g. Meinke]. The corresponding *mechanical* input impedance $\underline{Z}_E$ depends on the wave resistance Z_W, the terminating impedance ($\underline{Z}_{termination} \to \infty$, because of velocity $v = 0$), the conductor length l, on the frequency f and the phase velocity c. All of these system parameters can be attributed to the mechanical quantities by the analogy-laws: the string tension force Ψ, the string density ρ, the string length l and the string cross section area A.

$$\underline{Z}_E = \frac{Z_W}{j \cdot \tan \beta l}; \qquad \beta = \frac{\omega}{c} = 2\pi f \sqrt{\rho A / \Psi}; \qquad Z_W = \sqrt{\rho A \, \Psi} \qquad \text{Conduction}$$

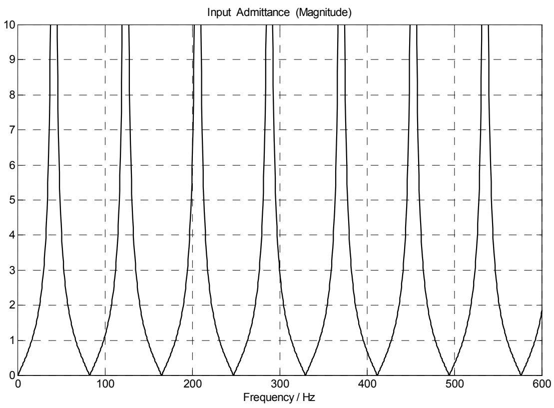
If one assumes a solid mounting for *both* ends of the string ($\underline{Z}_{el} = 0 \;\hat{=}\; \underline{Z}_{mech} = \infty$), the Eigen-frequencies (partial tone frequencies) of the string are the poles of the tangents-function, i.e. at integer multiples of the basic frequency $f_G = c/2l$. The reciprocal of the base frequency is the transit time over $2l$, i.e. from the beginning of the conductor to the reflecting end and back. In order to introduce the influence of the negative field stiffness into the conduction model, one divides the string into two consecutive conductors: a first conductor of length l_1 from the saddle to the pickup and a second conductor of length l_2 from the pickup to the bridge. The mechanical termination impedance $\underline{Z}$ of the first conductor is the sum of the input impedance $\underline{Z}_2$ of the second conductor and the stiffness impedance $\underline{Z}_S$: The input impedance $\underline{Z}_1$ of the first conductor (viewed from the saddle) is thereby given by:

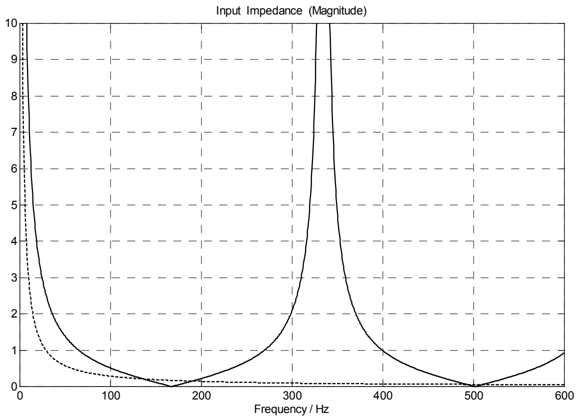
$$\underline{Z}_1 = \frac{\underline{Z} + jZ_W \cdot \tan \beta l_2}{1 + j\,\underline{Z}/Z_W \cdot \tan \beta l_2} \qquad \underline{Z} = \underline{Z}_2 + \underline{Z}_S \qquad \underline{Z}_S = \frac{s}{j\omega}$$

The Eigen-frequencies are located at the poles of the impedance function, i.e. at $\underline{Z}_1 \to \infty$.

Of course the input-impedance can also be calculated from the location of the bridge with the same result. For a first check the magnetic field stiffness can be taken to be zero ($\underline{Z}_S = 0$, $Z = \underline{Z}_2$) which indeed gives the partial tone frequencies at multiples of 82,4 Hz using the data of the E_2-string. As can be expected, a spring with stiffness zero cannot produce any changes. For every stiffness s different from zero the absolute value of $\underline{Z}_S$ will tend to zero with increasing frequency ($\underline{Z}_S = s/j\omega$), from which immediately follows, that the magnetic field stiffness can only detune low-frequency partials. As the field stiffness is negative the partial frequencies will *decrease*.

Figure 4.50 upper left shows the calculated (mechanical) input admittance of an E_2-string (82,4 Hz). The admittance is the inverse of the impedance; its zero points lie at the poles of $\underline{Z}_E$. The upper right picture shows the absolute value of the impedance of a 16 cm long part of the string located between bridge and magnet, as well as the absolute value of a magnetic field stiffness (–180 N/m). Its value was chosen to be unusually high to depict the effect more clearly. In the lower left plot the effects of the field spring on the admittance of the entire string are shown: especially the first and the second partial are detuned. For the calculation a characteristic wave impedance of 0.7 Ns/m was chosen; the length of the string is 65 cm and the magnet is located at a distance of 16 cm from the bridge. The frequencies of the partial tones are at the zero positions of the admittance. No dispersion was modeled.





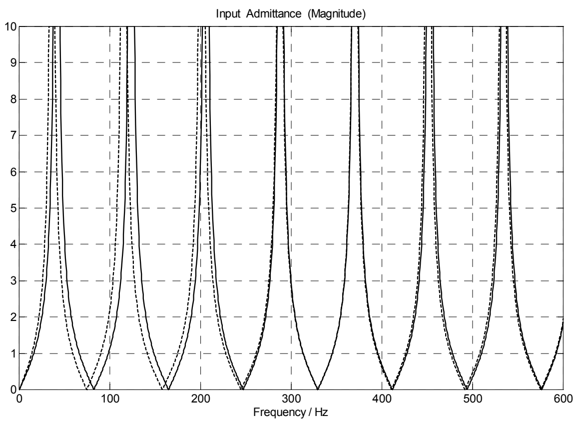


Fig. 4.50: Frequency dependences of the magnitudes. Upper left: Admittance magnitude of an E_2-string. Upper right: Impedance magnitude of the field stiffness (---), absolute value of the impedance of a 16 cm long part (—); both impedances are to be added.
Lower left: Admittance magnitude as in the upper left plot (—); Admittance magnitude including additional field stiffness (---).
Zero positions → frequencies of partial tones.

In addition to the calculations measurements of an E_2-string are shown in **Fig. 4.51**. A Fender E_2-string (3150, 1.1 mm diameter) was mounted in an Ovation solid-body guitar (EA-68, piezo-pickup) and the piezo-signal was analyzed. The magnetic forces were generated by an 18 mm long Alnico-5-Magnet (5 mm diameter) positioned relative to the string at a distance of 16 cm from the bridge.

Fig. 4.51 shows that a precise frequency analysis is problematic: the resulting detuning is only several Hertz, so that a frequency resolution smaller than 1 Hertz would be desirable. The string vibration, however, cannot be considered as stationary within the necessary time window (more than 1 s). The chosen DFT-windows represent a compromise between time and frequency resolution (analysis was done with the CORTEX-software *Viper*).

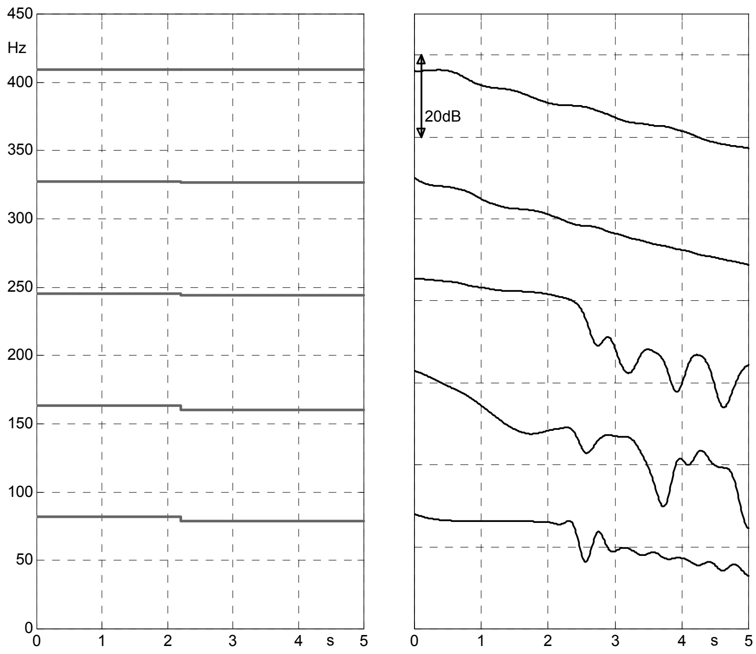


Fig. 4.51: Spectrogram (left) and partial tone level progress (right) of the vibration decay of an E_2 string. At 2.2s a magnet was approached to the vibrating string.
The frequencies of the first and second harmonic decrease from 2.2 s onwards . For the third harmonic one can mainly detect a level change, the fourth and the fifth harmonic remain unchanged (holds also for higher harmonics).

4.11.3 Field-Induced Amplitude Variations

The measurements and conduction model show, concordantly, that the permanent magnet will detune the lowest harmonics. The detuning will happen mainly for the fretboard-normal vibrations; field changes parallel to the fretboard and thus parallel to the magnetic pole surface only develop weakly. For the spatial vibration this means that there are two spatially orthogonal string vibrations with different frequencies which, after superposition, produce **beat frequency-like level changes**. If one denotes the fretboard-normal component with y and the fretboard-parallel component with x, one gets for the total amplitude ξ in vector-notation:

$$\vec{\xi} = \begin{pmatrix} \hat{x} \cdot \cos(\omega_1 t) \\ \hat{y} \cdot \cos(\omega_2 t + \varphi) \end{pmatrix}$$

$\hat{x}$ = Amplitude of the x-component
$\hat{y}$ = Amplitude of the y-component

For single frequency vibrations ($\omega_1 = \omega_2$) a point on the string moves according to the amplitude-relation $\hat{y}/\hat{x}$ and the phase shift φ along a line in an ellipse or a circle* (**Lissajous** figures). However, if both frequencies are not equal the figures above alternate with weak transitions. The time-dependent change of the curve is apparent when one transforms for small frequency differences:

$$\omega_2 t + \varphi = \omega_1 t + \Delta\omega t + \varphi = \omega_1 t + \varphi(t)$$

The x as well as the y-vibrations contain $\omega_1 t$, however, for the y-vibration an additional time-dependent (slow) phase-shift $\varphi(t)$ exists. A sensor that only detects the vibration exactly normal to the guitar body will, however, not be affected by the curve changes because $\hat{y}$ is time-invariant.

* Line and circle are special types of the ellipse.

Real sensors cannot be expected to exhibit such a perfect direction sensibility: common magnetic pickups are indeed the most sensitive for fretboard-normal vibrations; however, for fretboard-parallel vibrations the sensitivity will not be zero but approx. 1/10. The voltage which is generated is, thus, a combination of x and y vibrations which can, for the simplest case, be depicted as a **linear combination**:

$$u(t) = U\left(\cos(\omega_2 t + \varphi) + k \cdot \cos(\omega_1 t)\right) \qquad k = \text{relative } x\text{-ratio}$$

The commonly known formula for the beat frequency is obtained for k = 1 whereas for k << 1 the signal can be approximately regarded as a mixture of frequency and amplitude modulation. A cosine-like frequency modulation for a small modulation index can be represented, to a good approximation, by three spectral-lines [3]:

$$u_{FM} = U\left[\cos(\omega_2 t) - \frac{m}{2}\cos((\omega_2 + \Delta\omega)t) + \frac{m}{2}\cos((\omega_2 - \Delta\omega)t)\right]$$

If this FM signal should become amplitude modulated, the AM has to be applied to each of the three spectral components. By neglecting the $m^2/4$-terms (because m << 1), the lines at $\omega_2 + \Delta\omega$ compensate, while the lines at $\omega_2 - \Delta\omega$ add:

$$u = U\left[\cos(\omega_2 t) + m \cdot \cos((\omega_2 - \Delta\omega)t)\right] \qquad \omega_2 - \Delta\omega = \omega_1$$

This signal equates to the above mentioned linear combination for $\varphi = 0$; corresponding transformations are possible for other phase shifts. Hence it has been shown that for x and y vibrations with $k = 1$ a beat frequency, and for $k << 1$ a mixture of AM and FM, will result. This result can also be derived from the projection of the sum of two pointers with different frequency. If one assumes, for example, that the pickup for the y oscillations is eight times more sensitive than for the x oscillations ($k = 0.125$) then, for $\hat{y} = \hat{x}$, the amplitude of the pickup voltage changes by ±12.5%, or ± 1 dB. The modulation frequency corresponds to the difference frequency, which is the detuning caused by the magnet (e.g. 1 Hz). The amplitude relation $\hat{y} = \hat{x}$ means that the string vibrates at an angle of 45° with respect to the fretboard. The amplitude modulation effect will decrease for larger angles (normal to the fretboard) and for smaller angles (→ fretboard-parallel) it increases, until at arctan(1/8) = 7° a precise beat frequency is reached: The level change here is theoretically unlimited.

The linear combination is only a simple model for the description of time-variant level fluctuations. For the magnetic pickup the induced voltage depends on the **non-linear** relationships of the x and y velocities, which will result in additional sum and difference tones. However, as this will not result in completely different effects, we have dispensed with a precise investigation. An additional effect, which has also not been taken into consideration acts at both **string mountings** (bridge / saddle). Both mountings are idealized as rigid, but show a direction dependent compliance. As a consequence, the reflection factor has to be defined including all modes: a pure y vibration will also be reflected, to small extent, in the x direction and vice versa. For example, if the string is plucked exactly normal to the fretboard, after a certain time there will be fretboard parallel component which will yield amplitude variations in the pickup; the magnetic field can enhance or diminish them.

In addition, the (predominantly) fretboard normal **magnetic field** can induce a **rotation** of the vibrating plane when it is not exactly fretboard-normal or fretboard-parallel: for an inclined vibrating angle the string will experience a stronger pull at the turning point closer to the magnet than at the turning point further away. By reducing the magnetic force into coplanar and orthogonal parts one will get an angular force that tries to align the string (along fretboard-normal direction).

Finally, one has also to consider that the field stiffness is **non-linear**: the absolute value of the stiffness increases with decreasing distance. A simulation with a non-linear conduction model results in weak beat frequencies, even for exact fretboard-normal vibrations, whose fractional variation amplitude is dependent on the input signal amplitude.

In summary: Already without magnetic fields, sound level deviations are generated that develop differently for each partial tone. They originate from anisotropic mounting reflections, i.e. mounting impedances that depend on the oscillation direction and mode-coupling. The magnetic field detunes the fretboard-normal vibration component which might enhance or reduce the existing deviations. Non-linearities occurring in the mechanics and during electro-mechanic transformation will create additional sub-lines in the spectrum so that, in summary, a complicated level characteristic might develop for each partial tone.

Fig. 4.52 shows the selectively measured characteristic of the partial tones of the E_2-string. The recordings were performed with the built-in piezo pickup without a magnetic field. The differences between both pictures originate from the plucking technique as well as from the non-identical guitar positions and, possibly, slightly different guitar tuning and temperature. In the course of these first orienting investigations it became clear that the guitar should not be placed somehow on the thigh but must be supported in a defined way. Appropriate frame conditions are “in vivo” (guitar hanging from the guitar strap, fret-hand defined at the neck), and “in vitro” (guitar attached at the strap-pin, no damping at the neck).

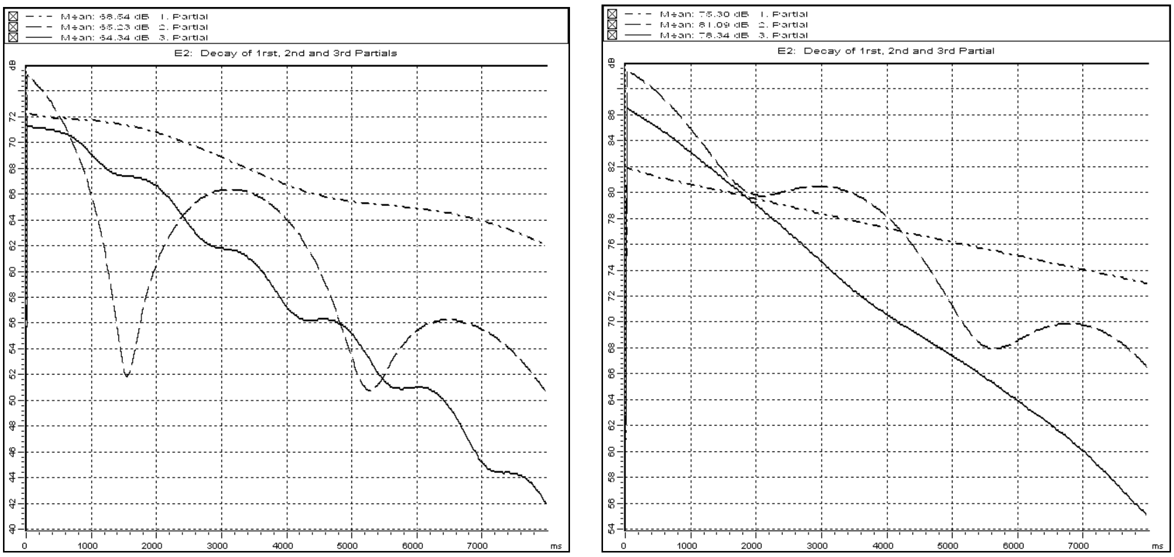


Fig. 4.52: Decay of the first three partial tone levels after plucking (left) or fretboard-normal excitation pulse (right) for an E_2-string with no magnet and an Ovation EA-68 piezo-pickup. The recordings were taken on different days.

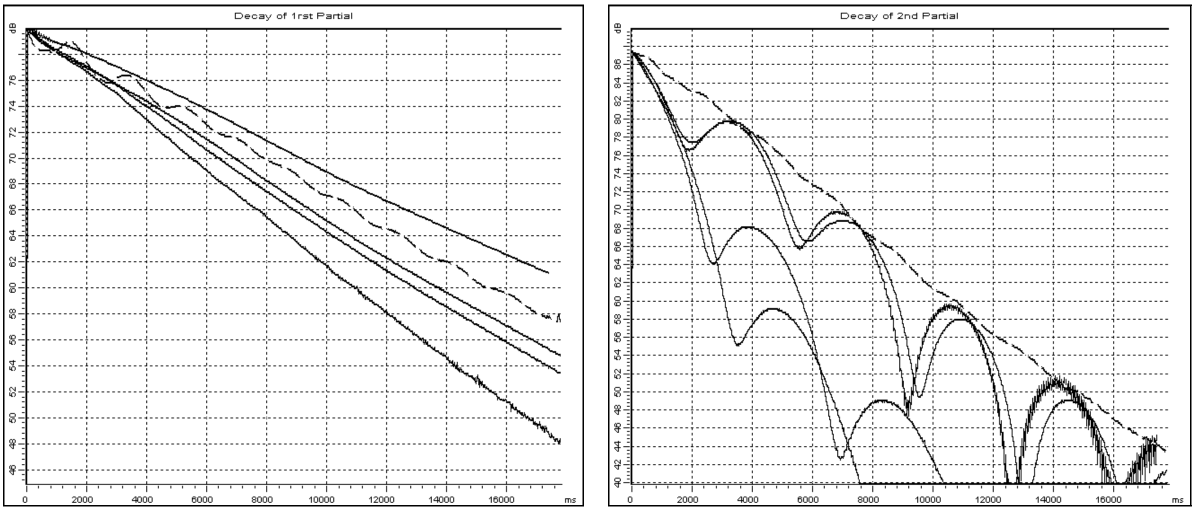


Fig. 4.53: Decay of the first two partial tone levels after fretboard-normal excitation pulse for an E_2-string, with an Ovation EA-68 piezo-pickup. Continuous lines: without magnet, dashed lines: Alnico-5-magnet in neck pickup position for a 2.5 mm distance between the string and magnet. Left: first partial, right: second partial.

Figure 4.53 shows the decay of the first two partial tones. The continuous lines were taken without a magnetic field. The upper curve, with the slowest decrease, shows the level decay of the undamped neck whereas the lower three continuous curves belong to measurements that were made with the fret-hand holding the neck in different ways without touching the strings. The dashed line was taken without neck-damping but with a magnetic field (Alnico-5-magnet placed 16 cm from the bridge). A strong influence of the fret-hand on the decay-characteristic (sustain) is observed in both measurements. The hand primarily acts as a damping resistance removing vibration energy. The level decays linearly with time for the first partial tone (left picture) without a magnetic field (exponential tension envelope curve), whereas a slight level oscillation occurs with a magnetic field. The second partial tone is completely different: There are intense level oscillations without a magnetic field, whereas there is a nearly oscillation-free decay with a magnetic field. **Fig. 4.54** shows similar results for fretboard-parallel excitations (both with magnetic field).

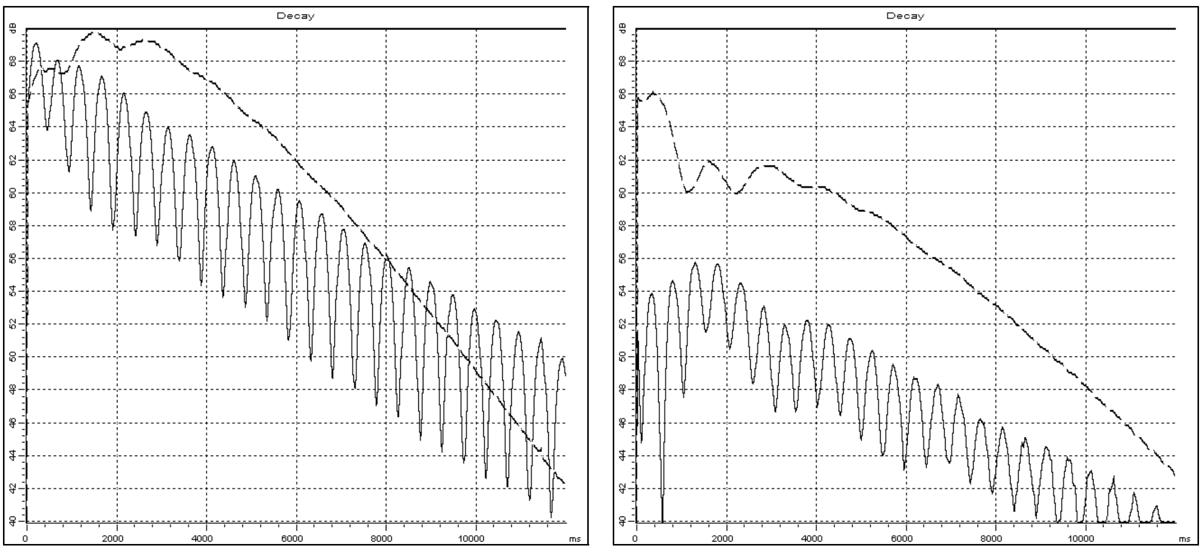


Fig. 4.54: Decay of the first (continuous) and third (dashed) partial tone after fretboard-parallel excitation using an Alnico-5-magnet in the neck pickup position. The only difference between both pictures is a slightly different plucking direction.

4.11.4 Field-Induced Damping

Pickup magnets are rumored to disturb the decay and to deteriorate the sustain of the string. Indeed, as shown in chapter 4.11.3, the magnetic stiffness can induce changes in the vibrational parameters; at small magnet distances these changes are also audible. However, an (ideal) spring is not able to extract energy from an oscillating system. If one pushes an ideal spring (with positive stiffness) it will store energy. However, after expansion this energy is returned entirely and without loss. In information technology one speaks of **reactive energy** in contrast to **active energy**, which is "lost" in a frictional resistance. The term "energy loss" can, of course, not just be viewed globally: in reality energy cannot be lost; however, it will be transformed irreversibly into thermal energy due to the frictional resistance and is no longer available for the oscillating system.

However, energetic considerations at a pickup are dangerous and may lead to the wrong conclusions: a pickup does not transform the vibration energy of a spring into electrical energy; rather it partakes from one component of the oscillation. Customary pickups mainly react to fretboard-normal vibrations. If a magnet would rotate the vibration plane of the string from fretboard-normal to fretboard-parallel this would not affect the vibration energy – nevertheless the pickup output voltage would decrease. Fortunately, this rotation occurs rather in the opposite direction (from fretboard-parallel to fretboard-normal); in this case the magnet will indeed increase the pickup output voltage, however, without increasing the vibrational energy.

At one place, however, real power is necessary: the voltage delivered by the pickup heats the ohmic resistors of the electrical circuit, and this real power has to be drawn from the vibration of the string, because the magnetic pickup is a passive transducer [3]. In addition, the so-called active pickups are passive with respect to their transformation process; in this case only the first amplification stage is located at a different place. The **ohmic resistors** in the electrical pickup load circuit are the volume potentiometer, the tone potentiometer, the amplifier input resistance and the coil resistance. The cut-off frequency of the 250 kΩ and 50 nF series connection (tone-pot) is 13 Hz, for higher frequencies the capacitor is approximately a short circuit. Both potentiometer resistances and the amplifier input resistance are in parallel for the standard circuit and, therefore, the result for the total resistance is 100 – 200 kΩ. Further, one has to add the coil resistance (4-15 kΩ). For the pickup/cable resonance one would have to consider a load transformation for the exact calculation, the following orienting calculation assumes 100 kΩ for simplicity. According to this calculation a pickup, that generates 100mV produces a real power of $P = U^2/R = 0.1$ µW. This is very small but must be viewed relative to the string energy.

The kinetic energy of a mass differential dm is $dmv^2/2$. Here, v is the velocity of the differential mass. The **kinetic energy of the string** will be highest at the transit through the rest position. Integration over the total length of the string (with sinusoidal length-dependent velocity) yields $W = mv^2/4$, with m = mass of the entire string and v = velocity at rest position.

A typical Stratocaster pickup will generate an effective voltage of U = v·0.186 V for a 0.66 mm solid string at a magnet-distance of 2 mm; the velocity v has to be inserted as an effective value in m/s. However, the velocity is not the one stemming from the energy-formula, rather is it the velocity of the string *above* the pickup. For an oscillation of the first partial the maximum of the velocity is located in the middle of the string (12th fret); above the neck-pickup v is only 0.69 times as big. In addition, one has to bear in mind that in the energy formula the amplitude of the velocity is depicted, whereas for the computation of the voltage the effective value of the velocity is necessary. This will yield for the mechanical energy W and for the electrical power P:

$$W_{mech} = \frac{1}{4} m \hat{v}^2 \qquad P_{el} = \frac{U^2}{R} = \frac{(0.186 \cdot 0.69 \cdot \tilde{v})^2}{100\,\mathrm{k}\Omega} \qquad \frac{P_{el}}{W_{mech}} = \frac{3.3 \cdot 10^{-7}}{m} \cdot \frac{\mathrm{kg}}{\mathrm{s}}$$

The power P is the quotient out of the energy loss dW and the duration dt (power is energy over time), the relative energy loss is thus $dW/W = Pdt/W$. Using 1.78 g for the mass of the string, the relative energy loss per second is 0.019 %. The time dependent damping value, the **decay-rate** D thus will be:

$$D = 10 \lg \frac{W}{W - \Delta W} \frac{\mathrm{dB}}{\mathrm{s}} = -10 \lg \left(1 - \Delta W / W\right) \frac{\mathrm{dB}}{\mathrm{s}} \approx \frac{10}{\ln 10} \cdot \frac{\Delta W}{W} \frac{\mathrm{dB}}{\mathrm{s}} = 4.34 \frac{\Delta W}{W} \frac{\mathrm{dB}}{\mathrm{s}} \quad ♣$$

Here, ΔW is the energy-loss over 1 s, which will be computed as $P \cdot 1$ s. With the above string one will get a decay rate of 0.0008 dB/s. This is the level decrease resulting from the electrical damping. Even if one assumes much more efficient pickups with e.g. ten times larger transformation coefficient, this effect is still minimal and can surely be neglected compared to other damping mechanisms.

This seems to result in very simple conditions: the magnetic field acts as a spring mainly on the lower partials and the electrical losses are negligible. However, it is not quite that simple. The problems are already present in the **measurement of the decay curves**. It is relatively simple to choose the appropriate DFT-windows that enable a sufficiently fast and selective measurement of single partials. For most of the measurements with the CORTEX-software *Viper* the 50-dB-Kaiser-Bessel-window with N = 4096 and zero padding = 2 turned out to be well suited. The decay lines of the partial tones, however, are often curved and, thus, hamper the modeling. In **Fig 4.55** the level trends for the E_2 string are depicted, taken without and with magnetic field. How can the decay (the sustain) be defined with one number? As a level change within the first second? Every time interval chosen appears to be arbitrary. The guitarist will not be fussed about the functional decay of the vibrational level, however, for basic research it will rather play an important role whether the level decay will be caused by dissipation or by exchange of vibrational energy. For the case of rapid beat frequencies (right picture) it seems to be relatively simple to derive a time-dependent envelope function from of the maxima. But if the beat frequency period lasts for ten seconds or longer, the measurement can become impossible: Until the next beat frequency maximum the oscillation may possibly have become too small due to other damping mechanisms. It is also not particularly practical to extract average values from a 30 second level decay because in music tones are seldom kept over this time period. OK, *A Day In The Life.* But that was one day. And not guitar but a piano!

♣ Approximation for $\Delta W <<« W$

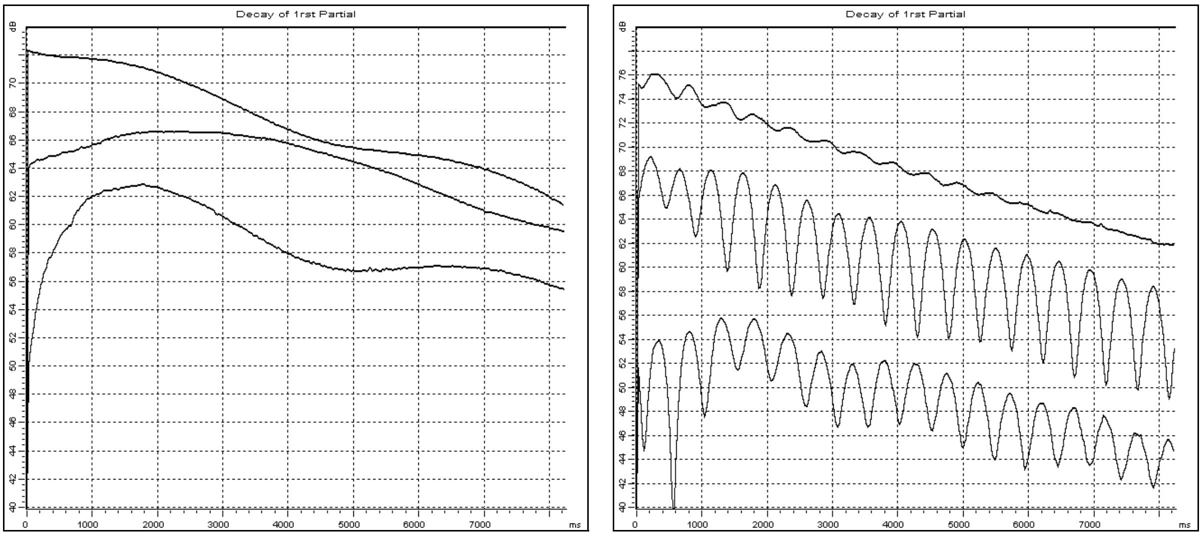


Fig. 4.55: Decay of the first partial of the E_2-string after different excitations; without magnetic field (left), with magnetic field (right).

Since it is discussed over and over again in guitarist circles, if and how the pickup-magnet may dampen the string vibration (and shorten the sustain), we will finally make an attempt at clarification. For this a guitar (Ovation EA-68) was suspended at the strap-button and the E2-string (Fender 3150) was reproducibly hit with a pendulum. An Alnico-5 magnet was attached at a variable distance to the string using a bridge placed over the last fret. The measured signal was generated by the piezo-pickup (**Fig 4.56**). Placing the magnet at 2.5 mm distance causes only a minor level reduction in the 1st harmonic, which hardly stands out. For lower magnet distances the level loss is considerable. At the 2nd harmonic there is an intense beat frequency without the magnetic field; a weak magnetic field (b) will increase the level, a strong magnetic field (c, d) will lead to a substantial level loss. Almost contrary is the 3rd harmonic: here a weak magnetic field (b) will yield intense beat frequencies. The differences in the higher harmonics are so small that they are of the same order as the reproducibility.

From these measurements it can be concluded that the magnetic field *changes* the decay of the partials; the term *dissipation* is conditionally justified only for the first two harmonics – the magnetic field is indeed extracting energy from them in a considerable amount. However, one has to take into account that, in practice, the neck-pickup-magnet is never brought as close as to a distance of 1 mm to the string: the string would otherwise impinge on the magnet. The **small distances** were chosen for the measurements in order to generate a distinct effect. Dissipation effects are only clearly visible for this atypical situation (Fig. 4.56 upper left, curve c and d). During the first seconds the level of the first partial decays much faster than later on. The cause for the higher fluctuation frequency of 4 Hz, as compared to curve b, is the higher negative field stiffness, which will lead to larger detuning. The time-dependent slope of the envelope curve has to be attributed to a non-linear dissipation effect or an amplitude-dependent damping. This is probably due to **hysteresis losses** in the string. As the magnetic field strength in front of the magnetic pole is very inhomogeneous (location-dependent), the field strength and flux density will change within the string during decay. The respective reorientation events within the microstructure are partially irreversible.

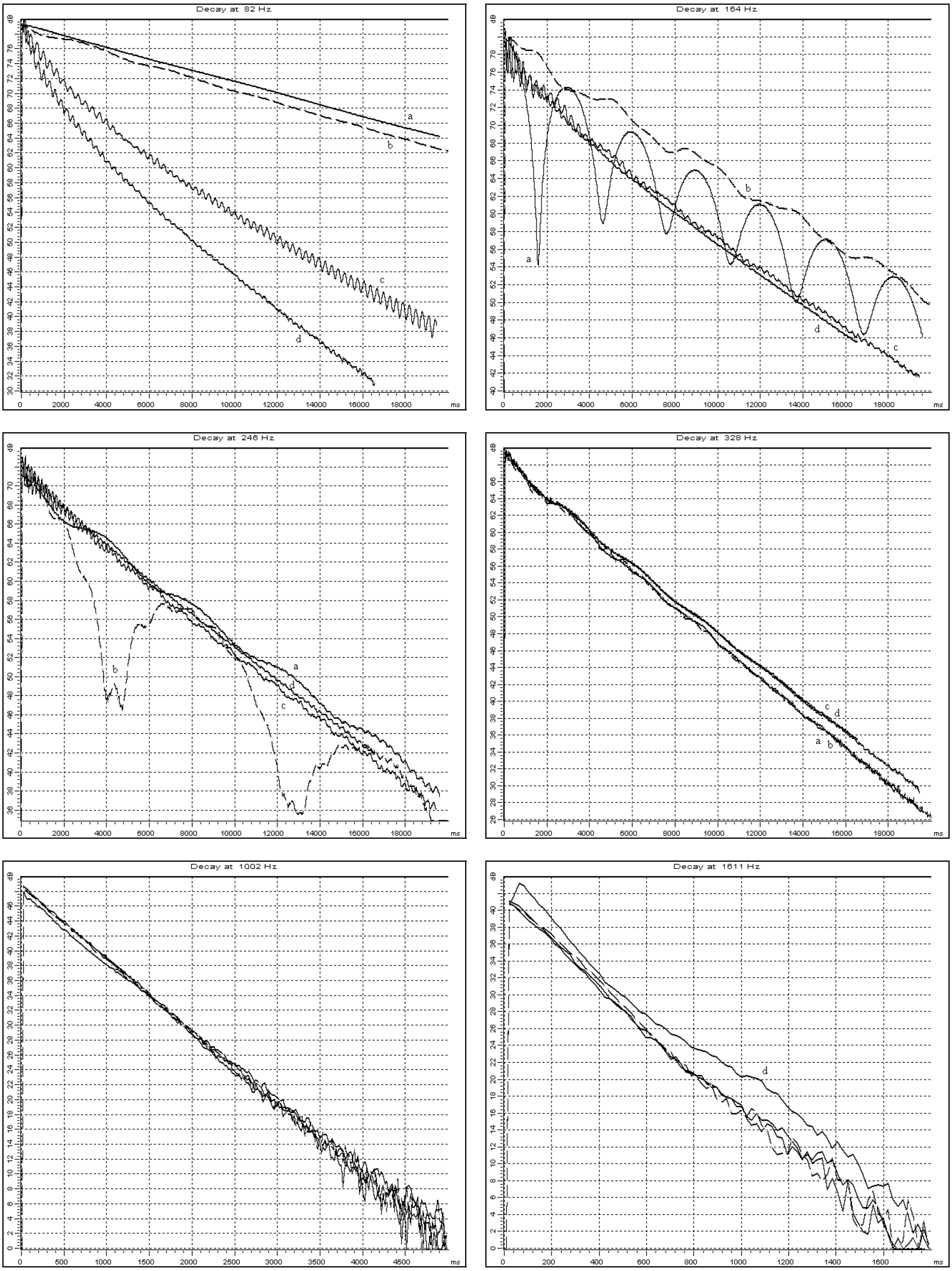
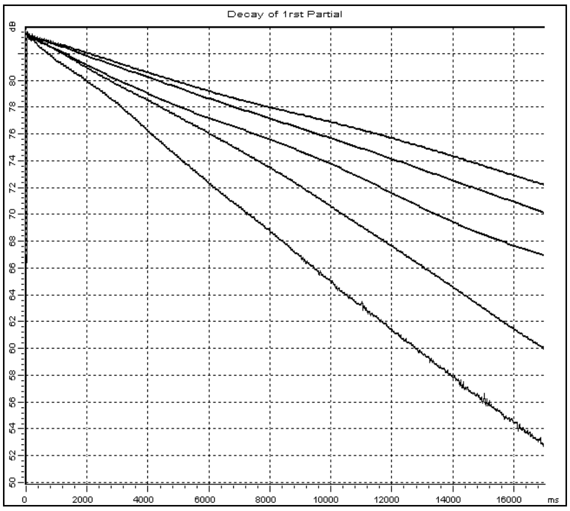


Fig. 4.56: Decay of the partial tone levels of the E_2-string after consistently comparable fretboard-normal excitations: Without a magnetic field (a), a magnet distance of 2.5 mm (b, dashed), a magnet distance of 1 mm (c) and a magnet distance of 0.8 mm (d). The higher d-level trend at 1611 Hz has to be attributed to a slightly different excitation, which shows up only at high frequencies. The results are typical for the guitar under investigation, its specific mounting and excitation; they should not be generalized for other guitars.

The hysteresis losses are proportional to the frequency, in the first approximation. With every cycle of the *BH*-hysteresis loop the magnetic field will lose an amount of energy ΔW; the higher the frequency the higher the number of cycles per second and the higher the dissipation losses. For the string, however, one has to consider that higher frequency partial tones are damped more strongly by other mechanisms and that the strength of the magnetic flux change depends on the displacement. However, the displacement decreases for higher frequencies. The lower pictures in Fig. 4.56 clearly show that the magnetic field does not have any effect in the high frequency range. In addition, for low frequency partials, one should not overestimate the field-induced dissipations. Finally, for comparison, the influence of the **fretting hand** on the decay of the partials is shown (**Fig. 4.57**, left picture). The upper curve shows a measurement in which the guitar was suspended from a steel wire at the strap button, whereas for both of the lower curves the guitar was clamped at the strap button. For the remaining measurements the fret hand surrounded the neck with different tightness but without touching the strings. All measurements were done without magnetic fields. One recognizes that even without magnetic field a variable dissipation is generated – the **heel of the hand** touching the neck has to be interpreted as damping resistance. Its energetic (!) influence on the sustain is considerably larger than that of a common pickup-magnetic-field (right picture).



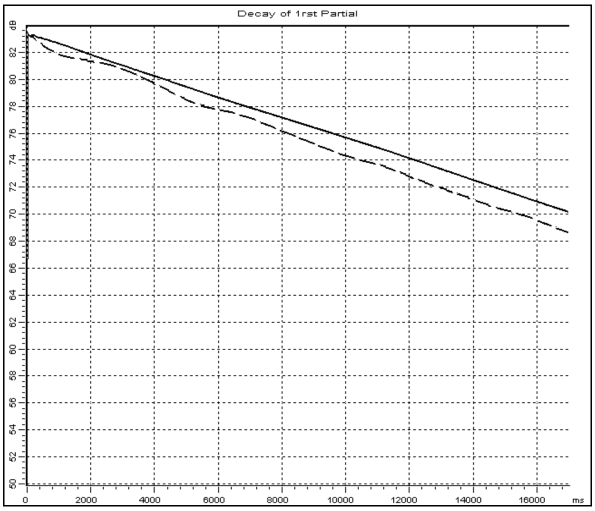


Fig. 4.57: Decay of the first partial tone for different manners of hand-damping (left). On the right, with identical scaling, the influence of an Alnico-5-magnet attached at a distance of 2.5 mm is depicted (neck-position).

4.11.5 Indirect Effects on Sound

In professional music magazines magnet-characteristics are often published without physical rationale. It is to be feared that the following citations are pure speculations resulting from findings after the replacement of an *entire* pickup. In addition, one can only hope that the author also did not replace the strings (… the new pickup delivers much more treble …). For an old Stratocaster pickup, for example, it is impossible to *solely* change the magnets; the coil rests directly on the magnets and as soon as one pulls them out one destroys the flimsy coil-wire. If, however, the whole pickup is replaced by another, the number of turns may change – and, consequently, it would be incorrect to attribute changes in sound only to the magnet.

In literature very different characteristics are attributed to the magnet material, as can be seen by the following collection of citations:

a) “For a pickup with the rather weak Alnico-2 magnet the tone seems to virtually die out after hard plucking. The output signal is not only more quiet but also seems to be less dynamic and perceptibly compressed in the treble range – which is actually appreciated by many guitarists.”
b) “As the magnetic field of an Alnico-II magnet is somewhat weaker than that of a common Strat-pickup (Alnico-V), the string vibration decays more open and more natural. The result is an improvement in sustain.”
c) “Alnico-5: Strong and clear sound.”
d) “Alnico-5: Fast responsiveness and slightly undifferentiated reproduction.”
e) “The stronger the magnet, the more treble.”
f) “As time goes on, older magnets lose some of their power. The less power the magnets have, the better the strings can vibrate. So maybe after 30 years, the magnets are at their 'ideal' power, thus producing 'ideal' tone.” ☺

One might add: “If someone has some Les-Pauls lying around that are older than 30 years – throw them away! Especially for the 50’s Les-Pauls the magnets are completely shot, all power lost, get rid of them. The author will accept these guitars for research sake, at a small waste disposal charge.”

Still, back to the physics. The pickup-magnet is part of a mechanic-electric transducer and as such it influences both the mechanical as well as the electrical partial system. Mechanically the magnet retroacts on the vibrational characteristics of the string; the result can be chorus-like beat frequencies and – to a minor extent – dissipation. The **electrical effect** of the magnet does not really belong in chapter 4.11 because the forces or mechanical effects are described there. The following listing is, therefore, only a precis: The reversible permeability of the magnet influences the inductance of the pickup coil and, consequently, the **resonance frequency**. If the resonance frequency is shifted, partials with different decays may influence the sound and the perceived “sustain”; however, this should not be mixed up with a more openly vibrating string – changes in the cable capacity would have a similar effect. Eddy currents within the magnet influence the **resonance quality factor** (Alnico conducts, ferrite is an insulator). Stronger magnets may increase the **output voltage** of the pickup and overdrive the amplifier in a different way; this may also change the sound and perceived sustain – as well as by changing the input gain. A replacement of the magnets may also change the **aperture** because the spatial flux distribution may change as a function of the (non-linear) string saturation and because the anisotropy of the new magnets may be different from that of the old ones.

The magnetic material can, thus, indeed influence the (“electrical”) sound of the guitar. A hindrance to the free string vibration, however, is not to be expected if the string/magnet-distance is chosen properly.

4.11.6 Coulomb-Force

An electric field with the field strength $E = U/d$ is generated between two electrodes at different potentials. Here U is the potential difference, also depicted as a voltage (or voltage difference, voltage drop) and d is the distance of the electrodes. 100 V at a distance of 10 mm yields E = 10 kV/m. If one inserts an electrical charge q into this electrical field, an electrostatic force F is generated which is called Coulomb-force, after its discoverer (Charles Augustin de Coulomb, 1736 – 1806).

$$\vec{F} = q \cdot \vec{E}$$ Coulomb-force

The coulomb-force does not play any role for guitar pickups; it may lead, however, to misinterpretations: other than for magnetic forces the coulomb-force also "acts" within the homogenous field. While a positively charged Styrofoam ball between two parallel electrodes is drawn to the cathode (negative electrode) an iron ball between two parallel poles of a permanent magnet will rest (more precise: 4.11.1). Indeed, within the magnetic field there are also attractive forces but they are balanced in this idealized example. The Coulomb-force is only mentioned here to point out its differentness. Analogy-considerations between electric and magnetic fields have model limits that have to be observed.

4.11.7 Lorentz-Force

With the Lorentz-force (Hendrik Antoon Lorentz, 1853 – 1928) we will explain another force that has no direct importance for the magnetic pickup (but indeed for the dynamic loudspeaker). Again, we want to eliminate misinterpretations. A force F acts on a conductor of length l carrying a current I when the conductor is carried into a magnetic field with flux density B. F is oriented normal to the plane defined by I and B. If I is directed parallel to B then $F = 0$. In vector notation one will get the vector product $(\times)$:

$$\vec{F} = l \cdot \vec{I} \times \vec{B}$$ Lorentz-force

If one points with the thumb of the right hand into the direction of the technical current flow (from plus to minus) and with the forefinger into the direction of the magnetic flux, the middle finger will point into the direction of the force (right-hand rule). The value of the force is given by the product $l \cdot I \cdot B \cdot \sin\alpha$, where α is the angle between the current and field directions. For the magnetic pickup the Lorentz-force, as given in the above form, does not play any role. A small alternating current does in fact flow through the coil which, however, with a value of 10 µA, will not exert any substantial force on it. A retroactive effect on the vibrating string is described by the Maxwell and not by the Lorentz-force, because the string is not carrying a current. If the string would be conductively suspended one could hypothesize an induced current in the neighboring string – however, the effect of its force would be negligible.

4.12 Magnetic Quantities and Units

The literature on magnetic fields refers to two different unit-systems: The MKSA-system as proposed by Giorgi and the CGSA-System.

The **MKSA-system** emanates from the four basic units *Meter, Kilogram, Second* and *Ampere* (SI-units, Système International). All other units are derived from them and occasionally linked with the names of outstanding scientists:

1 N	= 1 Newton = 1 kg m / s^2	1 J	= 1 Joule = 1 N m
1 W	= 1 Watt = 1 N m / s = 1 VA	1 Wb	= 1 Weber = 1 V s
1 T	= 1 Tesla = 1 Wb / m^2	1 V	= 1 Volt = 1 m^2 kg /(A s^3)

The **CGSA-system** uses the four basic units *Centimeter*, *Gram*, *Second* and *Ampere* and derives further units from them:

1 dyn	= 1 g cm / s^2	1 erg	= 1 dyn cm
1 Gb	= 1 Gilbert = 1 Oe cm	1 Oe	= 1 Oerstedt = 1 Gb / cm
1 Mx	= 1 Maxwell = 1 G cm^2	1 G	= 1 Gauß = 1 Mx / cm^2

The following table enables the conversion between both systems:

B	Flux Density Induction	T = V s / m^2	1 G = 10^{-4} T
H	Magn. Field Strength	A / m	1 Oe = 1000 / 4π · A/m = 79.577 A/m
BH	Specific Energy	W s / m^3	1 MGOe = 7.9577 kJ/m^3
Φ	Magn. Flux	Wb = V s	1 Mx = 10^{-8} V s
Θ	Amperes	A	1 Gb = 10 A / 4π = 0.79577 A
F	Force	N = kg m / s^2	1 dyn = 10^{-5} N
P	Power	W = VA = N m / s	1 erg / s = 10^{-7} W
E	Energy	J = N m = W s	1 erg = 10^{-7} J
R_m	Magn. Resistance Reluctance	1 / H = A / (V s)	1 Gb/Mx = 7.9577·10^7 1/H
Λ	Magn. Conductivity Permeance	H = Henry = V s / A Instead of H also Hy for Henry	1 Mx/Gb = 1.2566·10^{-8} H
μ_0	Abs. Permeability of the Vacuum	= 4π·10^{-7} H / m	= 1 G / Oe

4π = 12.566; 10 / 4π = 0.79577.