4.10.4 Magnetic Losses, magnetic skin-effect

The field enhancing effect of ferromagnetic materials is caused, on the one side by the internal shift of domain walls (Bloch walls, chapter 4.4.1), and on the other side by initially randomly oriented elemental magnets that are turned into a common direction by the external field. A small part of the energy that is necessary for shifting and/or rotating is irreversibly transferred into heat. The thermal energy that is produced by a kind of micro-friction is "lost" from the electromagnetic field and this is the reason why one talks about **loss of electromagnetic field energy**, or in short about magnetic losses; the designation **iron losses** is also common. Losses will decrease the voltage generated in the pickup – an effect which may mainly affect higher frequencies as brilliance loss.

The two most important loss mechanisms are eddy current losses and hysteresis losses. The field-energy per volume w can be derived from the relationship between the magnetic field strength H and magnetic flux density B, as given by the hysteresis curve:

$$w = \int_{B_1}^{B_2} H dB$$ Volume-specific magnetic field energy

If the hysteresis curve is a (curved) line, the magnetic energy would be increased by elevating the flux density from B_1 to B_2 and likewise would be diminished by the same value by decreasing from B_2 to B_1 – the process would be reversible. However, as each hysteresis loop consists of two different branches, a complete circuit leading back to the origin does not yield $w = 0$ but an energy density which is proportional to the enclosed area and which represents a measure for the energy loss. For guitar strings, the specific **energy loss** for a *boundary loop* circuit is about 10 µWs/mm³. If one multiplies this value with a 2 cm long 0.7 mm string one ends up with an energy loss of approximately 77 µWs for the total hysteresis circuit. However, the working point of a vibrating string does not follow the boundary hysteresis (from negative saturation to positive saturation and back) but only a small fraction of it. The fraction of it heavily depends on the distance of the string to the magnet and on the amplitude of the string deflection. The steady flux is also high in the regions of high alternating flux and – conservatively estimated – the alternating flux may reach about one tenth of the steady flux. In addition, if one considers that the small signal changes yield relatively small areas, lancet-shaped hysteresis loops (also called **Rayleigh-Loops**), it becomes clear, that the energy losses caused by the string are of only marginal significance. As an order of magnitude one can estimate 10 mWs for the string energy and 1 µWs for the iron losses per cycle. If the string vibrates with 150 Hz with this assumption it will lose 1.5% of its vibration energy, which would be negligible. A more precise computation of the iron losses would be laborious, because one has to deal with a three-dimensional inhomogeneous field, for which material tensor parameters would have to be known. In addition, measurements are difficult because one has to discriminate from other damping mechanisms. But, even for the case that the above approximation would be unrealistic and the string energy loss per second would be 26% instead of 1.5% this would be equal to a level decrease of 1 dB/s – insignificant against other damping mechanisms. The bottom line of these approximations is, therefore, (without proof): the **hysteresis losses** (magnetization-change losses) **emerging within a string are negligible**.

Other than in the string, hysteresis losses are also possible in the magnet or in nearby ferromagnetics. Although these losses are not generated within the string, nevertheless the energy necessary for changing the magnetization of these ferromagnetics has to be delivered by the vibrating string. The magnetic volume affected by relevant alternating fluxes for single coils is larger, by more than one order of magnitude, compared to the above considered volume of the string. However, the relative change in flux density inside the magnet is also one order of magnitude smaller than inside the string so, on the whole, again an effect of marginal importance. As long as one does not move a very strong magnet close to the string (which is in contradiction to a large string displacement) **the conclusion is: hysteresis losses are negligible**. This statement is indeed speculative but is supported by measurements which yield, without doubt, that string vibrations are damped more intensely by the fretting hand of the guitarist than by the magnetic field of the pickup (chapter 4.11).

A second source for losses are **eddy current losses**. The induction law discussed in chapter 4.10.1 generates a voltage and a current in the pickup coil but also in every conductor that is in close vicinity to the pickup. As metals represent electrical resistances, an effective electrical energy or thermal energy is produced which weakens the magnetic field or the string vibration. The electrical voltage that causes the eddy current is dependent on the *change* of the magnetic field and, therefore, eddy currents do not play any role at low frequencies. With increasing frequency they become more and more important, however, the **skin effect** has also to be taken into account as reverse effect (chapter 5.9.2.2): the magnetic counter field induced by the current flow forces the current more and more into the boundary regions and consequently increases the eddy current resistance (chapter 3.3.2)

Eddy current losses cannot generally be neglected, but indeed deteriorate the treble reproduction of every pickup; not only marginal but possibly by 5 dB and more, if thick low-ohmic metal plates are employed. One could interpret this fact as a sound characteristic which is deliberately chosen by the developer, but one should take into account that very dominant treble can be reduced easily by a potentiometer in parallel to the pickup, which is not possible the other way round. A pickup with less eddy currents may sound brilliant as well as dull; a pickup damped by eddy currents may only sound dull[♣]. Pickups that exhibit small eddy current losses are the ones with 6 Alnico magnets as sole metal pieces (USA-type Stratocaster). Soft-magnet pole-pieces with underlying bar magnets increase the eddy currents, as do tin covers. If one wishes to have a shielding case with small eddy current losses, thin-walled German silver cases are recommended. One pickup that sounds brilliant, despite having a metal case, is the Gretsch humbucker.

Eddy currents are not only present in magnets, pole-pieces and shielding cases but are also possibly in metal support plates and shielding foils. When replacing a plastic by an aluminum pick-guard one experiences a small treble loss. However, the loss can mostly be avoided by a small slit, which suppresses the circling eddy currents.

[♣] In general, sound filters built into guitar amplifiers cannot compensate for eddy current losses.

In order to obtain quantitative data for **eddy current losses**, a thin walled measuring coil was fabricated, into which cylinder-shaped ferromagnetics (∅ = 5mm) could be inserted. The 14 mm wide coil form was wound with 5500 turns of an 80-µ-CuL enameled copper wire (**Fig. 4.42**). In this representation the logarithmic impedance unit is depicted over the logarithmic frequency – unusual but convenient. 0 dB = 1 kΩ.

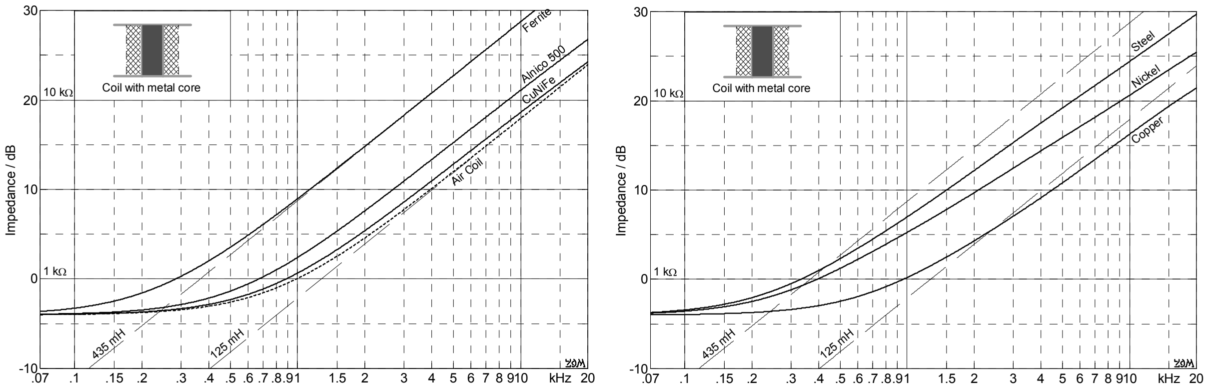


Fig. 4.42: Logarithmic impedance of a measuring coil ($N = 5500$); different core materials.

The wire resistance (630 Ω) is measured without a core ("air coil") at low frequencies and at high frequencies the impedance increase proportional to the frequency, the inductance (125 mH). The two terminal device is, thus, perfectly described by an *RL*-series connection in this frequency range. Insertion of an **Alnico**-500-magnet (5x14 mm) increases the inductance by 46%, insertion of a respective **ferrite** cylinder increases the inductance by a factor of 3.5. In both cases a frequency-proportional inductance increase happens at high frequencies, so that only one resistance is necessary in the equivalent circuit: the **wire resistance**♥. The inductance increase, however, does not mean that the relative permittivity of ferrite is only 3.5 (or for Alnico is only 1.46). These materials cover only a part of the field space, their effectiveness is, thus, substantially diminished. As an analogy one might think of two resistors in series, e.g. 1000 Ω and 10 Ω. The total resistance in this example is 1010 Ω. It decreases to 1001 Ω if the second resistor has only 1 Ω. At a 1V-source a current of approx. 1 mA will flow even if one will decrease the second resistor even further. This is similar for the magnetic circuit: the magnetomotive force is dominated by the low-conductive air field. With a little peculiarity: A change in the magnetic resistance of the core will also affect the shape of the field lines and, thus, the resistance of the air field.

The reason, why the impedance of Alnico and ferrite can be represented by an ordinary *RL*-two terminal device, is quite simple: in addition to the wire resistance no additional loss resistance has to be taken into account: eddy currents do not yet play a role♣. **Ferrites** are sintered out of oxide powder; they have a high electric resistance that prevents eddy currents. **Alnico**-alloys are, in comparison to ferrites, already quite good conductors. The fact, that they exhibit nearly no eddy current losses in the relevant frequency range, arises from their relatively small permeability (2 – 5). Good conductors with high permeability should, thus, produce enormous eddy current losses – and this is what they do, to be confirmed by the following measurements. To achieve this, we have inserted cylindrical cores made of different materials into the above mentioned coil: steel, nickel, copper (**Fig. 4.42** right)

♥ The denomination *copper-resistance* is disadvantageous here, because copper is also used for the coil core.
♣ The (nonlinear) remagnetization losses are also insignificant.

Copper is diamagnetic, its permeability differs only marginal from μ_0. **Steel** and **nickel** are ferromagnetic, their permeability is considerably higher than μ_0. Copper is a very good electrical conductor, Nickel has higher resistivity by a factor of 4, steel by a factor of 10 – 20. As can be seen clearly by the measured curves (Fig. 4.42, right) the high-frequency impedance increase with these metal cores is shallower than in air or in ferrites. The origin of this behavior is the **eddy currents**, which increasingly force the field lines out of the core with increasing frequency and, thus, decrease the inductance. **Fig.4.43** shows the internal field distribution for a steel cylinder (∅ = 5 mm, length = 14 mm) for three frequencies as well as the frequency-dependence of the magnetic impedance, which, as complex unit, consists of a real part (magnetic resistance) and an imaginary part (eddy current losses). The losses have to be imaginary because the magnetic resistance is generally defined to be real – different from *electrical* networks, where loss-resistances usually are defined to be real. However, these are only conventions, finally only orthogonality between effective and reactive power is necessary.

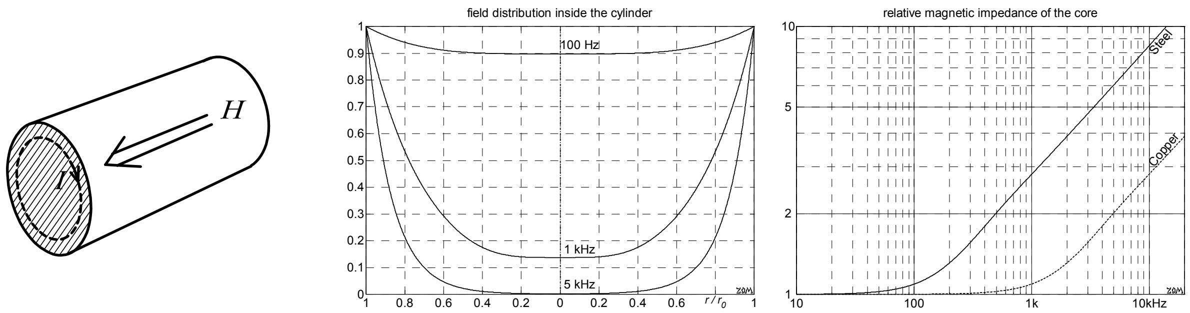


Fig. 4.43: An axial magnetic field *H* growing with time generates the eddy current *I* in the metal cylinder. This current will produce a circular magnetic field around itself, which is in opposite direction to the generating field and forces it out of the cylinder. The picture in the middle shows the radial distribution of the axial magnetic field in a steel cylinder (r_0=5mm), on the right the magnetic impedance normalized to low frequencies is depicted.

The basis of the calculations is **Maxwell'**s Laws in their differential form under the simplifying assumption that the electrical conductivity σ and the permeability μ are constant. For the conductivity this assumption is true, for the permeability actually not: the space and time-dependent flux-distribution leads to a space and time-dependent μ. The exact calculation in an anisotropic non-linear medium is, however, so complicated that simplification is necessary. Both Maxwell Laws now read as:

$$rot\,\vec{H} = \sigma \cdot \vec{E} \quad \text{and} \quad rot\,\vec{E} = -\mu \cdot \partial \vec{H} / \partial t$$ Differential form of Maxwell's Law

In cylinder coordinates *H* only exists in the axial direction, the field strength *E* exists only in the circular (azimuthal) direction and the rotation *rot* can, therefore, be simplified to:

$$\sigma \cdot E = -\partial H / \partial r \quad \text{and} \quad -\mu \cdot \partial H / \partial t = E / r + \partial E / \partial r$$ In cylindrical coordinates

Both formulas combined will yield **Bessel'**s Differential Equation, which can be solved for harmonic signals with complex units:

$$\frac{\partial^2 H}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial H}{\partial r} = \mu\sigma \cdot \frac{\partial H}{\partial t} \quad \rightarrow \quad \frac{\partial^2 \underline{H}}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \underline{H}}{\partial r} = j\omega\mu\sigma \cdot \underline{H}$$ Bessels Diff. Equation

The time differential operator $\partial/\partial t$ has been replaced by $j\omega$ (see system theory).

The **solution of Bessel's Differential Equation** for the radial distribution of the axially directed magnetic flux density $\underline{B}(r) = \mu\underline{H}(r)$ is:

$$\underline{B}(r) = \mu c \cdot J_0(kr) \qquad \text{with} \qquad k = (1-j)\cdot\sqrt{\pi f \mu\sigma} \qquad \text{or} \qquad k^2 = -j\omega\sigma\mu$$

Here c is an integration constant and J_0 is zero order Bessel's function of the first degree. The total magnetic flux that axially passes through the cylinder is given by the area integral over the cross section with r_0 = cylinder radius:

$$\underline{\Phi} = \int_0^{r_0} \underline{B}\cdot 2\pi r\cdot dr = 2\pi\mu c\cdot\int_0^{r_0} r\cdot J_0(kr)\cdot dr = \frac{2\pi\mu c}{k^2}\cdot\int_0^{kr_0} kr\cdot J_0(kr)\cdot dkr \qquad \text{Total flux}$$

The integration of Bessel's Function is carried out with $\int x\cdot J_0(x)\cdot dx = x\cdot J_1(x) + C$, where J_1 is a first order Bessel's Function of first degree. For the magnetic flux this yields:

$$\underline{\Phi} = \frac{2\pi c}{-j\omega\sigma}\left[kr\cdot J_1(kr)\right]_0^{kr_0} = j\frac{2\pi c k r_0}{\omega\sigma}\cdot J_1(kr_0) \qquad \text{Total flux}$$

The magnetic resistance is defined as quotient out of magnetomotive force and flux, the **length-specific** magnetic resistance R'_m is the quotient out of field strength and flux:

$$R_m = V_m/\Phi; \qquad R'_m = R_m/l = H/\Phi; \qquad \text{Magnetic resistance}$$

The length-specific magnetic resistance is calculated by dividing the field strength $H(r_0)$ by the flux Φ along the cylinder barrel; the result is complex and is, therefore, called the **length-specific magnetic impedance**:

$$\underline{Z}'_m = \underline{H}(r_0)/\underline{\Phi} = \frac{-j\omega c\sigma}{2\pi c k r_0}\cdot\frac{J_0(kr_0)}{J_1(kr_0)} = \frac{k}{2\pi r_0\mu}\cdot\frac{J_0(kr_0)}{J_1(kr_0)} \qquad \text{Length-specific impedance}$$

For very low frequencies k tends to zero and, using a series expansion of Bessel's Function, one will get as a (real) limit value $\underline{Z}'_m \to 1/r_0^2\pi\mu$, or the inverse of the cylinder cross-section and the magnetic conductivity. This means, that for low frequencies, there is no field displacement at all, the flux density is independent of position for the entire cross-section. However, with increasing frequency the magnetic flux is forced from the center to the boundary area (casing vicinity), the magnetic resistance (impedance) increases and the cylinder will become 'less magnetic' (see also chapter 5.9.2.4).

The field displacement calculated by Bessel's functions can qualitatively explain the impedance/frequency relation shown in Fig. 4.42. However, precise quantitative data are not possible because the (possibly tensor) magnet data are not known precisely and the metal cylinder is not percolated exactly axially. In contrast to the pickup calculations for metal cylinders, a finite elements (FEM) computation would be possible but, also for this case, the problem of insufficient material data remains.

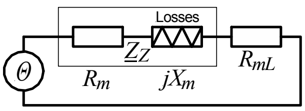
The (magnetic) field lines of the coil shown in Fig. 4.42 run partly in metal and partly in air. As already shown in chapter 4.6, one may visualize these magnetic networks by block diagrams, in **analogy** to electrical networks, in which resistors are displayed by rectangles. For the depiction of magnetic *loss* resistors there are no commonly defined symbols; in the following they are represented by rectangles enclosing a zigzag line (**Fig. 4.44**). The **magnetic impedance** Z_m (the inverse of which is the magnetic admittance Y_m) consists of real and imaginary parts: $\underline{Z}_m = R_m + jX_m$. It has to be kept in mind that the depicted loss resistors are imaginary – different from an electrical network. In order to project the networks onto one another using this analogy one has to define the flux quantity and the potential (difference) quantity [3]. The **flux quantity**♣ in electrical networks is the current, in magnetic networks it is the magnetic flux. The **potential quantity** is the electric voltage and the magnetomotive force, respectively. The flux quantity divides at nodes, where Kirchhoff's first law (or Maxwell I, respectively) is valid; in analogy, Kirchhoff's second law is valid for the potential quantity (or Maxwell II. respectively). Analogies that project flux quantities onto flux quantities create an **isomorphic** (equal structure) network; the projection of a flux quantity onto a potential quantity generates a **dual** network. Which is valid for **electromagnetic analogies**? Using the transformation mechanisms predominant for pickups (chapter 5) as orientation, one may find a projection of the magnetic flux to the voltage and of the current to the magnetic field strength – or **duality**. Written as equations:

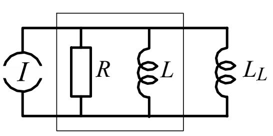
$$U = N \cdot d\Phi / dt \qquad \text{and} \qquad N \cdot I = \oint H \cdot ds \qquad \text{Electromagnetic transfer formulas [3]}$$

The first formula represents the law of induction, the second the law of magnetomotive force (Ampere's law). Consequently, a magnetic series circuit will become a parallel circuit in the electrical block diagram. The differential occurring in the law of induction will be replaced by a multiplication with $j\omega$ for complex (sinusoidal) signals, which yields:

$$\underline{\boldsymbol{Z}}_{\mathbf{el}} = \frac{\underline{U}}{\underline{I}} = \frac{j\omega \cdot N \cdot \Phi}{\Theta / N} = j\omega \cdot N^2 \cdot \underline{\boldsymbol{Y}}_{\mathbf{m}} \qquad \Theta = \oint H \cdot ds = \text{Magnetic flux}$$

The magnetic and the electric impedance are thus reciprocal: The higher the permeability, the lower is the magnetic impedance and the higher the electric impedance. A real magnetic resistor will be projected into an imaginary electrical resistor (inductance, $\underline{Z} = j\omega L$), an imaginary magnetic (loss-) resistor will be projected into a real electrical resistor. The series connection of the magnetic real and imaginary part of the impedance $R_m + jX_m$ will become the parallel connection of the electrical resistance R and the inductance L; <u>both are frequency-dependent</u>. The magnetic in-series connection of the air resistance R_{mL} will become the parallel lying inductor L_L.





$$\underline{Y}_{el} = \underline{Z}_m / (j\omega N^2)$$
$$R = \omega N^2 / X_m$$
$$L = N^2 / R_m \; ; \quad L_L = N^2 / R_{mL}$$

Fig. 4.44: Dual analogy between the magnetic (left) and the electric network (middle). $\underline{Z}_Z$ = magnetic (metal-) cylinder-impedance, R_{mL} = magnetic air resistor.

♣ For the electromechanical FI-analogy [3] the electrical flux quantity "current" will be projected to the mechanical flux quantity "force" in equal structure; the FU-analogy projects dually.

The magnetic cylinder impedance $\underline{Z}_Z$ for a **metal core** with length l and radius r_0 located in the center of the coil is:

$$\underline{Z}_Z = \frac{k \cdot l}{2\pi r_0 \cdot \mu} \cdot \frac{J_0(kr_0)}{J_1(kr_0)} \qquad \text{with} \qquad k = (1-j)\cdot\sqrt{\pi\mu\sigma \cdot f} \qquad \text{Magnetic cylinder impedance}$$

Here, μ is the (absolute) permeability of the core and σ is the electrical conductivity. Both the argument (kr_0) as well as the resulting Bessel function are complex. **Fig. 4.45** depicts the frequency dependence of the real and the imaginary parts of the magnetic cylinder impedance. If the metal cylinder were the only magnetic resistor in the (closed) magnetic loop, one would obtain 5.9 H for the low frequency case, as shown in the Fig. 4.46 (left). However, as for the cylinder coil under consideration, the field lines close over a long air distance and an air resistor also has to be taken into account.

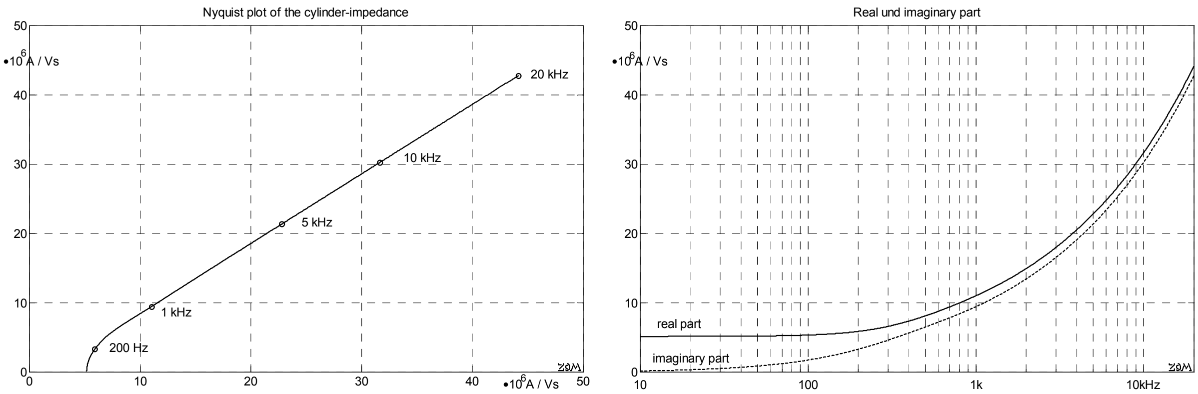


Fig. 4.45: Frequency-dependence of the complex magnetic cylinder impedance $\underline{Z}_Z$, $\mu_r = 110$, $\sigma = 5e6$ S/m.

If one considers, in a simple magnetic equivalent circuit, a series connection of core and air resistance (Fig. 4.44), this will reduce the absolute value of the inductance as well as its frequency dependence (**Fig. 4.46**). This simple model is well suited as long as the ferromagnetic metal core can sufficiently focus the field running through the coil. For small μ_r, however, a considerable part of the inner magnetic field flows within a kind of **hollow cylinder**, i.e. between core and average coil diameter. The magnetic resistance of this hollow cylinder is located parallel to $\underline{Z}_Z$ in the magnetic block circuit, hence, in the electrical equivalent circuit in series with the parallel connection of R and L (**Fig. 4.47**).

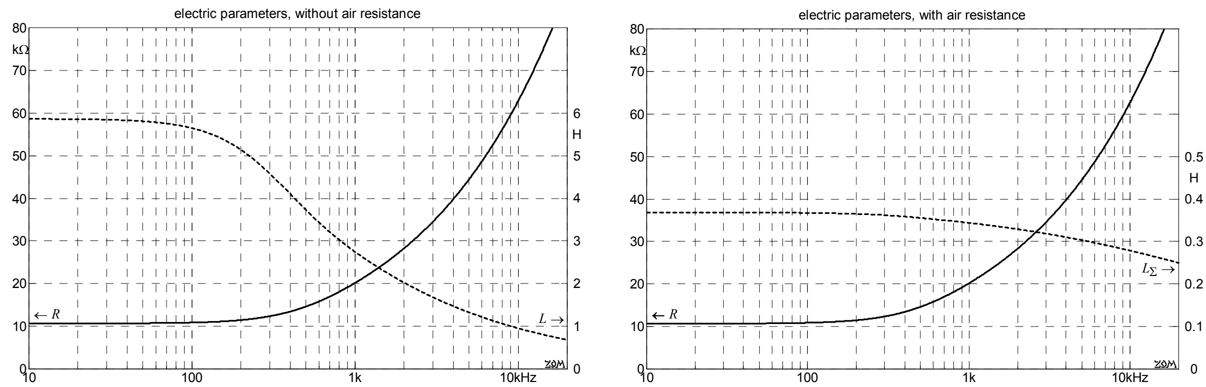


Fig. 4.46: Frequency dependence of R and L (left) as well as R und L // L_L (right) from Fig. 4.44 ($N = 5500$).

It is possible to explain every impedance/frequency curve in Fig. 4.42 with a good precision using this extended equivalent circuit diagram (**Fig. 4.47**). The magnetic resistance of this "hollow cylinder" is real and it is mapped onto the inductor L_{HZ} – by definition its *electrical* impedance is purely imaginary. The values of *R* and *L* are, as explained by Fig. 4.44, frequency-dependent.

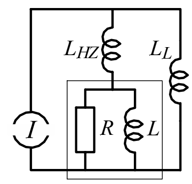


Fig. 4.47: ECD

For the **magnetic pickup** the magnetic eddy current losses have the following consequences: 1) The pickup resonance is damped not only by the wire resistance of the coil but also the ferromagnetic core inside the coil. 2) The inductance of the coil is frequency-dependent and decreases towards higher frequencies. A higher order *RL*-circuit can be employed in the block diagram as an alternative to the frequency-dependent inductance (see chapter 5.9.2.3). The different geometries and the diversity of the material parameters produce different damping and inductance frequency-dependencies. Using this, the pickup designer can purposely influence the frequency transfer characteristics.

Fig. 4.48 shows the impedance/frequency curves taken with a measuring coil (N = 5500, Fig. 4.44). The highest inductance is created by the ferrite rod, whose isolated elemental magnets do not allow eddy currents in this frequency range. The permeability of the humbucker screw, made of undefined steel, is practically the same for low frequencies (300 Hz), however, due to high eddy currents, its inductance decreases. The humbucker cylinder ("slug") has a somewhat smaller inductance for lower frequencies but also smaller eddy-current losses. Alnico-magnets are practically free of eddy currents; the magnetically weaker Alnico 2 has a higher permeability as compared to Alnico 5 (Alnico 500), resulting in a lower pickup resonance (for otherwise equal parameters).

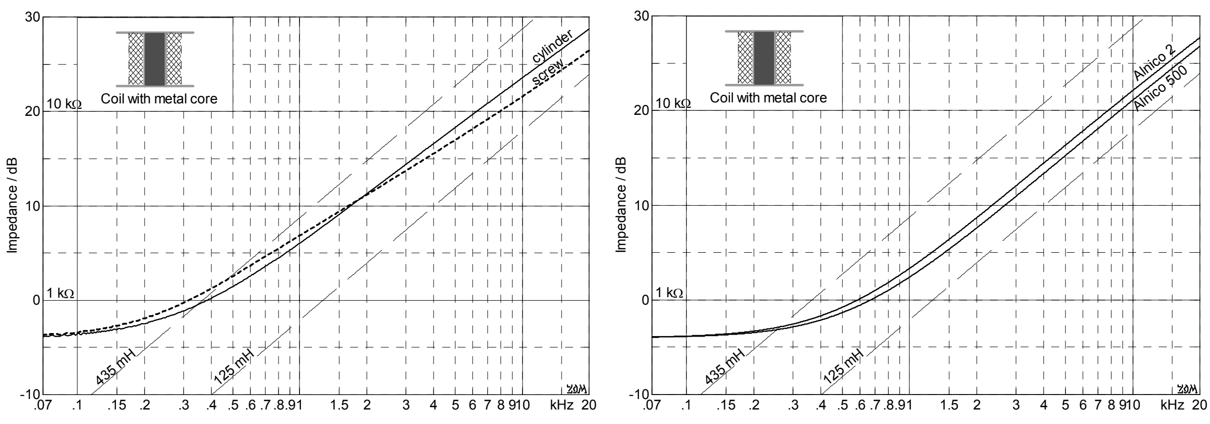


Fig. 4.48: Impedance/frequency curves, taken with the measuring coil; core dimensions 5x14 mm. "Cylinder" means the metal cylinders (= slugs) commonly used for humbuckers, "screw" depicts the humbucker screw (5.9.2.6).

Elaborate details for the construction of single-coil and humbucker pickups, as well as their technical data are summarized in chapter 5.