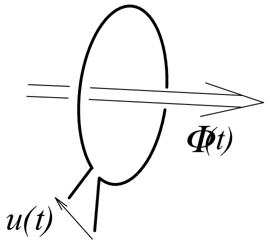
4.10 Magnetodynamics

Dynamics is derived from the Greek word dynamos = force, which is why dictionaries like to explain this word as “tenet of the force”. However, magneto-*dynamics* does not primarily deal with forces but, contrary to magneto-*static*s, it deals with systems whose signals or system variables are experiencing variations. In this case it can be changes of location or movement as well as temporal changes of fields in a stationary system. Ultimately, forces may also be involved, but they are not in the foreground.

4.10.1 Magnetic Voltage Induction

An electric voltage is induced in a conductor loop, which correlates with the temporal change of the percolating magnetic flux. This is the basic principle of voltage generation in a magnetic pickup. The flux is the product (in general the integral) of the flux density and the area. A change can therefore arise as a change of the flux density and/or area. In a pickup the conductor is formed by many loops of coiled copper wire. For high quality manufacturing the single loops are glued together in such a way that the coil area remains constant so that the only changes are in the flux density. The source and origin of the flux change is the vibrating string, the effect of which can be described in two different ways: changes of location of the string alter the magnetic resistance in the magnetic circuit, resulting in flux density changes (magnetic transducer). Alternatively, the string can be considered as being magnetized by the pickup; movements of the string are, therefore, relative movements between the string magnet and the coil (dynamic transducer).



$$u(t) = N \cdot d\Phi/dt$$ Induction Law

Fig. 4.34: If the magnetic flux *Φ(t)* increases with time, a positive voltage *u(t)* will be induced.

In **Fig. 4.34** a wire loop is depicted which is penetrated by a flux that increases with time. The voltage *u(t)* shown forms as a consequence of the change in flux. Sometimes, the induction law is also written with a **minus sign**, depending on the reversed definition of the arrow.

For a **guitar pickup** there is not only *one* loop, rather the wire is wound to a **coil** with N = 5000 – 10000 turns. If one calculates from $\tilde{u} = 1\text{V}$ back to the magnetic parameters, one will get a change in flux density of 1 mT (peak value $\hat{B}$) for f = 2 kHz, N = 6200 turns and a magnet area of 18 mm^2. The relative change in the flux density caused by the vibrating string is, thus, only approximately 1% compared to the static flux density of the permanent magnet (approx. 100 mT).

More accurate considerations show that it is difficult to compute the induced voltage in the pickup coil. Not only does the string change its position in space, it also warps (bends) while it vibrates. For the calculation of the flux change one would have to perform a three-dimensional field calculation, including the non-linear *B/H* behavior of magnet and string. Further, one has to take into account that not every single loop of the coil is penetrated by the same magnetic flux: the field that is generated by the string diverges and loops that are located closer to the string will experience a larger flux than those further away. Despite these difficulties one can, with restrictions and approximations, realize a reasonable agreement between theory and observation.

Two questions are particularly pertinent when using the **Induction Law**: How big is the induced voltage and which type of curve is generated? As the movements of the strings are non-linear projections of the change of flux, sine-like vibrations will not yield sine-like voltages. Chapter 5.8 will address pickup-distortion in more detail. We will only investigate the small signal behavior here. The flux change will, therefore, be assumed to have a single frequency, be sine-like and be described by the frequency *f* and by the effective value of the flux density *B*. The time differential, thus, simplifies to a product:

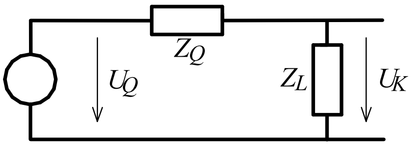
$$u = 2\pi f \cdot N \cdot B \cdot S$$

Induction Law for a sine-like flux density

where N is the number of turns, B is the effective flux density, and S is the area of the coil; u is the effective value of the induced voltage. Since all turns of a coil are wound in the same direction, the voltages induced in every single loop are adding up to the total coil voltage (typically some 100 mV, max. approx. 5 V).

4.10.2 Self-Induction, Inductance

The voltage that is induced in the coil should be interpreted as a **source voltage**, not as a terminal voltage. It evolves, so to say, in the interior of the pickup, just as if an alternate voltage source is built in there. The voltage measured at the clamps (pickup cable) only equals the source voltage for the case without **load**, i.e. in **open source** mode. Once a load is applied, "the terminal voltage breaks down", i.e. it becomes smaller than the source voltage. This behavior can also be observed in the lighting main: If one switches on a 2 kW furnace, the light dims. The reason for the voltage decrease is the voltage drop across the internal resistance, which results in a **voltage divider** (**Fig. 4.35**):



$$U_K = U_Q \cdot Z_L / (Z_Q + Z_L)$$

Fig. 4.35: Voltage divider between load impedance Z_L and source impedance Z_0.

For the general case the load and source resistors are frequency dependent, which is why one speaks of load impedance Z_L and source impedance Z_Q. The induced voltage is U_Q and at the terminals U_K builds up. Both voltages are only identical for an infinite load impedance (open circuit).

Figure 4.35 depicts a circuit mesh in which a current can flow. The required **energy** is delivered by the ideal voltage source, which is drawn as a circle. Of course, the energy cannot emerge from nothing. If the pickup should deliver energy, it must be provided with energy. And it is: energetically, the source of the pickup terminal voltage is the vibrating string. Its vibration energy is diminished a little bit by the pickup (and the connected resistances), where electrical energy is transformed into heat. In other words, the electrical resistances in Fig. 4.35 retroact on the string and change its vibration so that the source voltage is load dependent. However, as this effect is only very small, it is neglected here. The source voltage U_Q is considered to be imprinted, it equals the induction voltage defined in the preceding chapter.

The **load impedance** is composed of the coil capacity (see pickup parameters), the guitar electronics (volume and tone potentiometers), and of the guitar cable and amplifier. Simplifying, this can be described by a parallel circuit of 300 – 1000 pF and 100 – 350 kΩ. Further, the **source impedance** is formed by the coil. Here, one first deals with approximately 1 km of thin enameled copper wire with a DC resistance of approx. 5 -15 kΩ. Another effect has to be considered for AC, and only AC flows as consequence of the induced alternating voltage: as already shown in chapter 4.1 every alternating current produces a magnetic field in its vicinity: an *alternating* current produces an alternating field, i.e. a field with alternating polarity. This field also percolates through the pickup coil and induces a voltage. As this field is produced by the pickup *itself* (in contrary to the field produced by the string), the voltage which is built up as a result is called the **self-induction voltage**. The self-induction voltage superimposes itself inversely phased on the voltage originating from the string vibration and weakens it (Lenz's rule). It is obvious that this voltage superposition cannot be in phase because otherwise the current would increase with increasing voltage, yielding voltage increase, yielding current increase … and the system would become unstable. In order to take into account the self-induction induced voltage decrease, one could include an additional regulated voltage source in Fig. 4.35, whose source voltage is dependent on the current flowing in the mesh. However, it is common to instead draw in a component, whose voltage drop is equal to the self-induction voltage. In the circuit it is symbolized either by a black, filled rectangle, or by a symbolic representation of the wire coils (**Fig. 4.36**); this component is an **inductive two-terminal device** (= **inductor**), the unit symbol for an inductor is *L*.

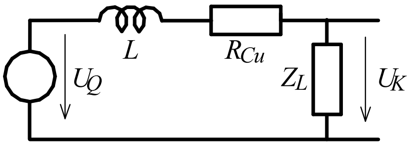


Fig. 4.36: Equivalent circuit of the coil with inductor *L* and resistance of the copper wire R_{Cu}.

In a Gedankenexperiment (thought experiment) we will now allow a constant current to flow through the circuit depicted in Fig. 4.36. This constant current can, however, not be generated by induction as U_Q, but it could be fed in at the clamps depicted on the right. As consequence of this constant current, a constant magnetic field would be generated, whose flux differential (with time) yields the induced voltage. The differential of a constant is, of course, zero –

consequently, the constant voltage drop at an ideal inductor must be zero too. However, if an AC current flows, an induced voltage develops, the quantity of which is depending on the current change:

$$u(t) = L \cdot di(t)/dt; \qquad \underline{U} = j\omega L \cdot \underline{I}$$ Two terminal equations

A voltage drop *u(t)*, which is proportional to *L* and to the current change with time, will be generated in an inductor *L* in which a current *i(t)* flows. A representation with rotating complex pointers is convenient for sinusoidal oscillations. Using this, the time differential will transform into the factor $j\omega$. The imaginary unit *j* will yield a rotation (phase shift between voltage and current) of 90°, $\omega = 2\pi f$ is the angular frequency. The derivative *d/dt* and multiplication with $j\omega$ are linear operations; they do not destroy the proportionality between current and voltage. For direct current the proportionality coefficient between *U* and *I* is called the **resistance** ($U = RI$), for alternating currents it is called **impedance** instead. The impedance of the inductor is $j\omega L$; for direct current it is zero, with increasing frequency it increases in proportion to the frequency. The impedance of an inductor is a positive imaginary quantity (precisely: not negative), one could also say: the resistance of an inductor is positively imaginary.

The **unit** of the inductance is **Henry**: 1H = 1Vs/A.

The letter H should not be mixed up with the formula symbol *H*, which stands for the magnetic field strength! The quantity of inductance of *L* can be deduced from the geometry of the coiled wires. Typical values for a guitar pickup are *L* = 2 - 10 H.

Equations for the **calculation** of simple coil inductivities are quoted in every book on magneto-dynamics. Simple formulas are obtained for the toroidal coil and the long cylinder coil. However, for the magnetic pickup the conditions are more complicated: the magnetic field generated by the vibrating string is inhomogeneous, i.e. dependent on the position. Thus, every turn of the pickup coil will be penetrated by different magnetic fluxes and an analogy, as in Fig 4.35, with one single voltage source and one single inductor is not possible in the first instance. This effect is substantial, it cannot simply be ignored: for a Stratocaster pickup the magnetic fluxes in a turn near the string and in a turn away from the string differ by a factor of 10 (chapter 5.4.3). For the calculation of the quantity of the induced voltage one has to perform appropriate suitable averaging. In addition, one has to consider that the magnetic field will be focused (enhanced) by ferromagnetic materials. The Alnico-magnets placed inside the coil are ferromagnetic and focus the magnetic flux, which yields a higher inductance in comparison to a coil which is free of magnetic fields (see chapter 4.10.3).

Since the alternating magnetic flux has its maximum strength in close vicinity to the string, for efficient conversion (‘loud pickup’) it is recommended to locate the coil as near as possible to the string. One may find Fender pickup designs with magnets ending right at the edge of the flange facing the string; they are ‘loud’. However, there are also pickups (the ones with ‘*staggered magnets*’) with magnets extending up to 4 mm; they produce (as a rough approximation) only half of the voltage. Naturally, this rule of thumb presumes that all other parameters remain constant. In particular, the contour of the coil can deliver an additional degree of freedom: of equal height or conically wound.