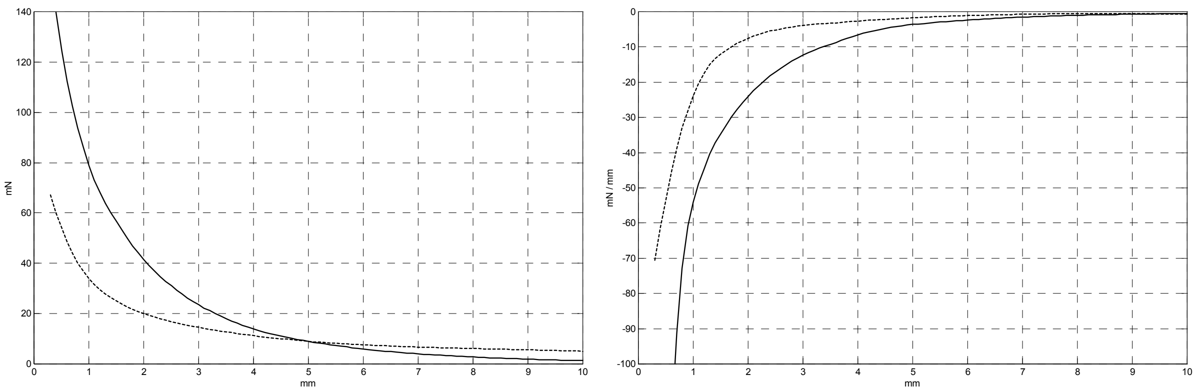

Fig. 4.49: Magnetic force as function of the clearance. Alnico-5, singlecoil (——), Gibson-humbucker (---). In the right picture the differential stiffness is shown as a function of the clearance.

However, the **negative magnetic field stiffness** not only affects the vibrations normal to the fretboard; for vibrations parallel to the fretboard the string/magnet distance is practically constant, the magnetic field stiffness is therefore negligible♣. For vibrations normal to the fretboard the negative field stiffness generates a decrease of the mechanical stiffness and, hence, a decrease of the partial tone frequency of the string. The effect is not dramatic, but audible for strong magnets: if the magnet is moved closer the tone frequency drops. However, every vibration of the string will occur as spatial wave, not as plain transversal wave. Even if a *plain* vibration is prevalent shortly after plucking, mode coupling in the supporting points and, last but not least, the magnetic field will cause a rotation of the original vibration plane. The rotation frequency is low (some Hertz) and beat-frequency-like **amplitude variations** will evolve in the pickup signal (4.11.3).

In this way the magnet does not just change the tone frequency but also the tone color of the vibrating string. Whether this is good or bad depends on subjective assessment criteria. Many guitarists have the opinion that the chorus-like beat frequencies of a Stratocaster belong to the typical sound of this guitar – as long as they are not too dominant. The assumption (or "certainty" of expert authors) expressed in several books that "the harmonics are slightly detuned in comparison with the fundamental*s*" is incorrect: the **fundamental** will be detuned the most. One can ask, since 2001, why it is suddenly referred to in the plural, and who feels that the announcement: "*further handbooks are in preparation*" is mere threat. However, this is just how they are, those string fundamentals.

4.11.2 Field-Induced Deviations of the Tone Frequency

When a magnetic pickup approaches a string, three effects can be anticipated: The **tone frequency** decreases, chorus-like **beat frequencies** evolve and the **amplitude** changes. The frequencies, especially the fundamental, will decrease due to the negative field stiffness, which can be audible for large values. The detuning between the fretboard-normal and fretboard-parallel vibrations induces beat frequencies; the altered frequency relationship between the partial tones in the subsequent non-linear systems causes additional partial tones, which can further increase the chorus impression. True damping, i.e. removal of vibrating energy, occurs only to a negligible extent. Firstly to the tone frequency:

♣ If the string is located substantially beyond the magnet axis, both vibrating planes are affected.

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The resonance frequency of a vibratory mass-spring-system depends on the square root of the spring stiffness. The stiffness caused by the magnetic field is negative because approaching the magnet does not need a force in the direction of the movement (as it is for every common spring) but in the opposite direction: the magnet does not need to be pushed towards the string with force but, to the contrary, must be held back. The negative stiffness which is acting thereby decreases the total stiffness of the string and reduces the frequency. The frequency-dependence of this effect or which partial tones may be affected can be investigated with the **conduction-analogy**. Here, the mechanical system is described by an analogous electrical circuit with the analogies: force/current, velocity/voltage, spring/coil, mass/capacitor [3]. The principle effect can be investigated for an undamped plain transversal wave which is ideally reflected at a solid mounting. The string corresponds to an electrical conductor shorted at the end and whose length is large in comparison with the wavelength [e.g. Meinke]. The corresponding *mechanical* input impedance $\underline{Z}_E$ depends on the wave resistance Z_W, the terminating impedance ($\underline{Z}_{termination} \to \infty$, because of velocity $v = 0$), the conductor length l, on the frequency f and the phase velocity c. All of these system parameters can be attributed to the mechanical quantities by the analogy-laws: the string tension force Ψ, the string density ρ, the string length l and the string cross section area A.

$$\underline{Z}_E = \frac{Z_W}{j \cdot \tan \beta l}; \qquad \beta = \frac{\omega}{c} = 2\pi f \sqrt{\rho A / \Psi}; \qquad Z_W = \sqrt{\rho A \, \Psi} \qquad \text{Conduction}$$

If one assumes a solid mounting for *both* ends of the string ($\underline{Z}_{el} = 0 \ \hat{=} \ \underline{Z}_{mech} = \infty$), the Eigen-frequencies (partial tone frequencies) of the string are the poles of the tangents-function, i.e. at integer multiples of the basic frequency $f_G = c/2l$. The reciprocal of the base frequency is the transit time over $2l$, i.e. from the beginning of the conductor to the reflecting end and back. In order to introduce the influence of the negative field stiffness into the conduction model, one divides the string into two consecutive conductors: a first conductor of length l_1 from the saddle to the pickup and a second conductor of length l_2 from the pickup to the bridge. The mechanical termination impedance $\underline{Z}$ of the first conductor is the sum of the input impedance $\underline{Z}_2$ of the second conductor and the stiffness impedance $\underline{Z}_S$: The input impedance $\underline{Z}_1$ of the first conductor (viewed from the saddle) is thereby given by:

$$\underline{Z}_1 = \frac{\underline{Z} + jZ_W \cdot \tan \beta l_2}{1 + j\,\underline{Z}/Z_W \cdot \tan \beta l_2} \qquad \underline{Z} = \underline{Z}_2 + \underline{Z}_S \qquad \underline{Z}_S = \frac{s}{j\omega}$$

The Eigen-frequencies are located at the poles of the impedance function, i.e. at $\underline{Z}_1 \to \infty$.

Of course the input-impedance can also be calculated from the location of the bridge with the same result. For a first check the magnetic field stiffness can be taken to be zero ($\underline{Z}_S = 0$, $Z = \underline{Z}_2$) which indeed gives the partial tone frequencies at multiples of 82,4 Hz using the data of the E_2-string. As can be expected, a spring with stiffness zero cannot produce any changes. For every stiffness s different from zero the absolute value of $\underline{Z}_S$ will tend to zero with increasing frequency ($\underline{Z}_S = s/j\omega$), from which immediately follows, that the magnetic field stiffness can only detune low-frequency partials. As the field stiffness is negative the partial frequencies will *decrease*.

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Figure 4.50 upper left shows the calculated (mechanical) input admittance of an E_2-string (82,4 Hz). The admittance is the inverse of the impedance; its zero points lie at the poles of $\underline{Z}_E$. The upper right picture shows the absolute value of the impedance of a 16 cm long part of the string located between bridge and magnet, as well as the absolute value of a magnetic field stiffness (–180 N/m). Its value was chosen to be unusually high to depict the effect more clearly. In the lower left plot the effects of the field spring on the admittance of the entire string are shown: especially the first and the second partial are detuned. For the calculation a characteristic wave impedance of 0.7 Ns/m was chosen; the length of the string is 65 cm and the magnet is located at a distance of 16 cm from the bridge. The frequencies of the partial tones are at the zero positions of the admittance. No dispersion was modeled.

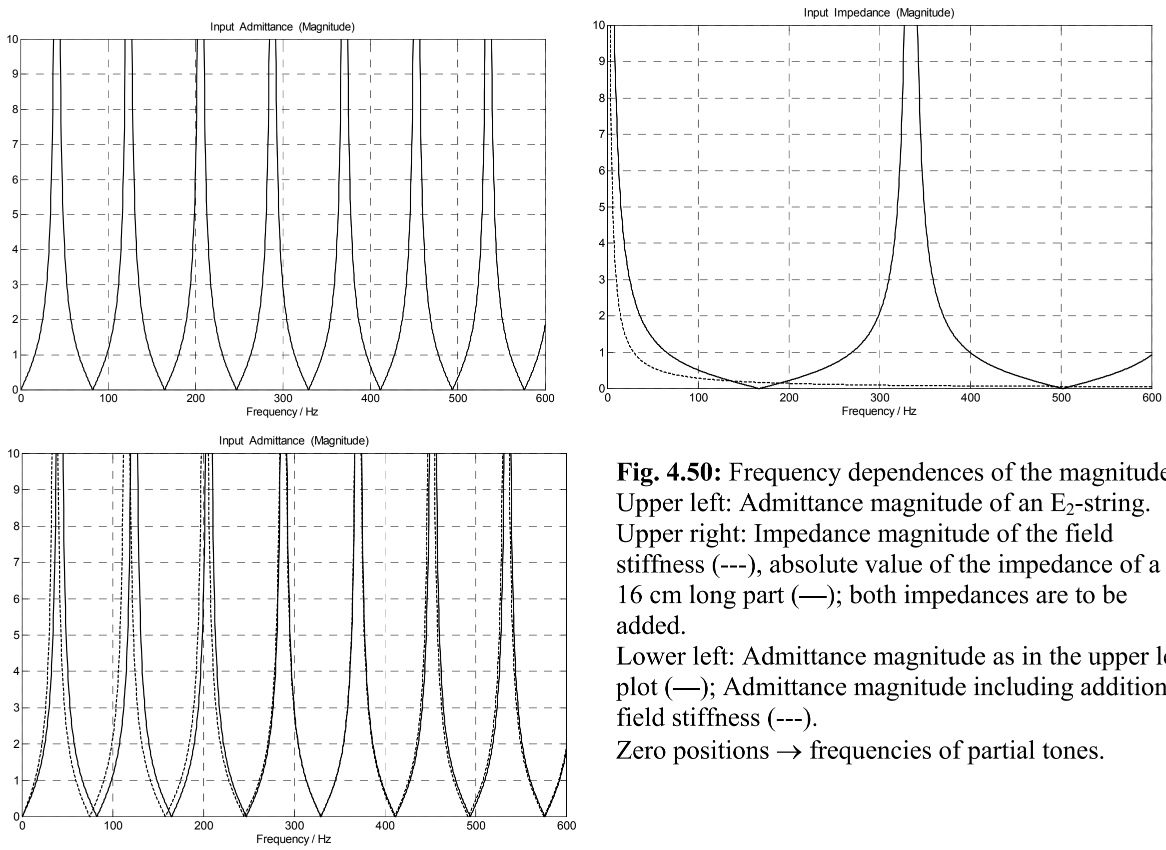

Fig. 4.50: Frequency dependences of the magnitudes.
Upper left: Admittance magnitude of an E_2-string.
Upper right: Impedance magnitude of the field stiffness (---), absolute value of the impedance of a 16 cm long part (—); both impedances are to be added.
Lower left: Admittance magnitude as in the upper left plot (—); Admittance magnitude including additional field stiffness (---).
Zero positions → frequencies of partial tones.

In addition to the calculations measurements of an E_2-string are shown in **Fig. 4.51**. A Fender E_2-string (3150, 1.1 mm diameter) was mounted in an Ovation solid-body guitar (EA-68, piezo-pickup) and the piezo-signal was analyzed. The magnetic forces were generated by an 18 mm long Alnico-5-Magnet (5 mm diameter) positioned relative to the string at a distance of 16 cm from the bridge.

Fig. 4.51 shows that a precise frequency analysis is problematic: the resulting detuning is only several Hertz, so that a frequency resolution smaller than 1 Hertz would be desirable. The string vibration, however, cannot be considered as stationary within the necessary time window (more than 1 s). The chosen DFT-windows represent a compromise between time and frequency resolution (analysis was done with the CORTEX-software *Viper*).

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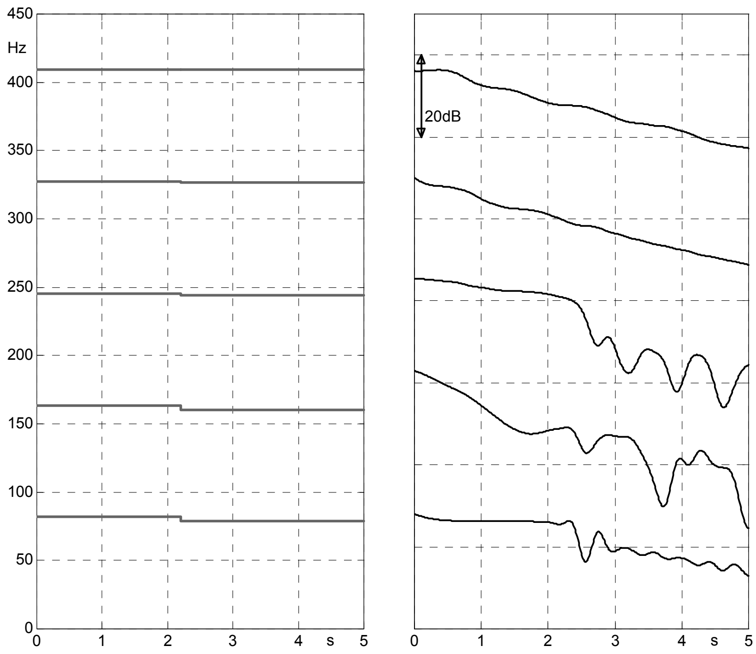

Fig. 4.51: Spectrogram (left) and partial tone level progress (right) of the vibration decay of an E_2 string. At 2.2s a magnet was approached to the vibrating string.
The frequencies of the first and second harmonic decrease from 2.2 s onwards . For the third harmonic one can mainly detect a level change, the fourth and the fifth harmonic remain unchanged (holds also for higher harmonics).

4.11.3 Field-Induced Amplitude Variations

The measurements and conduction model show, concordantly, that the permanent magnet will detune the lowest harmonics. The detuning will happen mainly for the fretboard-normal vibrations; field changes parallel to the fretboard and thus parallel to the magnetic pole surface only develop weakly. For the spatial vibration this means that there are two spatially orthogonal string vibrations with different frequencies which, after superposition, produce **beat frequency-like level changes**. If one denotes the fretboard-normal component with *y* and the fretboard-parallel component with *x*, one gets for the total amplitude ξ in vector-notation:

$$\vec{\xi} = \begin{pmatrix} \hat{x} \cdot \cos(\omega_1 t) \\ \hat{y} \cdot \cos(\omega_2 t + \varphi) \end{pmatrix}$$

$\hat{x}$ = Amplitude of the *x*-component
$\hat{y}$ = Amplitude of the *y*-component

For single frequency vibrations ($\omega_1 = \omega_2$) a point on the string moves according to the amplitude-relation $\hat{y}/\hat{x}$ and the phase shift φ along a line in an ellipse or a circle* (**Lissajous** figures). However, if both frequencies are not equal the figures above alternate with weak transitions. The time-dependent change of the curve is apparent when one transforms for small frequency differences:

$$\omega_2 t + \varphi = \omega_1 t + \Delta\omega t + \varphi = \omega_1 t + \varphi(t)$$

The *x* as well as the *y*-vibrations contain $\omega_1 t$, however, for the *y*-vibration an additional time-dependent (slow) phase-shift $\varphi(t)$ exists. A sensor that only detects the vibration exactly normal to the guitar body will, however, not be affected by the curve changes because $\hat{y}$ is time-invariant.

* Line and circle are special types of the ellipse.

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