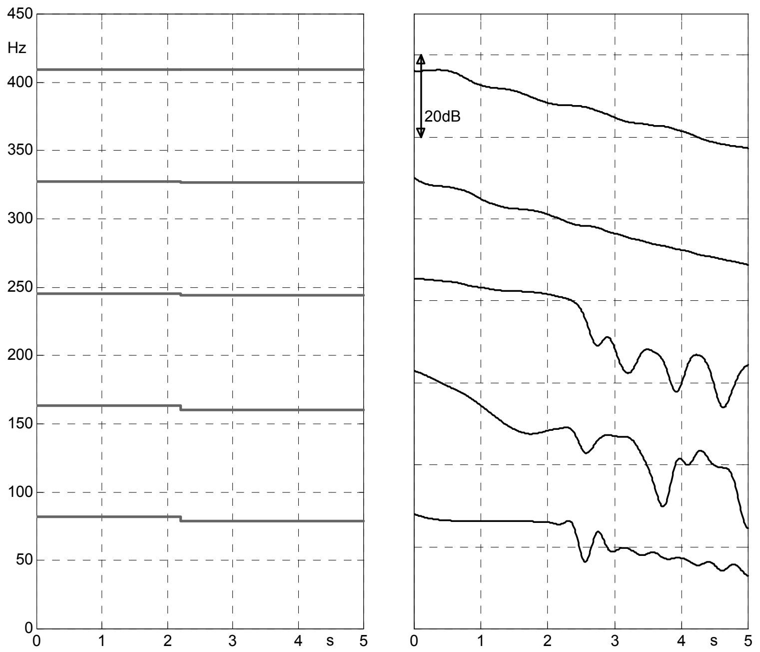


Fig. 4.51: Spectrogram (left) and partial tone level progress (right) of the vibration decay of an E_2 string. At 2.2s a magnet was approached to the vibrating string.
The frequencies of the first and second harmonic decrease from 2.2 s onwards . For the third harmonic one can mainly detect a level change, the fourth and the fifth harmonic remain unchanged (holds also for higher harmonics).

4.11.3 Field-Induced Amplitude Variations

The measurements and conduction model show, concordantly, that the permanent magnet will detune the lowest harmonics. The detuning will happen mainly for the fretboard-normal vibrations; field changes parallel to the fretboard and thus parallel to the magnetic pole surface only develop weakly. For the spatial vibration this means that there are two spatially orthogonal string vibrations with different frequencies which, after superposition, produce **beat frequency-like level changes**. If one denotes the fretboard-normal component with y and the fretboard-parallel component with x, one gets for the total amplitude ξ in vector-notation:

$$\vec{\xi} = \begin{pmatrix} \hat{x} \cdot \cos(\omega_1 t) \\ \hat{y} \cdot \cos(\omega_2 t + \varphi) \end{pmatrix}$$

$\hat{x}$ = Amplitude of the x-component
$\hat{y}$ = Amplitude of the y-component

For single frequency vibrations ($\omega_1 = \omega_2$) a point on the string moves according to the amplitude-relation $\hat{y}/\hat{x}$ and the phase shift φ along a line in an ellipse or a circle* (**Lissajous** figures). However, if both frequencies are not equal the figures above alternate with weak transitions. The time-dependent change of the curve is apparent when one transforms for small frequency differences:

$$\omega_2 t + \varphi = \omega_1 t + \Delta\omega t + \varphi = \omega_1 t + \varphi(t)$$

The x as well as the y-vibrations contain $\omega_1 t$, however, for the y-vibration an additional time-dependent (slow) phase-shift $\varphi(t)$ exists. A sensor that only detects the vibration exactly normal to the guitar body will, however, not be affected by the curve changes because $\hat{y}$ is time-invariant.

* Line and circle are special types of the ellipse.

Real sensors cannot be expected to exhibit such a perfect direction sensibility: common magnetic pickups are indeed the most sensitive for fretboard-normal vibrations; however, for fretboard-parallel vibrations the sensitivity will not be zero but approx. 1/10. The voltage which is generated is, thus, a combination of x and y vibrations which can, for the simplest case, be depicted as a **linear combination**:

$$u(t) = U\big(\cos(\omega_2 t + \varphi) + k \cdot \cos(\omega_1 t)\big) \qquad k = \text{relative } x\text{-ratio}$$

The commonly known formula for the beat frequency is obtained for k = 1 whereas for k << 1 the signal can be approximately regarded as a mixture of frequency and amplitude modulation. A cosine-like frequency modulation for a small modulation index can be represented, to a good approximation, by three spectral-lines [3]:

$$u_{FM} = U\left[\cos(\omega_2 t) - \frac{m}{2}\cos\big((\omega_2 + \Delta\omega)t\big) + \frac{m}{2}\cos\big((\omega_2 - \Delta\omega)t\big)\right]$$

If this FM signal should become amplitude modulated, the AM has to be applied to each of the three spectral components. By neglecting the $m^2/4$-terms (because m << 1), the lines at $\omega_2 + \Delta\omega$ compensate, while the lines at $\omega_2 - \Delta\omega$ add:

$$u = U\big[\cos(\omega_2 t) + m \cdot \cos\big((\omega_2 - \Delta\omega)t\big)\big] \qquad \omega_2 - \Delta\omega = \omega_1$$

This signal equates to the above mentioned linear combination for $\varphi = 0$; corresponding transformations are possible for other phase shifts. Hence it has been shown that for x and y vibrations with $k = 1$ a beat frequency, and for k << 1 a mixture of AM and FM, will result. This result can also be derived from the projection of the sum of two pointers with different frequency. If one assumes, for example, that the pickup for the y oscillations is eight times more sensitive than for the x oscillations ($k = 0.125$) then, for $\hat{y} = \hat{x}$, the amplitude of the pickup voltage changes by ±12.5%, or ± 1 dB. The modulation frequency corresponds to the difference frequency, which is the detuning caused by the magnet (e.g. 1 Hz). The amplitude relation $\hat{y} = \hat{x}$ means that the string vibrates at an angle of 45° with respect to the fretboard. The amplitude modulation effect will decrease for larger angles (normal to the fretboard) and for smaller angles (→ fretboard-parallel) it increases, until at arctan(1/8) = 7° a precise beat frequency is reached: The level change here is theoretically unlimited.

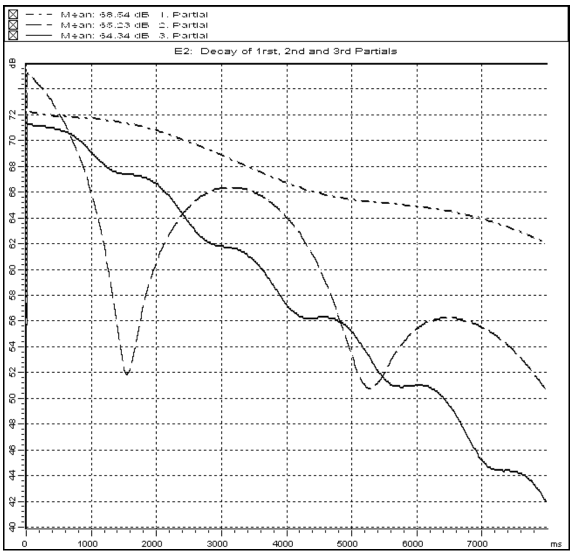
The linear combination is only a simple model for the description of time-variant level fluctuations. For the magnetic pickup the induced voltage depends on the **non-linear** relationships of the x and y velocities, which will result in additional sum and difference tones. However, as this will not result in completely different effects, we have dispensed with a precise investigation. An additional effect, which has also not been taken into consideration acts at both **string mountings** (bridge / saddle). Both mountings are idealized as rigid, but show a direction dependent compliance. As a consequence, the reflection factor has to be defined including all modes: a pure y vibration will also be reflected, to small extent, in the x direction and vice versa. For example, if the string is plucked exactly normal to the fretboard, after a certain time there will be fretboard parallel component which will yield amplitude variations in the pickup; the magnetic field can enhance or diminish them.

In addition, the (predominantly) fretboard normal **magnetic field** can induce a **rotation** of the vibrating plane when it is not exactly fretboard-normal or fretboard-parallel: for an inclined vibrating angle the string will experience a stronger pull at the turning point closer to the magnet than at the turning point further away. By reducing the magnetic force into coplanar and orthogonal parts one will get an angular force that tries to align the string (along fretboard-normal direction).

Finally, one has also to consider that the field stiffness is **non-linear**: the absolute value of the stiffness increases with decreasing distance. A simulation with a non-linear conduction model results in weak beat frequencies, even for exact fretboard-normal vibrations, whose fractional variation amplitude is dependent on the input signal amplitude.

In summary: Already without magnetic fields, sound level deviations are generated that develop differently for each partial tone. They originate from anisotropic mounting reflections, i.e. mounting impedances that depend on the oscillation direction and mode-coupling. The magnetic field detunes the fretboard-normal vibration component which might enhance or reduce the existing deviations. Non-linearities occurring in the mechanics and during electro-mechanic transformation will create additional sub-lines in the spectrum so that, in summary, a complicated level characteristic might develop for each partial tone.

Fig. 4.52 shows the selectively measured characteristic of the partial tones of the E_2-string. The recordings were performed with the built-in piezo pickup without a magnetic field. The differences between both pictures originate from the plucking technique as well as from the non-identical guitar positions and, possibly, slightly different guitar tuning and temperature. In the course of these first orienting investigations it became clear that the guitar should not be placed somehow on the thigh but must be supported in a defined way. Appropriate frame conditions are “in vivo” (guitar hanging from the guitar strap, fret-hand defined at the neck), and “in vitro” (guitar attached at the strap-pin, no damping at the neck).



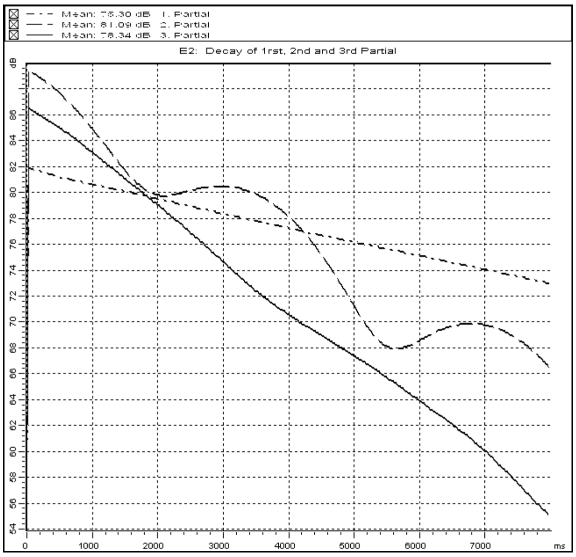


Fig. 4.52: Decay of the first three partial tone levels after plucking (left) or fretboard-normal excitation pulse (right) for an E_2-string with no magnet and an Ovation EA-68 piezo-pickup. The recordings were taken on different days.

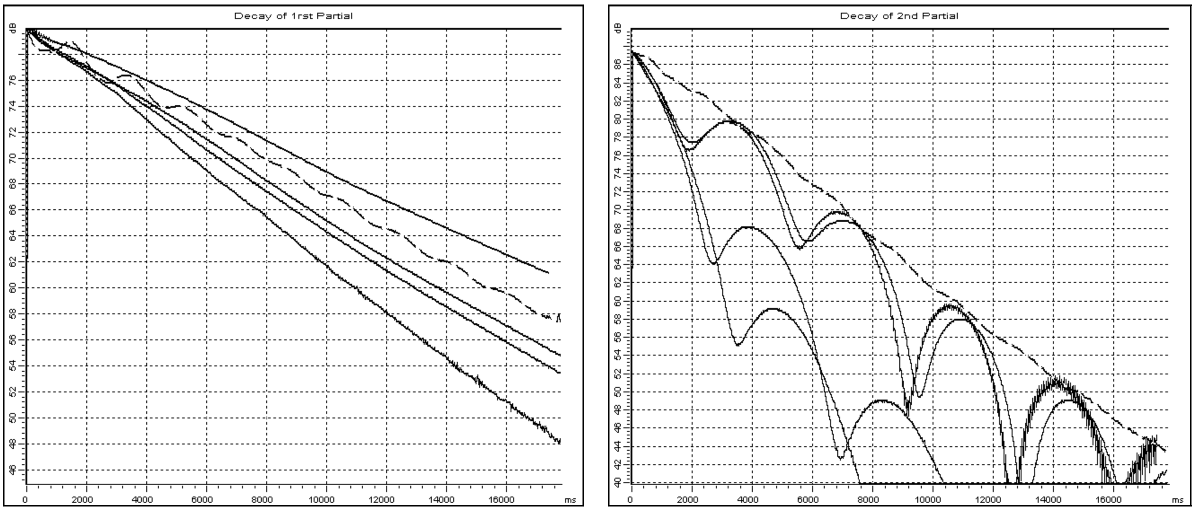


Fig. 4.53: Decay of the first two partial tone levels after fretboard-normal excitation pulse for an E_2-string, with an Ovation EA-68 piezo-pickup. Continuous lines: without magnet, dashed lines: Alnico-5-magnet in neck pickup position for a 2.5 mm distance between the string and magnet. Left: first partial, right: second partial.

Figure 4.53 shows the decay of the first two partial tones. The continuous lines were taken without a magnetic field. The upper curve, with the slowest decrease, shows the level decay of the undamped neck whereas the lower three continuous curves belong to measurements that were made with the fret-hand holding the neck in different ways without touching the strings. The dashed line was taken without neck-damping but with a magnetic field (Alnico-5-magnet placed 16 cm from the bridge). A strong influence of the fret-hand on the decay-characteristic (sustain) is observed in both measurements. The hand primarily acts as a damping resistance removing vibration energy. The level decays linearly with time for the first partial tone (left picture) without a magnetic field (exponential tension envelope curve), whereas a slight level oscillation occurs with a magnetic field. The second partial tone is completely different: There are intense level oscillations without a magnetic field, whereas there is a nearly oscillation-free decay with a magnetic field. **Fig. 4.54** shows similar results for fretboard-parallel excitations (both with magnetic field).

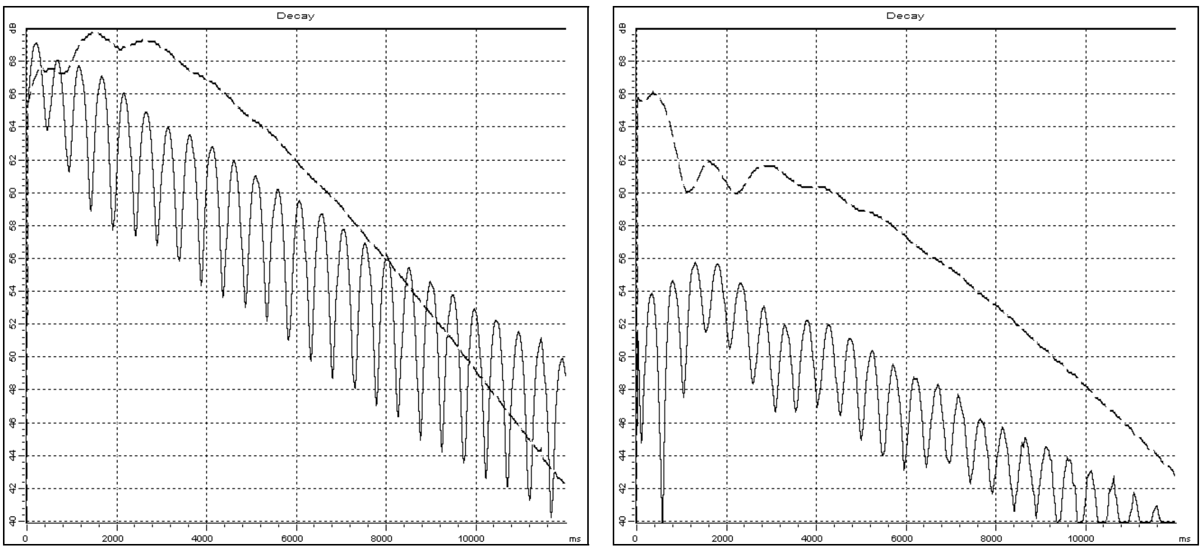


Fig. 4.54: Decay of the first (continuous) and third (dashed) partial tone after fretboard-parallel excitation using an Alnico-5-magnet in the neck pickup position. The only difference between both pictures is a slightly different plucking direction.