4.3 Matter in Magnetic Fields

It has been found to be useful to describe fields, in analogy to fluxes of matter (water circuit), by **potential** and **flux quantities.** The expressions *flow* and *flowing* are to be used in a figurative sense. There is no real flow in the magnetic circuit, contrary to the water circuit. The scalar pressure is the quantity of drive in the water circuit. If the pressure is not equal in the entire fluid, but varies as a function of position, there are vector pressure differences or gradients and forces acting on the fluid particles which, as a consequence, move or flow in the opposite direction to the gradient. Hence, the pressure can be interpreted as a scalar potential in which its gradient would be comparable with the field strength. The velocity of the fluid, however, cannot be deduced directly within this scheme. Other characteristics of the fluid, like viscosity and inertia as well as the boundary conditions, have to be considered.

An **electric** circuit is quite similar. The gradient of the electric scalar potential is the electrical field strength and its line integral is the electric voltage. The material quantity "impedance," or the admittance, has to be known in order to deduce the current flow from the voltage or the electrical field. This is not different for the **magnetic circuit**. The magnetomotive force or, alternatively, the magnetic field strength, is the quantity of drive and the magnetic resistance determines the amount of magnetic flux. As already mentioned this flux is immaterial and, like all the other magnetic quantities, is not visible. As long as the entire magnetic field is confined in a single material, the introduction of a magnetic flux could be dispensed with. The introduction of the flux quantity is advantageous if several materials have to be considered. The **continuity condition** is especially useful. It means that the entire incoming node flux is zero for an incompressible liquid. If, for instance, a node is formed by three tubes and in the first tube the incoming flux is 5 m^3/s, in the second tube the incoming flux is 4 m^3/s, then, consequently, the incoming flux in the third tube has to be -9m^3/s, i.e. the outgoing flux at the node is +9 m^3/s. This law is also known as Kirchhoff's (nodal) rule or Kirchhoff's first law. The electric current divides itself at a conductor node and the magnetic flux at a material node based on the same principle.

The flux quantity already yields clear descriptions without the presence of nodes. If the cross-section of a tube with impermeable walls (!) varies, or the flow resistance depends on location, there is still the same flux through every cross-section, given that the fluid is incompressible. This is equivalent to electrical engineering: The same current flows through serial resistances even though their ohmic values might be different.

The potential quantity is defined in integral and differential form. The integral quantities in the water circuit are the pressure and the local pressure difference. The differential value is the pressure gradient. The flux quantity is also defined as integral and differential, as the total flux in the water circuit, e.g. m^3/s, and as the **flux density**, i.e. flux per transverse section (m/s). The relationship between the differential potential and the flux quantity is established via the **specific conductivity** or the reciprocal **specific resistance**. In this case "specific" means material specific as well as volume specific.

The quotient of pressure gradient and flux density is the specific flow resistance in the water circuit. If the flow resistance is large, the water flow is low. A higher fluid viscosity leads to a higher specific resistance and a slower current. For the electrical current, the quotient of the electrical field strength (in V/m) and the current density (in A/m^2) yields the specific resistance (in Ωm). Poor conductors have a high specific resistance, i.e. they are "highly resistive". The specific conductance is defined reciprocally to the specific resistance. In an electric circuit it is the quotient of current density and electric field strength.

The specific conductivity in the **magnetic circuit** is called the **permeability** μ, which is the quotient of the magnetic flux density B and magnetic field strength H:

$$B/H = \mu = \mu_r \cdot \mu_0 \qquad \mu_0 = 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}} = 1.257\,\mu\text{H/m} \qquad \text{Permeability } \mu$$

In many cases the permeability μ is divided into two factors, the absolute permeability μ_0 and into the dimensionless relative permeability μ_r. The absolute permeability, which is also called the magnetic field constant, has the unit Vs/Am, or Henry / Meter (H/m). Care has to be taken here. The italic H is the equation symbol for the field strength, the non-cursive H stands for the unit Henry (1H = 1Vs/A). Sometimes the unit Henry is also abbreviated by Hy to avoid confusion. µH means one **microhenry**, which is 10^{-6} H. Again one has to differentiate: The italic μ is the quantity of permeability, the upright µ is a prefix which means "one millionth".

The relative magnetic permeability of the vacuum is 1. Thus μ_0 can be interpreted as the permeability of the vacuum. The absolute permeability μ_0 can also be applied to air with a high accuracy. For many materials the relative permeability μ_r shows only a minor deviation from 1. These are called non-magnetic materials. In physics one further distinguishes between paramagnetic and diamagnetic materials but this discrimination is not necessary here. For **magnetic materials** (magnet materials) $\mu_r >> 1$ is valid. This holds for all iron and steel parts and the permanent magnets of an electric guitar. Magnetic materials that can be magnetized by weak magnetic fields are called **magnetically soft**. The opposite expression is **magnetically hard**. The limit at which a material becomes magnetically hard can only be described approximately (H_C > 1kA/m, see later).

The **permeability** μ is the magnetic conductivity. A material with large μ has a high magnetic conductivity and the magnetic flux density B can become very high even at low field strength. In an electric circuit one would talk about a highly conductive, low resistance material. If materials with different magnetic conduction are located next to each other in the flow direction (parallel), the material with higher conductivity will carry the larger part of the flux. In two parallel resistors the one with the lower resistance will carry the higher electrical current, and if two parallel layers of iron and air are considered, almost the entire magnetic flux will be focused in the iron, because its μ is considerably higher than 1.

The electrical current passing through a transverse area S is $J \cdot S$, or electrical current density multiplied by the area. Likewise, the magnetic flux is the product of magnetic **flux** density and the area. A scalar product has to be formed if the area is not located transverse to the flux density. If the flux density depends on the location, one has to integrate:

$$\Phi = \int_S \vec{B} \cdot d\vec{S} \qquad \vec{B} = \frac{d\Phi}{dS} \cdot \vec{e}_\Phi \qquad \text{Magnetic flux } \Phi$$

The flux density is the quotient of the flux and of the area it flows through. The flux density vector points into the direction of the Φ-unit vector if the area tends to zero.

The permeability μ is a scalar constant only in very simple cases. μ shows a strong nonlinear dependence on H in most cases for which μ deviates considerably from 1. Very large values of μ can be obtained (above 10000) for small values of H. The material will become "magnetically saturated" with increasing field strength and μ_r decreases. Consequently, the magnetic field can no longer be considered as a linear system, which has far-reaching consequences: **non-linear distortions** emerge, the superposition principle is no longer valid and there is no transfer-function and impulse response. In addition, time invariance can no longer be assumed because the memory of permanent magnetic materials yields a hysteresis: for increasing field strength the flux is different from the decreasing case. Finally, one has to consider that, at least in strong magnets, the permeability becomes orientation-dependent; μ_r will become a tensor in these anisotropic materials.

The material is isotropic and linear for the simplest case. Then μ_r is a constant and the field directions of the $\vec{B}$- and $\vec{H}$- vectors are the same: $\vec{B} = \mu \cdot \vec{H}$. However, an approximately linear description is also possible for a non-linear *B/H* relation (linearization, tangent approximation, Taylor series) for small deviations from linearity. If the amplitude of the signal can no longer be considered as small, an **isotropic/non-linear** model has to be used; in that case μ is defined as an H-dependant series of curves.

For the **anisotropic/linear** model, μ is indeed independent of H, but depends on the spatial orientation (relative to the crystal axes).

$$\vec{B} = \begin{pmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{pmatrix} \cdot \vec{H} \qquad \rightarrow \qquad \vec{B} = \begin{pmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{pmatrix} \cdot \vec{H}$$

The μ-tensor can be simplified by choosing a suitable coordinate system, so that only 3 elements remain. This can be achieved by orienting the coordinate system along the main axes of the material (which is the direction of the largest μ); the other two corresponding μ-values are then smaller and often equal.

Anisotropic/non-linear materials can only be described with enormous effort. For the simple case, every one of the three μ-components is depicted as H-dependent curve or series of curves. However, this case does not include the existence of non-linear couplings between the three spatial directions. An exact modeling most often fails due to imprecise measurements and a too large a diversity of parameters.

Materials with a large μ_r are called **ferromagnetic** because, in most cases, iron (Ferrum) is the root cause for the magnetisability. Cobalt and Nickel as well as some rare earths and special alloys also show magnetic behavior. A single crystal of iron will show anisotropic behavior. Its μ_r yields the largest values in the direction of the cube edge. However, since all magnetic domains are pointing into different directions in the unmagnetised (virgin) state of iron, the macroscopic magnetic field can be considered as isotropic (quasi-isotropy). An anisotropic behavior can be grown using particular production procedures, e.g. cool down within a magnetic field or crystallization on a quenching plate.

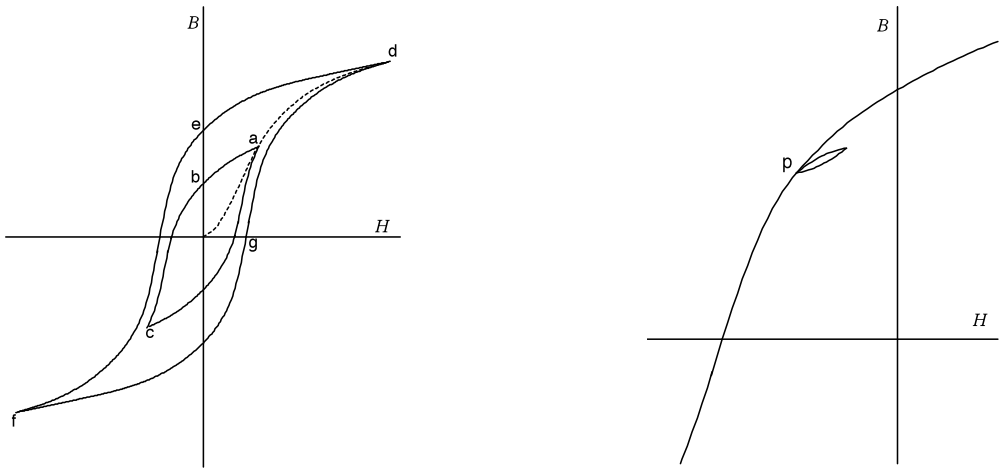

Fig. 4.6: Ferromagnetic demagnetization curves (left).In the demagnetized state, the field strength *H* and the flux density *B* are zero (at the origin). Increasing *H* e.g. towards the point **a**, *B* will increase according to the dotted initial magnetization curve. However, if *H* is set to zero again, *B* will not return to zero but rather to the value at point **b**. Applying a negative magnetic field, one will reach e.g. point **c**, and by reversing the field again point **a**. Further increasing the field, one will reach point **d** via the initial magnetization curve. If now *H* is set to zero a remanent flux density at point **e** will remain. The picture on the right shows reversible changes at very small amplitudes (reversible permeability).

In **Fig. 4.6** the nonlinear relationship between *B* and *H* for a ferromagnetic material is depicted. This so called "**hysteresis**" is not only curved, it also splits into two sections: approaching a certain value of the field strength by increasing *H* (from the left) will result in a smaller *B*-value than by decreasing the *H*-value (from the right). The loop in Fig. 4.6 can only be run through counter-clockwise.

Both the increasing and the decreasing section of the curve converge against a common asymptote for high absolute values of the field strength – the material is magnetically saturated. If the field strength is set to zero from one of these saturation points then a permanent flux density remains at the crossing point with the ordinate axis. This is called the remanent flux density or **remanence**. In Fig. 4.6 the remanence point is depicted by **e**. In order to reduce the flux density to zero a counter field strength must be applied, which is called the **coercivity** or **coercive field strength**. In older literature it is sometimes named the "coercive force". In Fig. 4.6 the coercivity point is the abscissa section of the outermost hysteresis curve and is marked with **g**.

The flux density follows the curves in Fig. 4.6 only if *H* changes monotonically. If *H* is decreased from positive values to the point **p,** as shown in **Fig 4.6** (right), and is successively increased again by a small amount, the return run will not take place on the large section of the drawn hysteresis but rather on the lower part of the slanting branch. The return to **p** will be realized on the upper part of the branch. For very small changes around the working point **p** the branch sections will approximately coincide and their slope will yield the **reversible permeability**. It is not given by the differential quotient of the *B/H* curve, but is smaller (see magneto dynamics).