4.7 The Representation of Magnetic Fields

Magnetic fields are spatial vector fields. Alternatively, to be more precise: the magnetic flux density $\vec{B}$ and the magnetic field strength $\vec{H}$ are vector quantities of a three-dimensional field. In a graphical representation the location, as well as the field quantities, have to be visualized three-dimensionally – impossible on a two-dimensional sheet of paper. Spatial depth can be affected with perspective charts, but distances and angles are very difficult to realize correctly. This is the reason, why cross-sections (and, for special cases, curved areas) are used to draw the field profile. This method is especially suited for parallel-plane or rotational-symmetric fields. For a parallel-plane field, the coordinates only depend on two Cartesian coordinates (x, y). The field of an infinitely long, straight current carrying conductor (Fig. 4.1) is an example if the axis of the conductor is chosen to be the z-direction. However, the same field can also be considered to be rotationally-symmetric with the axis of the conductor to be the symmetry line and with cylinder coordinates (r, φ) instead of Cartesian coordinates. Still there remains the problem of representing the three-dimensional field quantities in this cross-section. Every point in the represented area indeed stands for a point on the cross-section – in fact no space is left to draw the value and direction of the field quantity. So, compromises have to be found and less important information has to be omitted in favor of more important ones. For instance, the field quantity might be depicted only at discrete locations and not throughout the entire (continuous) area. Alternatively, one codes the value of the field quantity by an assigned color and forgoes the direction representation.

4.7.1 Field Strength and Flux Density

The following field representations refer to **two-dimensional fields**. The field quantity is depicted by an **arrow**, whose length characterizes the value and whose direction describes the orientation of the field quantity. Scaling is necessary, e.g. 1 cm ≙ 1 T for the values. The field quantity depicted by an arrow is associated with its root point. This can easily lead to misinterpretations, as the drawing area now has two functions: it represents the position and also the field quantity. The observer is tempted to establish a local relation between the tip and root points of the arrow, although only the root point is assigned to a point on the cross-section. **Fig. 4.22** explains this difficulty with the help of an example of rotating arrows:

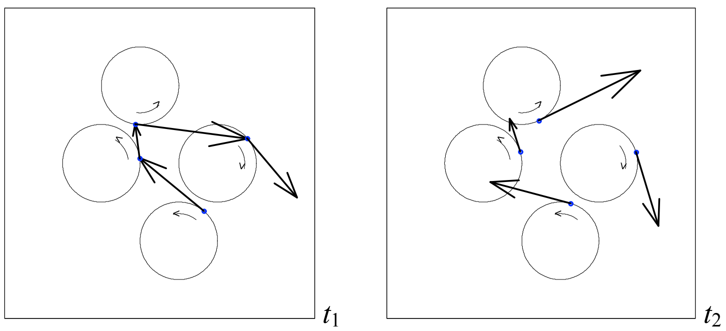

Fig. 4.22: Velocity vectors, drawn at two different times $t_2 > t_1$. In the left picture a connected polygon line is drawn, however, without physical meaning.

The human brain converts optical information into visual impressions. As a result, the immense amount of data is reduced considerably and structured by **shape** laws. Thus, similar objects in close vicinity are combined into higher-level units with smooth gradients. In the left picture of Fig. 4.22 the tips of the arrows point to the bases of the next arrows in each case. The perceived line is, however, irrelevant, as shown by the picture on the right taken at a later point in time. **Fig. 4.23** shows the upper left vectors of the magnetic field strength. An electric current flows into the image plane at the point [0.5, 0.5], yielding a concentric field. All arrows are tangents to a concentric array of circles; however, due to the large distances it is hard to identify these circles. On the upper right picture the conductor has been moved slightly to the upper right resulting in a giant arrow. This is correct because the base of the arrow is now located very close to the wire and the field strength is actually very high – but this representation is not very clear.

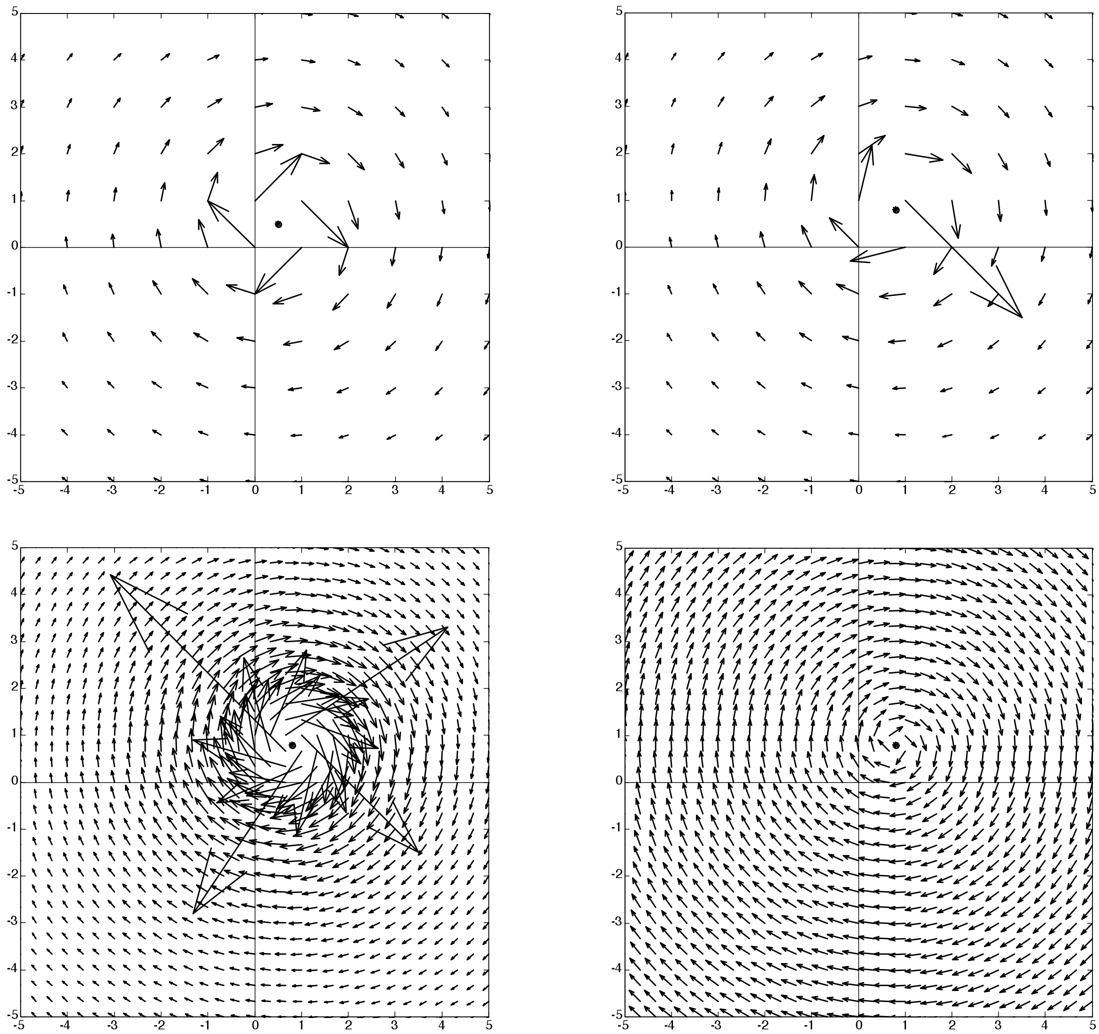

Fig. 4.23: Field strength vectors around a current carrying conductor.

In the lower left picture the density of arrows has been increased in order to increase the resolution of the circles – not a good idea, either. In the lower right picture all arrows are drawn with identical length; the visual impression here is the best but indeed the value information is lost.

If the viewer of the lower right picture of Fig. 4.23 comes to the conclusion that the field is slightly rotated clockwise because there is a slanting characteristic in every picture frame, they perceive an optical illusion: the connection of single arrows to contiguous lines is physically not meaningful for this arrow lattice. Moreover, a rotationally symmetric field cannot be twisted!

Figure 4.24 depicts the field strength vectors of a two-wire conductor. Here, the clarity can also be increased considerably if the value depiction is omitted. If there is a possibility for a color print the value can be shown as colored arrow – on a black and white copy, however, nothing more is visible.

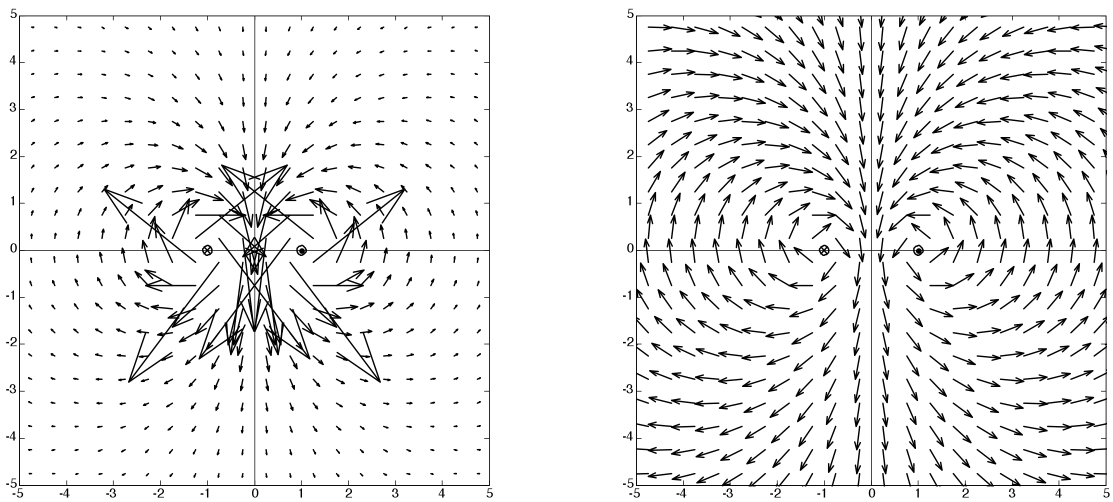

Fig. 4.24: Field strength vectors of a current carrying two-wire conductor (left) and the normalized version (right). In the right image, asymmetries between the upper and the lower part of the picture can be perceived, again based on the misinterpretations shown in Fig 4.23.

In addition to the vector characterization, **field line images** convey a descriptive impression of the spatial field characteristics. Field lines do not show locations of equal field strength – they should not be mixed up with the isobars of a weather chart or the contour lines of a map. Rather a curve will become a field line through the field strength vector $\vec{H}$ defined as tangent vector at every point of this curve. The direction is defined at every point in space as the differential quotient of the field strength. If looked upon geometrically, the integration of this spatial differential equation means the connection of infinitesimally small direction arrows into integral curves, i.e. into field lines (Chapter 4.1).

The field lines of a current carrying conductor are concentric circles. In this simple case one is successful with this equation-analytical description. However, with more complicated real fields, an FEM computation is necessary. **Fig. 4.25** depicts the concentric field: for a current flowing into the image plane the field lines proceed clockwise. The direction of the field strength vector can easily be deduced as tangent to the field lines; however, its value cannot be determined from a field line. Yet an estimate can be deduced from the distance of the field lines: the closer the neighboring field lines, the higher the value of the field. The value is depicted as gray tone on the right of Fig. 4.25, with limited success. The dynamic range that can be presented is not sufficient for a linear relationship; for the 1/r decrease a special color map would have to be defined.

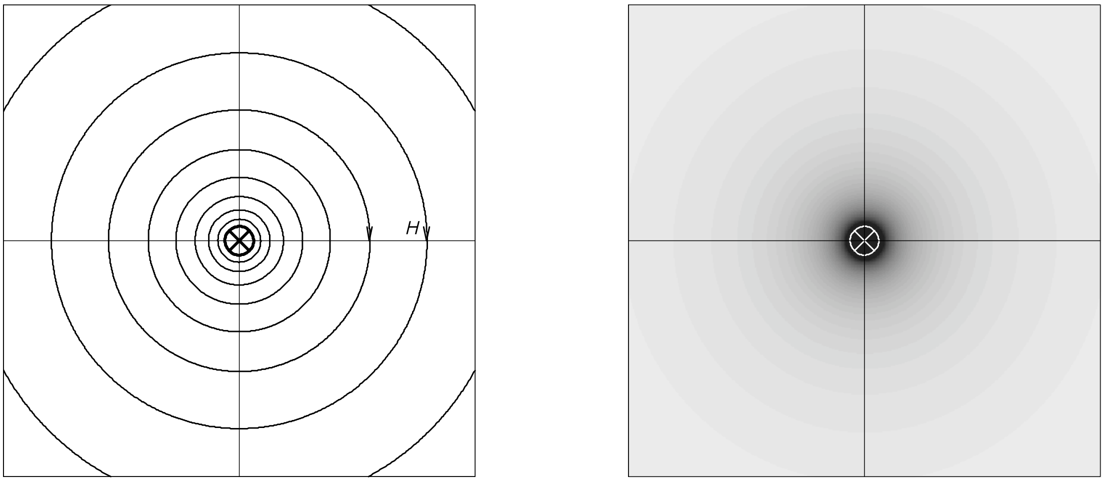

Fig. 4.25: The magnetic field of a current carrying conductor. Left: Field lines. Right: Gray tone coded. The gray tones produced by printers and copiers are not able to resolve the radial field decrease with sufficient precision.

An analytical field-description is also possible for an infinitely long two-wire conductor (**Fig. 4.26**). Assuming opposite current flow, the field lines are eccentric circles with centers located on the x-axis. This could already be inferred from viewing the arrow-description (Fig. 4.24), but in the line description it is obvious. *Between* the wires the field strength is the highest (= maximum line density) and with increasing distance *H* decreases rapidly. A **contour-plot** can be obtained, if all points of equal field strength are connected by lines. This representation is known from maps: A contour line connects all points of equal altitude. However, the expression “contour-line” only means that all points with equal functional values are connected by lines; it has to be specified which value is shown. On an *isobar* the pressure is constant, on an *isotherm* the temperature is constant, for the magnetic field one could call it *iso-field-strength-line*, but this expression is not used. Instead, one talks about **curves of equal field strength**. If one considers curves of equal flux density, sometimes they are called **iso-flux lines**.

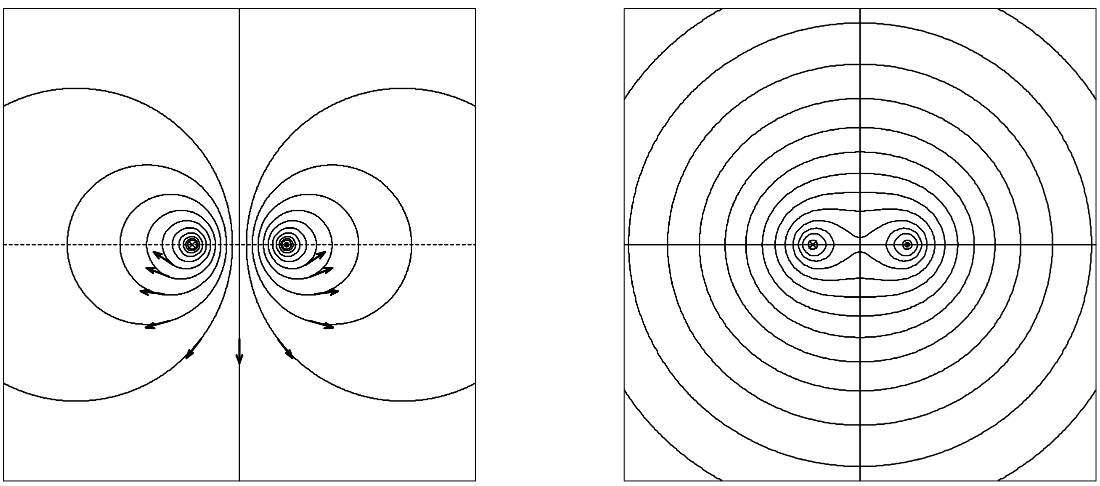

Fig. 4.26: Current carrying two-wire conductor. Field lines with direction arrows (left), contour lines of equal field strength (right). Lines of equal field strength are not equipotential lines (→ Fig. 4.27).