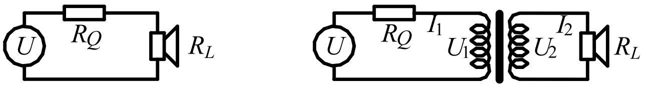
10.6 Output transformer

Typically, your customary power tube will have an optimum load-impedance in the kilo-ohm-range i.e. about 1000 times the impedance of a loudspeaker. If, for example, an 8-Ω-load-impedance were to be connected to a source having an internal impedance of 8000 Ω, then 99,9% of the generated power would be dissipated via the internal impedance, and only 0,1% would arrive at the load-impedance. That is of course not acceptable. Tubes operate at high voltages (400 V) but can digest only small currents (0.2 A). With loudspeakers, the situation is exactly the other way round: a 4-Ω-loudspeaker requires 16 V to take on 64 W, with a current of 4 A flowing through it. The output transformer (OT) has the task to **match** the high-impedance tube circuit to the low-impedance loudspeaker. As a matter of principle, the OT at the same time works as a filter that rejects high and low frequencies, and it generates special non-linear distortion. While the matching function of the OT is relatively easily calculated, the non-linear distortion eludes an exact description. The corresponding models are therefore either inadequate, or not at all readily understood, or both. The following elaborations try to give a clear picture on the basis of specific measurements. For the latter, genre-typical output transformers were used – they do, however, not represent any selected sample-median.

10.6.1 The linear model

Impedances (complex resistances) are only defined within the linear model [20], and therefore the impedance transformation can be calculated only for a linear output transformer. The AC-source is the tube circuit that is assumed to be a voltage-source with a (series-connected) source-impedance R_Q. The load is given by the loudspeaker-impedance R_L (**Fig. 10.6.1**), and both source- and load-impedance taken to be purely ohmic for our first investigations.



$$U_1/U_2 = TR = I_2/I_1$$
$$U_1 \cdot I_1 = P = U_2 \cdot I_2$$

Fig. 10.6.1: AC voltage-source with load-impedance; with & without an ideal matching transformer.

The transformer shown here is of **ideal** characteristics, and completely described by the two equations given above; $TR = N_1/N_2$ is the **turns-ratio**, also termed **transformer-ratio**. The windings shown in the schematic therefore must not be interpreted as inductances but have a purely symbolic character. The **idealization** mentioned above may be in sharp contrast to reality: the ideal transformer can transmit DC – something impossible for a real transformer. For our first forays into transformer-land, this discrepancy is not a problem – we can (and will have to) expand the **model** as needed. According to the idealization, the transformer is also loss-less: $U_1 \cdot I_1 = U_2 \cdot I_2$. In the interior, energy is not stored, nor dissipated into heat. This is another difference to the real transformer: its windings do generate heat – which is not (yet) considered in this simple model. The latter is not able to simulate the non-linearity (magnetic hysteresis) caused by the iron core, and the same holds for winding capacitances and leakage flux. All these specific characteristics will need to be incorporated in a realistic model, and we can already now anticipate how complex this is likely to become.

 Translated by Tilmann Zwicker

The power matching, on the other hand, may very well be shown using the ideal transformer: the source (voltage-source with source impedance) “sees” as load the input-impedance R_E of the output transformer (**OT**):

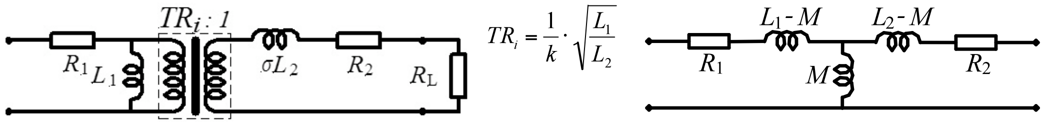
$$R_E = \frac{U_1}{I_1} = \frac{U_2 \cdot TR}{I_2 / TR} = \frac{U_2}{I_2} \cdot TR^2 = R_L \cdot TR^2$$ Impedance-transformation

The secondary load-impedance (R_L) is mapped (transformed) via the OT into the primary input-impedance of the OT. If this input-impedance R_E is very small relative to R_Q, the major part of the power is fed to R_Q, and not to R_L. Conversely, if R_Q is large, almost all power is fed to R_L, but due to $P \sim 1/ R_E$, this power becomes smaller as R_Q becomes larger. Therefore, equal internal- and load-impedance is often sought as an **optimum for matching**: $R_Q = R_E$. With internal impedance and load-impedance known, the transformer-ratio can easily be calculated from this simple condition: $TR = \sqrt{R_Q / R_L}$. Given R_Q = 7200 Ω and R_L = 8 Ω, we get, for example, a transformer-ratio (turns-ratio) of *TR*= 30 (tube amplifiers Chapter. 10.6.2).

So, how exactly does the output transformer accomplish this transformation, how does it generate the secondary quantities from the primary ones? This is done via the magnetic coupling of two windings the turns-ratio of which corresponds to the transformer-ratio TR. The primary current I_1 flowing through the primary coil generates a **magnetic field** that, in an ideal transformer, entirely permeates the secondary winding and induces the secondary voltage U_2. If the transformer has a load coupled to its secondary winding (as it is normally the case), there is also a current in the secondary circuit that itself generates a magnetic field entirely permeating the primary winding (in the ideal transformer) and inducing a voltage there. Both coupled processes (current → field → voltage) can and need to be superimposed; this is the basis for the calculation of the general case [4, 7, 17, 18, 20]. However, a wire configured as a winding needs to be represented in the **equivalent circuit diagram (ECD)** at least via a resistor (copper-resistance) and an inductance (magnetic field) – which leads to a first extension of the ideal transformer-schematic. Since the magnetic coupling of the two windings is an indispensable basis, it needs to find its way into the transformer-ECD, too. How this ECD is derived from the physical interrelations shall not be elaborated here explicitly – extensive literature already exists for this (see above). Basically, the real transformer can be represented by a special ideal transformer and several supplementary two-poles. The special ideal transformer is fully described by its transformation ratio *TR*, and what has been stated in Fig. 10.6.1 does hold for it. The supplemental two-poles approximately model the characteristics in which the real transformer differs from the ideal one. Still: these are approximations the applicability of which needs to be checked in each individual case.

The most important characteristics modeled by the supplemental two-poles are: resistive losses, inductances, and flux-leakage. Losses are due to the copper wire and the magnetic core, inductances result from (coupled) windings, and flux-leakage happens because, in the real transformer, not the whole magnetic flux generated by one winding permeates the second winding, but a part misses it. The **leakage-factor** σ defines the extent of the flux-leakages; alternatively, the **coupling-factor** $k = \sqrt{1-\sigma}$ can be given. A leakage-factor of σ = 0% corresponds to complete coupling (= ideal tight coupling), while a leakage-factor of 100% indicates non-coupled windings. There are different equivalent circuit diagrams; the individual factor *TR* may deviate from the physical turns-ratio.

Two of the most important ECD's are shown in **Fig. 10.6.2**. R_1 and R_2 represent the ohmic components of the winding-impedances and model the copper-resistances. L_1 and L_2 are the inductances of the primary and the secondary windings, respectively. For a secondary open-circuit, the measurement of the primary input-impedance yields $R_1 + j\omega L_1$. For a primary open-circuit, the measurement of the secondary output impedance yields $R_2 + j\omega L_2$. The inductance designated M in the right-hand ECD is the **mutual inductance**. The following relationships hold: $M = k\sqrt{L_1 L_2}$, $k = \sqrt{1-\sigma}$, $TR = N_1/N_2 = \sqrt{L_1/L_2}$.



$$TR_i = \frac{1}{k}\cdot\sqrt{\frac{L_1}{L_2}}$$

Fig. 10.6.2: ECD's for transformers. The transformer in the ECD on the left is ideal (and thus free of inductances). The inductances in the ECD on the right may become negative; this does not restrict the validity.

Besides the three ohmic resistances that can be easily determined from a DC-measurement, the ECD holds **three degrees of freedom**: L_1, L_2, and k. L_1 and L_2 may be ascertained e.g. via an impedance-measurement with contra-lateral open circuit. The coupling-factor can be determined with contra-lateral short-circuit. Measuring the primary DC-resistance R_1 of the OT is most unproblematic, while regarding the secondary resistance we need to bear in mind that it may by of very small magnitude (possibly $R_2 < 0.1\Omega$). When measuring the inductance, the fact that the ECD mentioned above has only limited applicability in practice requires consideration: stray- and winding-capacitances influence the impedance, as well (Fig. 10.6.4).

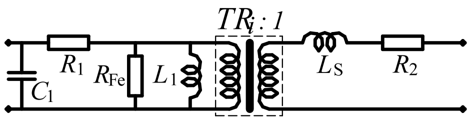
In both ECD's given in Fig. 10.6.2, the inductance in the parallel branch will short any DC voltages – the result is a **high-pass**. Accordingly, the parallel inductance needs to be as large as possible in order to allow for low-frequency operation. The inductance rises approximately with the square of the turns-number of the winding, and therefore a winding with a high turns-number would be desirable – however, this brings along mounting copper-resistance, and correspondingly increasing losses. To keep the copper-resistance low, the cross-section of the deployed wire needs to be large – requiring the dimensions of the transformer to be large, as well. **Simple conclusion: transformers that handle high power and low frequencies need to be large**. For the selection of the cross-section of the wire, the **current-density** supplies a first step of orientation: given an RMS primary current of 0.11 A, a 0.2-mm-wire would be suitable for 3.5 A/mm^2. The latter value is just for orientation: for large transformers, somewhat smaller current-densities will have to be assumed, especially if the surrounding air is heated up by the tubes. The current I_2 flowing in the secondary winding is larger than the primary current I_1 by the factor of TR; however, the secondary turns-number is $1/TR$-fold smaller than the primary turns-number; the product of current-strength and turns-number therefore is the same for primary and secondary winding. This holds at least for the ideal transformer – in real transformers there are small deviations that may, however, be disregarded for a first consideration. Given equal current-densities for primary and secondary winding, it follows from the equation $I_1N_1 = I_2N_2$ that the **cross-sectional areas of the windings** should be equal for both windings. The total cross-sectional area of the winding (amounting to e.g. 2.2 cm^2 for the M55-transformer) therefore is made available with 50% each to both primary and secondary winding. Depending on the application, transformers need to meet certain requirements, for example with a proof-voltage of more than 1000 V (and corresponding supplementary insulation layers), or a special low-capacitance winding (with different build), or additional taps (requiring more contact wires and thus space). This shows that transformers may have manufacturer-specific differences that are not obvious at first glance.

The M55-transformer cited as an example has a winding-surface of 2.2 cm^2 i.e. 1.1 cm^2 per winding. This value must, however, not be simply divided by the cross-sectional area of the wire because wire-insulation and -spacing also require space. Nevertheless, it should just about be possible to accommodate 2000 turns of 0.2-mm-wire. Applying the current (e.g. 0.11 A) as calculated from the current-density yields a magnetomotive force of 220 A, and a magnetic field-strength of 1.7 kA/m (as a first-order approximation). From a thermal point-of-view, this may be o.k. – from a communication engineering point-of-view, it is not: the materials normally used for cores in transformers are all but "saturated" at such high field-strengths, and the magnetic flux cannot increase anymore if the field-strength is further increased. Strong non-linear distortion would be the result. Schröder recommends in Vol. 1 of his book *Elektrische Nachrichtentechnik* a maximum magnetic field-strength of 0.1 kA/m. Consequently the overdrive found in our above example would be massive. Alternatively, the **maximum magnetic flux-density** could also be calculated:

$$\hat{B} = \frac{\sqrt{2} \cdot \tilde{U}_1}{2\pi f \cdot N_1 \cdot A_{Fe}}$$

Peak value of the magnetic flux-density.
N_1 = primary turns-number,
A_{Fe} = cross-sectional area of iron.

It is clear from the reciprocal dependency on frequency that, for a primary voltage U_1 sourced form a stiff voltage-source, the flux-density decreases with increasing frequency – therefore problems may result in particular for low frequencies. We will get back to the non-linear behavior in Chapter 10.6.4; first, the behavior for small drive-levels is under scrutiny. The (linear) ECD's introduced in Fig. 10.6.2 enable us to approximately describe impedances and transmission behavior of an output transformer. In the higher-frequency region, however, noticeably deficits remain because capacitive coupling among the windings and iron losses are not considered yet. Strictly speaking, every differential section of the winding is capacitively coupled to every other section, but a *single* **substitute capacity** is sufficient to model this infinite number of coupling capacitances. The **iron losses** (hysteresis- and eddy-current-losses) may be modeled via a resistor with good approximation, as well, and an extended equivalent circuit diagram shown in **Fig. 10.6.3** represents a good compromise between complexity and accuracy. Calculations with the approximation $TR_i \approx TR$ are always acceptable: the transformers considered here rarely have a leakage-factor of in excess of 1%.



C_1 = capacitance of the winding,
L_1 = primary inductance, $L_S = \sigma \cdot L_1 / TR^2$
R_1, R_2 = copper-resistances,
R_{Fe} = iron losses, $TR_i = TR / \sqrt{1-\sigma}$
L_S = leakage inductance.

Fig. 10.6.3: Equivalent circuit diagram of transformer♣ (linear model). Non-linear behavior: see Chapter. 10.6.4.

Fig. 10.6.4 shows comparisons between measurements and calculations carried out on the basis of the above model. Since all these transformers are used in push-pull output stages, the respective primary winding is divided in two halves. Calculation and measurement was respectively done for one half of the primary winding. For secondary open-loop operation, the primary impedances of the two winding-halves are practically identical; there are differences for secondary short-circuit, though – these are due to different coupling of the windings. For low-impedance loading (i.e. for loudspeaker-loading, as well) the push-pull drive-signal therefore is not symmetrical anymore in the higher-frequency region.

♣ The capacitance may also by connected on parallel to L_1; the differences are small.

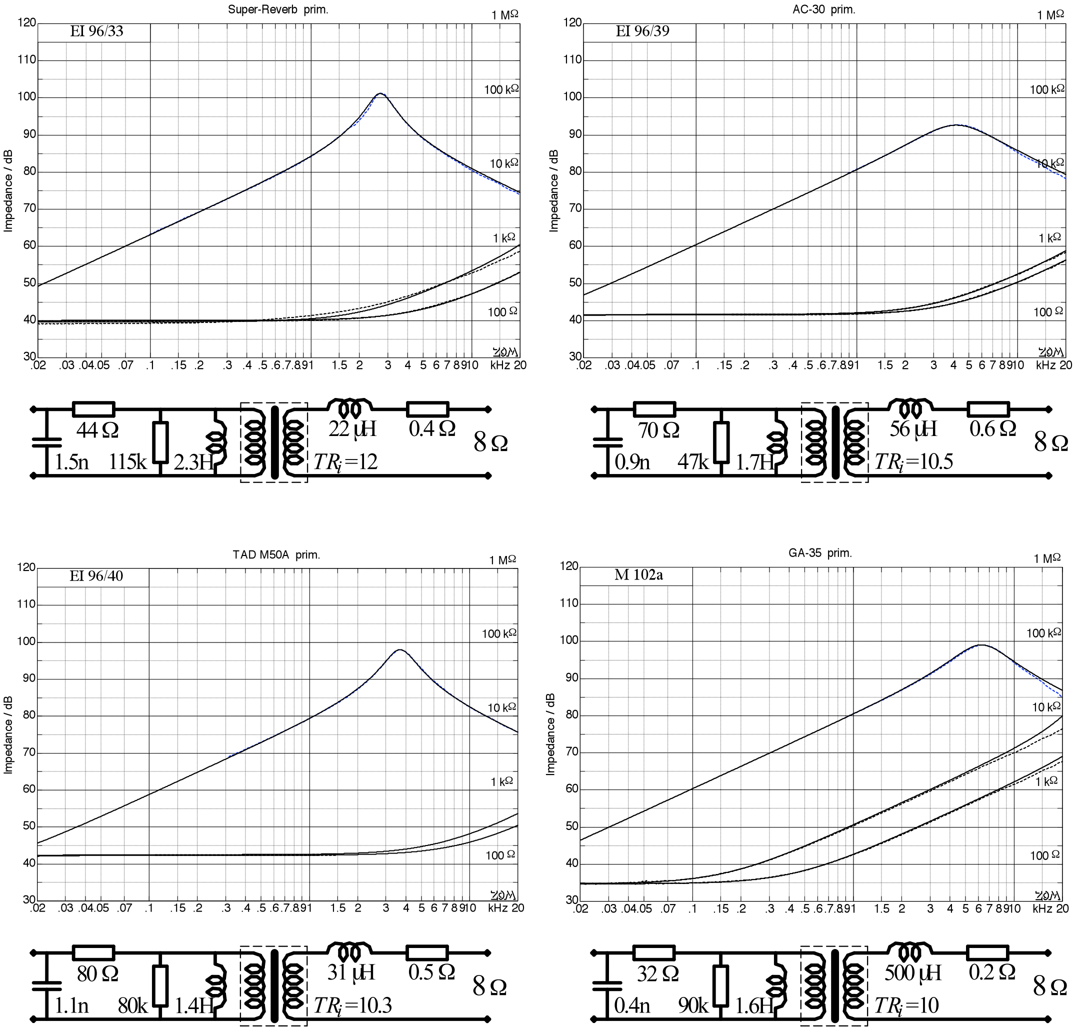


Fig. 10.6.4: Comparison of impedance measurements (------) and model calculations (———), each for one half of the primary winding (R_a). The two open-loop impedances are practically identical; the short-circuit impedances differ due to different coupling-factors.

Measurements and calculations in Fig. 10.6.4 are practically identical over a wide range but there are some sections in which differences become apparent. In principle it would not be difficult to extend the model by a few further components such that a good correspondence would be achieved across the whole frequency range. However, in the interest of general applicability, the ECD as developed above shall remain unchanged. The divergences are rather limited, anyway.

We can also see from Fig. 10.6.4 that – at least for the transformers investigated here – the ECD is well suited to model the primary load-impedance (i.e. the strain on the power tubes) **for linear operation**. However, output transformers work linearly only for *very* small output power, typically $P < 1$ mW. For your regular output power, the **parallel inductance** (L_1), in particular, depends very strongly on the drive-level. As simple as the linear equivalent circuit diagrams are, their applicability still remains strongly limited. For this reason, Chapter 10.6.4 will elaborate more extensively on the non-linear behavior.

 Translated by Tilmann Zwicker