4. The Magnetic Field

Macroscopic magnetic effects were already known in ancient times: Magnetite attracts iron particles. This force effect can be described by a vector field in which a defined field intensity is assigned to every point in space, characterized by its strength and direction. Every magnet produces a magnetic field in its vicinity which decreases rapidly with distance. Energy conservation is of course valid: No energy needs to be added for retention of the field (!). If there is a displacement of a piece of iron by the magnetic force, mechanic energy is "gained"; at the same time the magnetic field is weakened. If, conversely, the piece of iron is detached from the magnet, the same amount of energy has to be added which will increase the energy of the field accordingly.

Materials which generate and sustain a permanent magnetic field are called **Permanent Magnets**. This characteristic is predominant for Magnetite (Fe_3O_4) and some other metals. The root cause of the magnetic field is electrons moving around the atomic nucleus and their own intrinsic spin. According to the Bohr-Rutherford atomic model, electrons move in stationary orbits without energy dissipation but produce a magnetic field. A more or less intense magnetic field effect evolves in macroscopic space according to the direction and strength of these fields and the coherence effects of neighboring atoms.

In the same way an electric current flowing through a wire will produce a magnetic field. This field will further increase if the wire is wound to form a coil. However, contrary to the permanent magnet, the electromagnetic field disappears if the current is switched off. The name of this kind of magnet is derived from its operational principle: **Electromagnet**. Permanent magnets and electromagnets have the same effect. Both produce magnetic fields and forces on iron and similar metals. Energetically there seems to be a difference. A permanent current flow is necessary in order to sustain the magnetic field for the electromagnet, which means that energy needs to be supplied. However, one has to distinguish between the one-off portion of energy which is needed to build up the field and the continuous supply of energy which heats up the wire (current × voltage × time = energy). In an ideal conductor (superconductor), the magnetic field could be sustained permanently without the continuous addition of energy.

In addition to the force effect of magnetic fields, there is also the effect of **Magnetic Induction**. A change of the magnetic field over time produces (induces) a voltage in a wire coil. This effect is exploited in a magnetic **Guitar Pickup,** in which a vibrating steel string changes the magnetic field of a permanent magnet, inducing a voltage in the coil of the pickup. Knowledge from several areas is helpful to understand the principles of the pickup, in particular *Magnetostatics,* which describe the stationary magnetic circuit (magnetization of the string), *Magnetodynamics,* which describe time-variant changes in magnetic fields (induction effects), and the *Two-port* and *Systems Theories* which are needed to describe the transfer behavior as a function of frequency. The following chapters will introduce these three disciplines in detail.

4.1 The Basics of Magnetostatics

We will start the following considerations with an electromagnet because the causal relation between field-generating current and resulting magnetic field are clearly visible. Electromagnets do not play any role for pickups, but the results that are obtained can be easily transferred to the permanent magnets which are used in pickups.

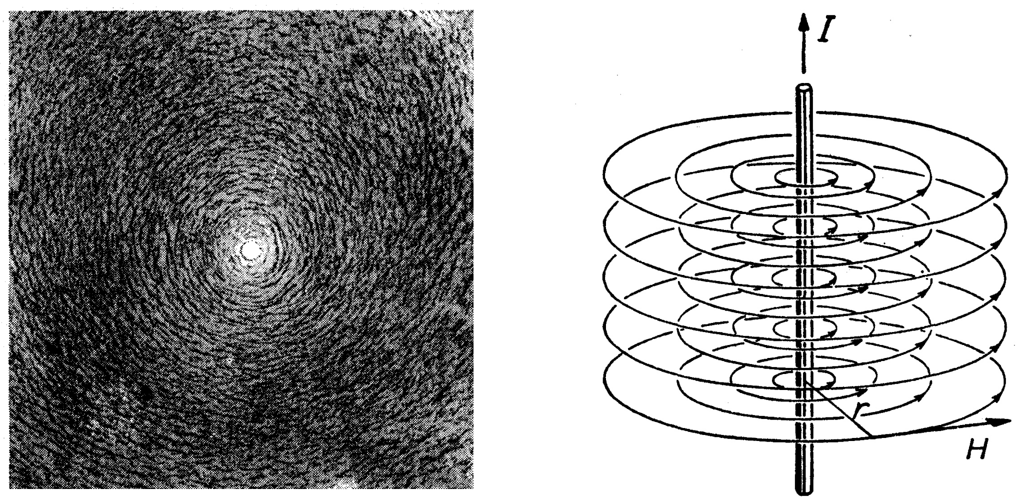


Fig. 4.1: The magnetic field surrounding a current carrying wire for iron filings (left) and field lines (right); [18, 19].

When an electric direct current flows through a very long, straight wire, a circular magnetic field is generated around it. The effect of the magnetic field can be visualized by elongated iron filings which are introduced into the space surrounding the wire. The filings line up into circular lines, concentrically wrapped around the wire (**Fig. 4.1**). In this visualization method, the circular lines are not perfectly aligned but easily recognizable by eye. Using the iron filings, a method to visualize the invisible magnetic field had been discovered. The lines marked by the iron filings (circular in this example) were designated **field lines**. The magnetic field does, of course, not only exist within the field lines, rather it fills the entire space. The line-like description is a discrete visualization of a (continuous) vector quantity equally distributed in space.

The evolution of circular structures has two origins. The elongated filings are oriented in a tangential direction by the magnetic field (normal to the position vector) and they arrange themselves into groups connected together at their ends. Iron filings are a good medium to visualize the effects of the otherwise invisible magnetic field. However, an exact quantitative description of the field is not possible with this method. Nevertheless, the empirically deduced circular form is the basis of an abstract analytical description of the field, called the **magnetic field strength** $\vec{H}$. The word "magnetic" is sometimes omitted if no confusion with the electrical field strength is possible.

In the example of a long wire with current flow in one direction, the vector of field strength points in the direction of the field lines, tangential to the circles or normal to the position vector. The value of the field strength vector decreases proportional to 1/r with increasing distance. However, before we start with the exact calculation we must first define the reference systems.

The magnetic field is a **vector field** and the descriptive field-parameter $\vec{H}$ has a value and a direction. Not every field has a vector character. For example, a spatial temperature distribution is described by a **scalar field** with every point having a value but no direction. The **direction** of a vector is given by an angle deviation with respect to a **reference system**. In a two-dimensional scheme **polar coordinates** are particularly suited for the description of the direction. The direction of a vector originating from zero is defined relative to the abscissa, with angle deviations being counted positive in the counter-clockwise (CCW) direction. The spherical coordinates are defined in a similar way in three dimensions. The definition of the positive CCW direction fits into other coordinate systems (Cartesian, complex e-function and Euler) but, ultimately, it is arbitrary: coordinates based on the clockwise direction would also be possible. However, once the **sense of direction** is defined, it has to be maintained throughout the following considerations. The direction of a magnetic field, i.e. the direction of the magnetic field vector, is defined by the tangent to the magnetic field line at every point in space. A tangent, however, is a straight line and not a ray. Consequently, there are two possible reference directions 180° to each other.

The directional reference system for magnetic fields valid today has an historical foundation. It is derived from the needle of a compass. The **Earth** is a huge permanent magnet, producing a weak magnetic field between the North and South Poles. If a compass needle (a little bar-shaped permanent magnet) is suspended so that it can move without restriction, the magnetic force will turn it to be parallel to the field lines. The part of the compass needle that points to the geographic North Pole was defined as magnetic north pole of the compass needle. At the same time it was deliberately defined that the field lines emerge from this **Magnetic North Pole**. This definition, however, yields that the geographic North Pole[§] must be a magnetic South Pole! In the following the North Pole is always the Magnetic North Pole. As for the relationship between current and magnetic field direction we also have to define direction conventions. In metallic conductors the term current flow designates the flow of free electrons (electrical current = charge displacement over time). The direction <u>opposite</u> to the electron flow is called the **technical current direction** (plus to minus within the electrical load). In graphical representations this technical current flow direction is often depicted by an **arrow**. The relationship between the above mentioned current and magnetic field direction can readily be visualized with the **right hand rule**: if the thumb points into the direction of the current flow the other (fisted) fingers will point in the direction of the circular magnetic field.

The field lines of an infinitely long straight conductor are concentric circles centered on the axis of the conductor. This field is called **parallel-plane**, because the same circular field line schemes will evolve on all planes which are in parallel to each other. The computation of this simple scheme is easy but has one disadvantage in that it does not exist in reality because an infinitely long conductor does not exist. Real magnetic fields can have considerably more complicated structures, which can usually be described by, mostly rough, approximations. Finite element modeling (FEM) programs, that divide the fields into small sections, may provide solutions, but will soon reach their limits in the case of fields relevant in, and around, pickups. In the following chapters we will first describe the basic relationships in an idealized manner. The particularities of pickups will be addressed at the end.

[§] Between the Geographic North Pole and the Magnetic South Pole there is a distance of about 1400 km. In central Europe the magnetic field lines have an inclination angle of approx. 65° with reference to the surface. The magnetic flux density is approx. 45μT.

The magnetic field originating from a single wire carrying a current is relatively weak. A strong magnetic field emerges, by superposition (addition) of the individual fields, only if the wire is wound to a coil. The superposition principle is depicted in **Fig. 4.2**. In this case, we have two parallel wires with an equal *amount* of current flowing in *directions* that are opposite to each other. Usually the *technical* current flow is defined from positive to negative. In the cross section, a current flow into the picture plane is represented by a cross ⊗, and the opposite direction out of the picture plane towards the viewer's position is marked by a circle with a point ⊙.

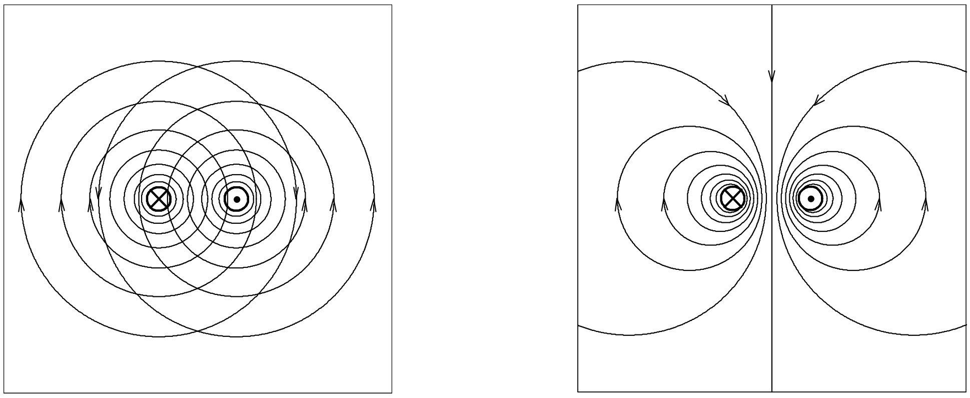


Fig. 4.2: Magnetic field of two parallel wires; anti-parallel current direction. Individual fields (left), superposition (right)

Every wire produces a circular magnetic field propagating with the speed of light. The delays related to the propagation speed are virtually negligible for the small dimensions of a pickup (< 10 cm) and the low frequencies (< 20 kHz), Thus, a quasi-stationary magnetic field can be assumed and no magnetic wave equations are necessary. The magnetic fields of both wires have to be vector-added at every point in space resulting in the eccentrically circular lines. Instead of the "superposition of magnetic fields," we can also speak of the "vector summation" of the magnetic field strength originating from both wires. However, this superposition is only valid for a **linear system**. Air permeated by a magnetic field has linear characteristics, iron has not. First, we will address the linear systems.

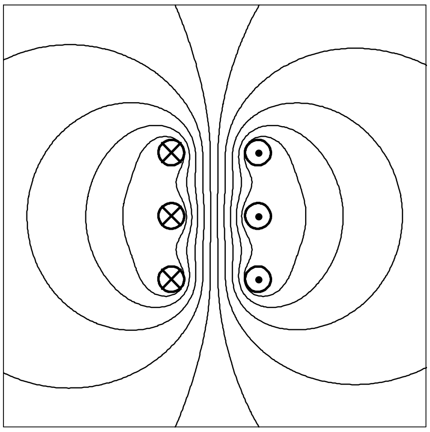
The magnetic field around a current carrying conductor has some characteristics which can be immediately and intuitively understood. It is proportional to the current, decreases with increasing distance and has rotational symmetry with respect to the conductor. Formally the scalar value of the vector of the magnetic field at the point of measurement can be determined by

$$H = \frac{I}{2\pi r},$$

The magnetic field strength outside a straight conductor

in which H is the scalar value of the field strength, I is the current strength and r is the shortest distance of the measuring point to the conductor axis. The formula is only valid for the space outside an infinitely long straight conductor. Again, it should be stressed that, in the case of two wires (Fig. 4.2), the scalar values cannot simply be added. Rather the field strength has to be a *vector* sum. If, for example, two equally large field vectors are normal to each other, the scalar value of the total field strength is not doubled, but only increases by a factor of $\sqrt{2}$.

The scalar value of the magnetic field strength can be increased if the current is increased or if several wires are acting together. **Figure 4.3** depicts several parallel current carrying wires. It is clearly visible that the field lines in between the wires are focused into a channel. A similar, but not identical picture can be obtained, if *one single* wire is wound up into three screw-like coils.



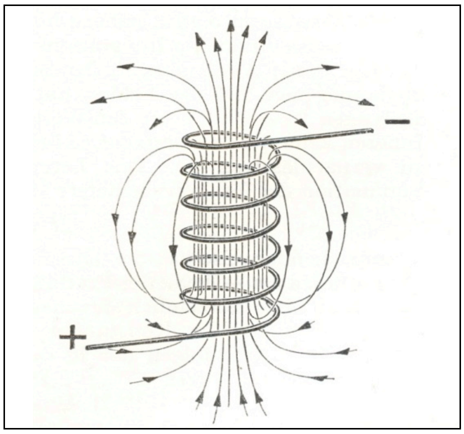


Fig. 4.3: Cross section through the spatial magnetic field of 6 current-carrying parallel wires (left). The spatial magnetic field of a current carrying coil (right) [19].

In Fig. 4.2 and 4.3 the field lines are used as visualization of the invisible magnetic field. The tangent to the 3-dimensional **field line** marks the *direction* of the field strength and the distance between the drawn field lines marks the *magnitude* of the field strength. The shorter the distance between neighboring field lines, the higher the magnetic field strength. The scaling factor can be chosen deliberately: Whether, the lines are drawn, e.g. with a distance of 1 mm or 5 mm for a magnetic field strength of 500 A/m, only depends on the clarity of the description of the total field distribution. The real magnetic field is of course not restricted to the drawn field lines but is continuously distributed in space.

Thus, the field lines do not represent points of equal field strength, so they should not be confused with the isobars of a weather chart or the lines of a contour map. Rather, a curve becomes a field line because the vector of the field strength $\vec{H}$ is a tangent vector on every point of the curve. The *direction* of the field is defined for every point in space by the differential quotient of the vector of field strength. From a geometrical point-of-view, the integration of this spatial differential equation represents the connection of differentially small direction arrows into integral curves, i.e. field lines.

Field lines are curves of equal field strength only in very simple cases such as in Fig 4.1. In general, the value of the field strength changes when moving along a field line. Thus, it stands to reason to examine the line integral over $\vec{H}$ because the field strength is a line-specific quantity. Calculation of the line integral means following a field line and integrating the product of the field strength and the differential (small) line-length ds. The field strength is the tangent vector to the field line along the line and, therefore, $\vec{H}$ is always parallel to ds. The quantity which is calculated by the line integral is called the **magnetomotive force *V*** in analogy to the case of the electric field. If one chooses to evaluate the line integral not along the field line, but on a general space curve, one has to calculate the scalar product of $\vec{H}$ and $d\vec{s}$.

In contrast to electric field lines, magnetic field lines do not have an origin and an end. In most cases they form closed loops, but infinitely long complex space curves are also possible. The integral along a closed loop field line, called the **contour integral,** yields the **magnetomotive force**. This force corresponds to the electric current confined by the field line, in other words the source of the magnetic field. This relationship can be easily seen in Fig. 4.1: In an infinitely long wire in which a current I is flowing the field strength at a distance r from the wire is $H = I\,/\,(2\pi r)$ and the contour integral along the circumference of a circle with radius r yields I.

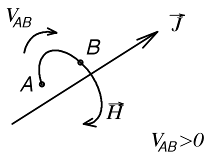
Even in the case that the contour integral does not run along a field line, but along an arbitrary *closed* path in space, its value represents the enclosed current. In this case the scalar product has to be applied, since the field strength vector is not necessarily pointing in the direction of the closed loop. The current passing through the area defined by the contour path is given by the surface integral of the **current density $\vec{J}$**. This surface integral is called the **magnetic flux Θ**. With it, it is possible to establish a relationship between the electrical origin $\vec{J}$ of the field and the magnetic effect $\vec{H}$:

$$\Theta = \int_S \vec{J} \cdot d\vec{S} = \oint_s \vec{H} \cdot d\vec{s}$$

Magnetic flux law (Laplace Law)

In this equation $\vec{J}$ is the vector of current density (Amperes/Square Meter)[§], $\vec{H}$ is the vector of magnetic field strength (Amperes/Meter). The flux passes through a surface S defined by the contour line s. $d\vec{s}$ is an infinitely small linear element of this contour line; $d\vec{S}$ is an infinitely small surface element of the entire surface delimited by s. The surface element is defined as a vector: The scalar *magnitude* of this vector is the surface area; its *direction* is perpendicular to the surface element. The product of the vectors is the **scalar product**, which is defined as the product of the vector *magnitudes* multiplied by the cosine of the angle between the vectors. The circle on the integral symbol indicates that integration has to be carried out along the *closed* curve s, i.e. a *contour integration* needs to be applied. If the contour integral does not run along the entire (closed) circumference, but rather along a curve between two points A and B, one obtains the **magnetomotive force V**:

$$V_{AB} = \int_A^B \vec{H} \cdot d\vec{s}$$



Magnetomotive force

The magnetomotive force is derived from a scalar product and is, consequently, a scalar. Scalars do not have a direction but they do have an orientation (also called direction character). $V_{AB} = -V_{BA}$ is valid. Most often the orientation is depicted by an arrow: The sign of the magnetomotive force is positive if the potential (4.2) decreases with the direction of the arrow; in this case the direction is identical with the magnetic-field-strength direction. If one points with the thumb of the right hand into the $\vec{J}$ - direction (technical current direction), then the bent fingers point into the V-direction.

[§] Sometimes J is also used for the polarization or the magnetic dipole moment – this is not meant here. In addition the current density is sometimes called j; j is used for $\sqrt{-1}$ here.