4.6 The Magnetic Circuit

Magnetic fields permeate the whole space (Chapter 4.1). As they are invisible, one depicts their distribution with field lines and performs modeling in analogy to flowing (immaterial) currents. In contrast to electrical field lines, magnetic field lines do not have an origin and an end. As a rule (for which exceptions are possible), they are closed lines with limited length. The tangent to a field line is oriented in direction of the flux density propagation; vertical to it is the penetrated unit area. The corresponding descriptive **field quantities** are the (magnetic) field strength $\vec{H}$ and the (magnetic) flux density $\vec{B}$. The field strength is a length specific quantity (unit A/m) which is determined along its length direction (direction of the flux), the flux density is an area based quantity (Vs/m²), which is specified for the penetrated area.

The line integral along a space curve over $\vec{H}$ yields the magnetomotive force V, the area integral over $\vec{B}$ yields the magnetic flux Φ. If the formula symbol V is already used for volume, the magnetomotive force may also be denoted by V_m. For an infinitesimally small volume element, the differential material quantity **permeability** $\boldsymbol{\mu}$ depicts the relationship between the differential field quantities of field strength and flux density: $\vec{B} = \mu\vec{H}$. In the macroscopic region the integral field quantities magnetomotive force and flux are related by the **magnetic resistance** $\boldsymbol{R_m}$:

$$V = \Phi \cdot R_m = \Phi / \Lambda \qquad \text{Hopkinson's Law}$$

In contrast to the electric field there is no "magnetic insulator", even vacuum has a non-zero permeability (μ_0). The magnetic resistance is also called the **reluctance**. As the formula symbol R is also used for the electrical resistance, an index m is added sometimes. The reciprocal value of the magnetic resistance is the **magnetic conductance** $\boldsymbol{\Lambda}$. Here, no confusion is possible (the electric conductance is Y), so there is no index m. The magnetic conductance is also called permeance and, sometimes, the formula symbols P or λ are used instead of Λ.

A **magnetic circuit** is defined in analogy to the current circuit, which naturally does not have to be a circle. In fact, the magnetic flux flowing ("circling") around the closed field lines is what is meant. The impetus and cause of the magnetic flux is the magnetic **source**, e.g. a current-carrying wire or a permanent magnet. The boundary of the source is sometimes clearly visible (e.g. the surface of a permanent magnet) but sometimes chosen rather arbitrarily (e.g. a permanent magnet with pole pieces) or does even not exist: The external field of a current-carrying wire runs completely in air, which means outside the source. The part of the flux that is defined as flowing inside the source flows through the **source resistance** (source reluctance), the part flowing outside the source flows through the **load resistance** (load reluctance). In general, each of these resistances consists of sub-resistances, only in the simplest cases there is only *one* source with *one* source resistance and *one* load (**Fig. 4.15**).

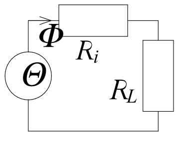


Fig. 4.15: Source with magnetomotive force Θ, source resistance R_i, load resistance R_L and flux Φ (cf. chapter 4.1).

A basic example is the horseshoe magnet (**Fig. 4.16**). Simplifying, one assumes that the magnetic flux is restricted to the magnet itself, to both air gaps and to the yoke (with no leakage flux). The internal, air gap and yoke resistances are successively penetrated by the same flux and are, therefore, represented by a series connection in the equivalent circuit diagram. The magnetic **potential drops** associated with every resistance will, when summed up, result in the magnetomotive force.

A division of the flux into two parallel resistances is necessary if one would like to consider that part of the flux, the **leakage flux**, bypasses the yoke (**Fig. 4.17**). The leakage flux is, of course, spatially distributed and the "channeled" representation in the equivalent circuit is a simplification. If, in addition, one would also like to consider the leakage flux in the magnet, one has to divide up the source (**Fig. 4.18**). This is also a simplification and, if necessary, more than two part sources and more than two part resistances have to be taken into account. **FEM-programs** divide the field structure into thousands of small cells (elements) which are then the basis for the numerical computations of flux and resistance, as well as for more accurate representations of the field lines.

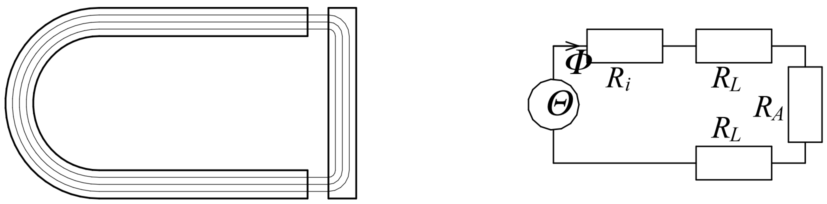


Fig. 4.16: Horseshoe magnet with yoke and no leakage flux.

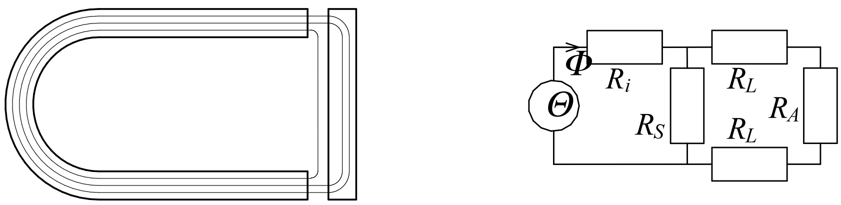


Fig. 4.17: Horseshoe magnet with yoke and load leakage fluxes (schematic).

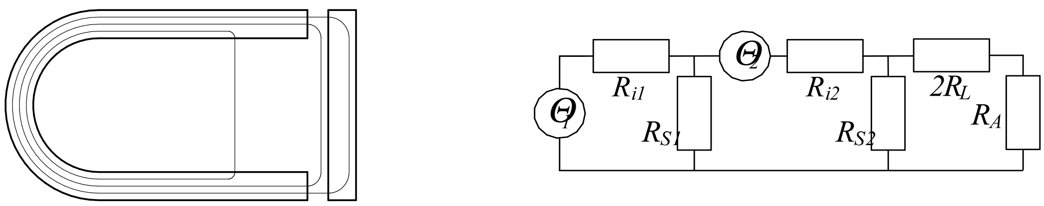
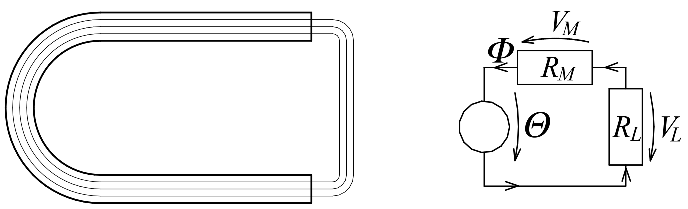


Fig. 4.18: Horseshoe magnet with yoke, source and load leakage fluxes (schematic). The series connection of the both air gap resistances is combined to a resistance 2 R_L.

The determination of the elements of the equivalent circuit of Fig. 4.16-18 is complicated and only approximately possible. For the magnetomotive force, one has to consider the product of coercivity and magnetic length. The source resistance of the magnet is non-linear; it can be determined from the hysteresis. The resistance of the air gap is linear, but it has to be calculated over an inhomogeneous (position dependent) field. The resistance of the ferromagnetic yoke is non-linear. A solution can only be determined iteratively: The spatial field distribution is dependent on the non-linear resistances, but their working point, on the other hand, is field dependent. In particular, it has to be stressed that the, otherwise so powerful, superposition principle cannot be applied here.

However, **Kirchhoff's mesh rule** can be applied, which means in its generalized form: *The flux quantity is equal everywhere in an undivided current circuit; the sum of all potential drops is zero.* When applied to a magnetic circuit this means that, in **Fig 4.19**, the magnetic flux is equal everywhere and the sum of the magnetomotive force and magnetic potential drops is zero: $\Theta + V_M + V_L = 0$. Here Θ is the magnetomotive force, V_M is the magnetic potential drop inside the magnet and V_L is the magnetic potential drop in air. For the sum to be zero, all arrows have to point into the same direction. Alternatively, there are other arrow systems, because each of the three circuit elements can be chosen to have a positive or negative sign. The conventional measurement technique common for permanent magnets yields an unusual sign convention: $\Theta = V_L - V_M$. The arrows of Θ and $V_{M,}$, on the one side, and V_L, on the other side, oppose each other. In addition, Θ and V_L are assumed to be negative. Two possibilities exist for the flux arrows; it has been defined in such a way that (also unusually) the arrows of Θ and V_L and Φ have the same direction inside the source. This means that, at the air gap resistance R_L, the potential and the flux arrows are opposite but at the magnet resistance R_M the direction are the same. Hopkinson's law is, therefore, written: $V_L = -\Phi \cdot R_L$, and $V_M = +\Phi \cdot R_M$. As mentioned, this needs getting used to.



$$\Theta = V_L - V_M$$
$$V_M = \Theta \cdot R_M$$
$$V_L = -\Theta \cdot R_L$$

Fig. 4.19: Horseshoe magnet without a yoke; no source leakage flux (simplified field line representation).

Hopkinson's law can be represented by a line through origin for the linear air gap resistance; according to the (arrow direction dependent) minus sign, the potential drop is negative for positive flux. The magnet resistance (source reluctance) is non-linear and the V_M /Φ relationship is described by the hysteresis curve. As both R_M and R_L are penetrated by the same flux, both functional graphs can be drawn in the same figure **(Fig. 4.20)**. The sum of V_M and V_L is Θ and this is the distance between the two vertical lines. If one lets the air gap resistance go to zero, as the limiting case, this will yield the remanence flux density. This would be the case when using a very high permeability material as a yoke instead of air. Otherwise, if the air gap resistance tends to infinity the flux density will become zero and one will get the coercivity point. However, a material with $\mu = 0$ may not be realized: no "magnetic insulator" exists.

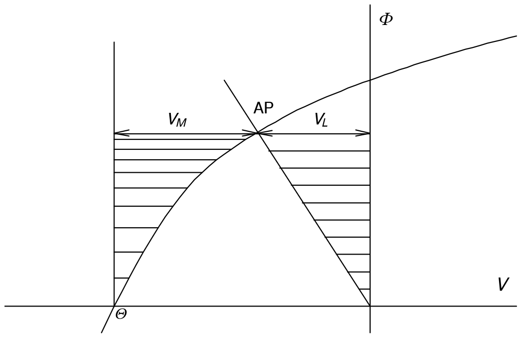


Fig. 4.20: Graphical solution of the non-linear flux equation. V_L increases proportionally to the left as a function of Φ (ordinate!) (straight load line, striped triangle on the right). V_M increases to the right with reference to the ordinate (!) (striped area curve on the left). The sum of both values represents the magnetomotive force Θ at the working point (AP).

Fig. 4.20 contains two functional graphs, which are also called the working characteristics. The non-linear *B/H* or Φ/V relationship is represented by the curved hysteresis line; the linear air gap resistance is depicted by the **load line**. The slope of the load line depends on the individual magnetic field geometry; its intercept with the hysteresis line marks the **working point** (AP). It is only possible to approximately calculate R_L since the air gap field fills the entire (infinite) space.

Cylinder shaped magnets are employed for simple pickup constructions, without any pole pieces (iron parts). Idealized, the magnetic flux will pass through the magnet cylinder in an axial direction, exits the end face, diverging and flowing through the entire air space before it re-enters at the other end face. In reality, however, a considerable source-leakage flux exists: The magnetic flux will also penetrate the lateral cylinder surface which means that the source must be partitioned (Fig. 4.18). The actual, effective air gap resistance, which is often depicted in reciprocal form as the **permeance**, can only be computed by FEM programs with sufficient precision. Indeed, the literature quotes permeance values [21-26] for some simply formed bodies, but their precision is only moderate. This is aggravated by the fact that the unit of permeance can be understood in the American literature only after some deliberation. The permeance *P* is the quotient of flux and magnetomotive force; it should have the unit Vs/A = H or Mx/Gb, respectively. Instead, *P* is given in the American literature with the dimension of length, (e.g. cm), which stems from the incorrect definition of μ_0. The correct value of μ_0 in the cgs-system is $\mu_0 = 1$ G/Oe. Instead, $\mu_0 = 1$ is used, with the consequence, that the units of the derived quantities are also wrong.

In addition to the absolute permeance, a dimensionless **standardized permeance** is also defined, which was introduced by Parker [22] as the **permeance coefficient** *p* = *unit permeance per centimeter* or elsewhere as a dimensionless *B/H* ratio, again under the assumption that $\mu_0 = 1$. McCaig [26] is more precise and speaks about $\mu_0 = 1$, but means the same. Cedighian [25] again prefers the *B/H* ratio without μ_0 but calls it the **demagnetisation coefficient** (see later). The *B/H* ratio indicates the slope of the load line (load characteristic). The dimensionless designation *B/H* = 12, for example, means that the a strength of 500 Oe corresponds to a flux density of 6 kG. Transformed to MKSA units this would mean that 40 kA/m correspond to 0.6 T. For Alnico magnets *B/H*-ratios of about 15 are optimal in order to run the load line through the point of maximum energy density (B/H_{max}-point). This would yield an optimum length/diameter ratio of about 4 for cylinder magnets if one uses the published permeance diagrams of [e.g. 22, 23, 25, 26,]. In fact, Parker [22] talks about *length-to-area-ratio*, but means *length-to-diameter*.

Many pickup magnets (e.g. Stratocaster) meet this optimum length to diameter ratio quite precisely, but one should not make a dogma out of it; the precision of the permeance data is not very high. The magnetic circuit theory, which is based on circuit analogies, is very well suited to gain insight into the qualitative relations. Nowadays, with FEM computations, more powerful and precise tools are available for quantitative conclusions.

The term **demagnetization** needs a further explanation. First of all, it means every process that shifts the working point away from the remanence point of the hysteresis curve further down to the left – this is the reason why sometimes instead of hysteresis curve one speaks of the demagnetization curve. In addition, demagnetization also means the irreversible destruction of the permanent magnetization, as will occur when the **Curie-temperature** is exceeded or under the influence of a strong field (e.g. inside a demagnetization tool). Demagnetization rarely refers to the aging processes, which are in fact also called aging, after-effect, losses or similar expressions. Rather unexpected, however, is the description of magnetomotive force drops at the source resistance as demagnetization: As R_M in Fig. 4.19 could not be zero, a positive magnetic voltage drop $\Phi \cdot R_M$ will result for every flux $\Phi \neq 0$ at R_M, which - in series with a negative (!) magnetomotive force Θ - will decrease the value of the magnetic voltage drop at the load resistance R_L. The field strength H will also be decreased; the magnetic circuit will thus be "demagnetized". However, one should not force the electromagnetic analogies too far, because otherwise one has to speak about "de-electrification" in conjunction with a charged car battery.

In **Fig.4.21** the load lines are given for typical Alnico curves. A flux density of 9 kG = 0.9 T results for the center of an Alnico-5 cylinder magnet for L/D = 4; if one takes an Alnico-2 magnet instead, B decreases to 5 kG. The standardized permeance values for Humbucker magnets are, with $p \approx 4$, surprisingly small; the flux density differences between Alnico-2 and Alnico-5 for are accordingly minor.

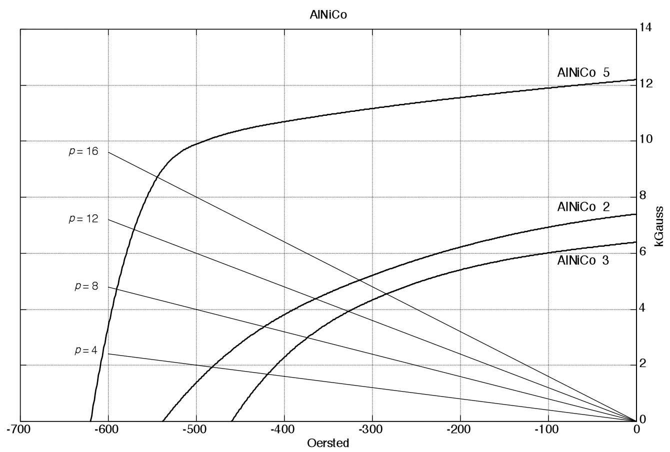


Fig. 4.21: Load lines for different standardized permeance values p = 8, 12 & 16. The flux densities emerging at the working point are achieved at the neutral surface, i.e. at the center of the cylinder magnet. A length/diameter-ratio of approx. 2.5 corresponds to p = 8 for cylinder magnets.
Accordingly: $p = 12 \rightarrow L/D = 3.2$
and $p = 16 \rightarrow L/D = 4$.
Alnico and ceramic magnets have somewhat different permeance values [23].