4.7.2 Magnetic Potentials

In chapter 4.2 we introduced the magnetic **scalar potential** and the magnetic vector potential. The negative gradient of the scalar potential ψ is the field strength: $\vec{H} = -\text{grad}\psi$. The scalar potential is a scalar quantity that only has a value and no direction. Hence, its characterization (and computation) is simpler than that of a vector. The spatial change of the scalar potential yields the field strength; the line integral over the field strength yields the scalar potential. If one considers very simple fields e.g., that of the current carrying conductor in Fig. 4.25, the field strength is constant on every one of the (circular) field lines. ψ increases proportionally with the angle for rotation with constant angular velocity. Thus, the connecting lines of equal scalar potential values, the so called **equipotential lines**, are rays originating from the center of the conductor outwards. The potential does not change along an equipotential line, perpendicularly it changes the most – this is the direction that the gradient points in. Expressed differently, the field lines and equipotential lines cross at an angle of 90°, the field strength vector is perpendicular to the equipotential line and its value corresponds to the spatial density of the equipotential lines. The field strength is large in areas where the equipotential lines are in close proximity.

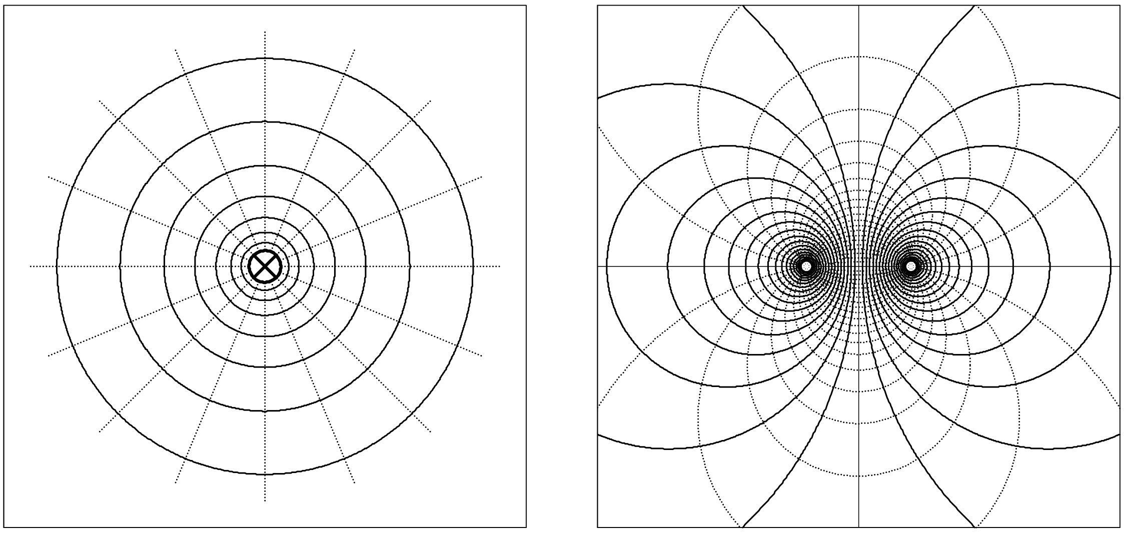

Fig. 4.27: Field lines and equipotential lines of simple fields: single wire (left); two-wire conductor (right).

The term equipotential line depicts curves at which the scalar potential is equal. Considering the **vector potential** $\vec{A}$, equality is much harder to achieve because, as consequence of its vector character, three components have to be equal. The vector potential of a two-dimensional magnetic field, however, has only one component which is normal to the field plane (Chapter 4.2); for a current carrying conductor it is, therefore, oriented parallel to the direction of the conductor. If one defines the field plane as the *x-y* plane, the vector potential consists only of an A_z component, with the partial spatial derivative yielding the flux density vector $\mu\vec{H}$:

$$\mu \cdot \vec{H} = \nabla \times \vec{A} = \text{rot}\,\vec{A} = \begin{pmatrix} \partial A_z / \partial y \\ -\partial A_z / \partial x \end{pmatrix}$$ 2-D vector potential

Considering the **parallel-plane** field, which is conveniently expressed in Cartesian coordinates, a flux density vector $\vec{B}$ can be assigned to every point in space. Both its components are:

$$B_x = \partial A/\partial y, \quad B_y = -\partial A/\partial x, \qquad A = A_z \qquad\qquad A_z = \text{2-D vector potential}$$

The direction of $\vec{B}$ coincides with the direction of the flux lines, $\vec{B}$ is the tangent to the flux line:

$$dy/dx = B_y/B_x \quad \rightarrow \quad B_x \cdot dy - B_y \cdot dx = 0 \quad \rightarrow \quad \frac{\partial A}{\partial y} \cdot dy + \frac{\partial A}{\partial x} \cdot dx = 0 \;\hat{=}\; dA$$

The equation on the right depicts a total differential *dA* which is zero. The total differential can be interpreted as increase in elevation above the *x/y*-plane, if a position change of *dx* or *dy* is made. Since, as discussed above, this increase in elevation for the vector potential is always zero if one runs along a flux line in the *x-y* plane, the value of *A* remains unchanged. Consequently, flux lines are associated with constant *A* values or the other way round: in the parallel-plane field, locations of constant vector potential are connected by flux lines. Hence, for the computation of flux lines (or, after division by μ, of field lines) the vector potential has to be determined and positions of equal vector potential have to be connected.

Cylindrical coordinates are more appropriate than Cartesian coordinates for **rotation-symmetric** fields The computation of the rotation yields somewhat different differential equations to those for the parallel plane field. The corresponding equation for a radius *r*, rotational angle φ and axis direction *z* is:

$$\frac{\partial(rA)}{\partial z} \cdot dz + \frac{\partial(rA)}{\partial r} = 0, \qquad A = A_\varphi \qquad\qquad A_\varphi = \text{2-D vector potential}$$

Such a field-symmetry would be adequate for a circular conductor; the vector potential would be circular as well. Flux lines, however, are not positions of *A* = const, but rather follow $r{\cdot}A = const$.

4.7.3 Spatial Fields

All real fields are three-dimensional and, only for special cases, are they either restricted to thin layers or are (in symmetric cases) characterized by a plane. Field or flux line projections onto a plane are not helpful for the case of a general spatial field evolution – as a rule, the spatial depth cannot be discerned. A last resort would be to define only cross-sections and to depict the flux density or the field strength by color coding within them. The relationship between the represented value and the associated color is given by color maps, which e.g. assign a color gradient from blue to green and yellow to red for increasing functional values.