5.5.2 Inductivity of the coil winding

As electric current flows through a conductor, a magnetic field surrounding this conductor is generated. Strictly speaking, a magnetic field is also generated within this conductor but this effect is mostly neglected. **Fig. 5.5.3** schematically shows a wire through which a current passes from bottom to top. The technical current direction (from plus to minus) and the direction of the magnetic flux (from north to south) are connected unequivocally: using your right hand and pointing with the thumb in the direction of the current will make the remaining (bent) fingers point in the direction of the magnetic flux.

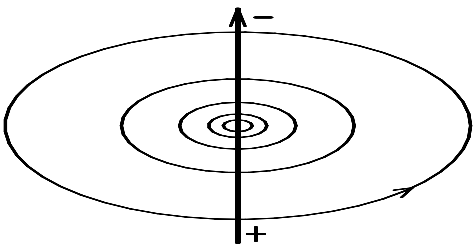

Fig. 5.5.3: Magnetic field around a conductor through which current passes

It has already been mentioned that magnetic flux must not be seen as a concrete means of transport. Flux and flux density are assumed as analogies from fluids. The same approach is found in other areas of physics (e.g. the flow of current). A straight wire of infinite length through which a current I flows generates – at a radial distance of R – the magnetic **field strength** of H and the magnetic **flux density** of B:

$$H = \frac{I}{2\pi R} \qquad B = \mu \cdot H \qquad \mu = \mu_r \cdot \mu_0 \qquad \mu_0 = 4\pi \cdot 100\text{nH/m}$$

The quantity μ_r is called **relative permeability** and identifies the magnetic property of the material penetrated by the magnetic field as a multiple of the **permeability of air** μ_0 (strictly speaking μ_0 is valid only for vacuum but the difference to air is negligible).

In **Fig. 5.5.4** we see a rectangular wire frame though which electrical current flows. Again, a magnetic field results which for this representation has an orientation perpendicular to the paper plane. Fields running in the viewing direction are customarily shown as crosses while the opposite direction is given by dots.

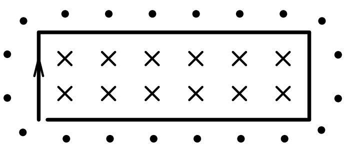

Fig. 5.5.4: Wire frame carrying a current, magnetic field. The field strength decreases with increasing distance.

The magnetic flux density B specifies the area-specific magnetic flux. Integrating B over the field-penetrated area S results in the overall **magnetic flux** Φ:

$$\Phi = \int_S \vec{B} \cdot d\vec{S} \qquad \vec{B} \cdot d\vec{S} = B \cdot dS \cdot \cos\alpha \quad \text{(scalar product)}$$

In air, there is a linear correspondence between the current I and the magnetic flux Φ generated by it. The coefficient characterizing this proportionality is the **inductance** L. Given material and topology, L can be calculated from the build and is (in linear systems) not dependent on the current. For the magnetic guitar pickup, the coil inductance L is the most important electrical parameter. It has a major influence on the sensitivity, the impedance frequency response and the resonance frequency.

If an **alternating current** is flowing through the wire frame shown in Fig. 5.5.4, a time-variant magnetic field results. The **law of induction** tells us that an electric voltage is induced in a current loop (through which a magnetic field is flowing). This voltage corresponds to the variation over time $d\Phi/dt$ of the flux penetrating the coil. For a sine-shaped current and with complex nomenclature the time-differential corresponds to a multiplication with $j\omega$:

$$\underline{U} = d\underline{\Phi}/dt = j\omega \cdot \underline{\Phi} = j\omega \cdot L \cdot \underline{I}, \qquad \text{with:} \quad \underline{\Phi} = L \cdot \underline{I}; \quad j = \sqrt{-1}; \quad \omega = 2\pi f .$$

The quotient of the voltage $\underline{U}$ and the current $\underline{I}$ is called **impedance** $\underline{Z} = j\omega L$ in the framework of complex calculation. $\underline{Z}$ is a system quantity and thus <u>in</u>dependent of the signal (as required in linear systems). The impedance of the wire frame in Fig. 5.5.4 is proportional to the inductance L and proportional to the frequency f.

Shown in **Fig. 5.5.5a** are <u>two</u> square wire frames through which the (same) current is flowing. The magnetic fields generated by these two frames should not superimpose which can be achieved either by a big distance between the frames or via fields with perpendicular orientation. In Fig. 5.5.5b the two frames are laid on top of each other such that the magnetic field penetrates both frames in the same way.

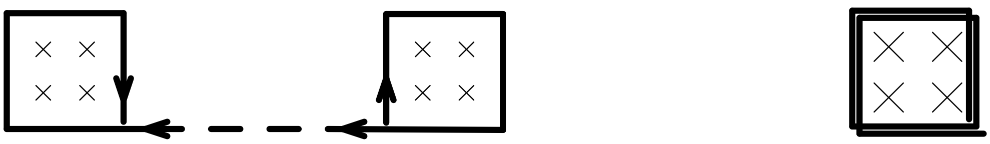

Fig. 5.5.5: two square wire windings connected in series carrying the same current.
a) separate location (left), **b)** on top of each other (right).

Each of the two frames in Fig. 5.5.5a generates the flux $\underline{\Phi}$, and in each frame the voltage $\underline{U}$ is induced; the overall voltage induced in the series connection of the two frames therefore is $2\underline{U}$. Relative to *one* frame of the same size, the inductivity has doubled (assuming the connecting wire to have no inductivity). In Fig. 5.5.5b the superposition of the magnetic flux generated by the two frames results in double the overall flux. The voltage induced in each of the two frames is double that found in the scenario of Fig. 5.5.5a, i.e. the series connection results in the quadruple overall voltage and – correspondingly – the quadruple inductivity. Thus, if a wire is wound with N windings, its inductivity may increase by a factor of N or by a factor of N^2 – depending on how the windings are **coupled**. Of course, wire windings can never share the exact some location – the individual turns will in reality have to have a certain distance and cannot be completely coupled. Still, real coils exhibit $L \sim N^k$ with $k > 2$ because an increase in the number of turns will also require an increase in the area.

The typical shape of the winding of a pickup is oblong (**Fig. 5.5.6**). Its inductivity can be calculated with good approximation [Hertwig]:

$$L = 4 \cdot N^2 \cdot \left[(x+y) \cdot \ln \frac{2xy}{b+h} - x \cdot \ln(x+d) - y \cdot \ln(y+d) + 2d - \frac{(x+y)}{2} + 0{,}447 \cdot (b+h) \right] \cdot 1\,\text{nH}$$

Here N is the number of turns and $d = \sqrt{x^2 + y^2}$ the diagonal. The dimensions need to be entered in cm. For a Stratocaster pickup (N = 7600, without magnets) the result is L = 1,7 H. With magnets, the inductivity rises by approx. 30% to 2,2 H.

The above formula makes it also possible to calculate the effects of changes in the number of turns. Since x, y, and h also changes, a power function with an exponent larger than 2 results: $L \sim N^{2,14}$. Increasing e.g. N by 10% pushes the inductivity by 23%. Since the resonance frequency is dependent $\sqrt{1/L}$, the resonance frequency decreases by 10% in this example (keeping the capacitance constant).

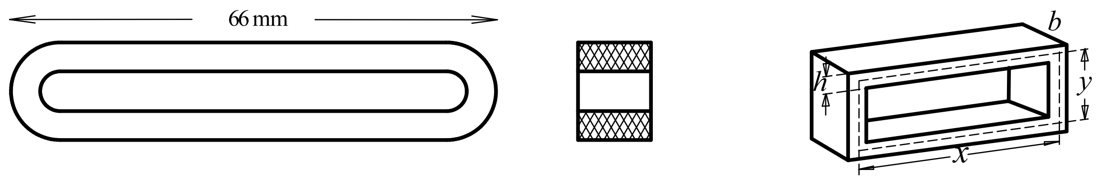

Fig. 5.5.6: Shape of a Stratocaster coil and cross section. Idealized rectangular coil.

The inductivity is dependent on number of turns of the winding and the geometry of the coil, but also on the magnetic conductivity of the space penetrated by the magnetic field. Most materials differ from air only marginally in magnetic terms; their magnetic conductivity, the permeability $\mu = \mu_r \cdot \mu_0$, is $\mu = \mu_0$ with very good accuracy, since the relative permeability of these (diamagnetic or paramagnetic substances) is almost exactly one. Ferromagnetic materials, on the other hand, react rather differently: their relative permeability is considerably larger than one and moreover not constant but dependent on the field strength. Alnico-pickups and polepieces, and also screws and shielding plates made out of ferromagnetic material (e.g. iron or nickel) are ferromagnetic. By means of their better magnetic conductivity (relative to air), such ferromagnetic materials decrease the magnetic resistance and thus increase the inductivity. A particularly strong increase in inductivity is possible if the complete magnetic field flows through the ferromagnetic material – this is, however, as a matter of principle not possible in guitar pickups (5.4). Due to the fact that the field running through air forms the largest part of the magnetic resistance, the magnets in the Stratocaster pickup can increase the inductivity by only 30%, for example.

The non-linear permeability μ of a ferromagnetic material bends the magnetic flux into such complex curves that an analytic description is not viable anymore. On top of this, **eddy-current-** and **skin-effects** aggravate any calculations even further since they contribute an additional inductive share which is dependent on frequency in a rather complicated manner. For pickups without cover which contain on top of the coil only alnico magnets (e.g. Stratocaster), stating *one single* inductivity is an acceptable compromise; here the losses in the magnets are rather small.

However, as soon as a pickup comprises polepieces and/or mounting panels, the **equivalent circuit diagram** contains either a single albeit frequency dependent inductance difficult to interpret, or several inductances. Characterizing such a pickup with a *single* inductance is a drastic simplification. For this reason, **inductance-measuring meters** are to be used only with great caution for magnetic pickups (chapter 5.6). Such instrumentation determines e.g. the inductive part of the complex impedance at one special frequency (e.g. at 1 kHz) and implying a series equivalent circuit: $\underline{Z} = R + j\omega L$. Since, however, polepieces subjected to the skin-effect do not result in an imaginary part with an ω-proportionality, this measurement approach is not suitable. More appropriate is to record a complete impedance-frequency-response $\underline{Z}(\omega)$ from which the components of a better suited equivalent circuit can be calculated using methods of network synthesis (chapter 5.9).

5.5.3 Coil capacitance

The capacitance is defined as the proportionality between electric charge and electric voltage. A small capacitance exists between two turns each of a coil; this capacitance is dependent on the length, the distance and the dielectric constant ε. In vacuum (or air) we find ε = 8.9 pF/m, insulators (such as the varnish and the bobbin) have 2 to 5 times that value. The overall capacitance of a pickup can only be calculated as an approximation, because there are capacitances between all turns of the coil. Given the height h, the width b (Fig. 5,5,1) and the average length ξ of one turn, the result for the coil capacitance C_W is:

$$C_W \approx 2\frac{b}{h}\cdot\frac{\xi}{\text{cm}}\cdot\text{pF}$$

Coil capacitance using regular varnished wire [17]

Customary pickup coils have capacities in the range of 10 ... 150 pF. The capacitance of wide, shallow coils (which seem to have a large surface area when observed from above i.e. from the direction of the string) is smaller than the capacitance of compact coils with approximately square cross-section of the winding. For example, Jazzmaster- or P90-pickups have a smaller capacitance than Stratocaster pickups. For machine-wound pickup coils the individual turns are closer together which results in a slightly higher capacitance compared to hand-wound pickups. Increasing the thickness of the varnish layer has the opposite effect: the individual windings have a larger distance, and the capacitance decreases.

Installing the pickup in the guitar leads to an increase of the capacitance. The main reason is the **pickup connecting cable** the capacitance of which can vary between a few picofarad (unshielded two-wire cable) and several hundred picofarad (old Gibson cables). The second reason for the increased capacitance is the presence of **stray capacitances** towards metal parts which are close-by, in particular towards shielding sheets.

Working in conjunction with the coil inductance, the capacitance is the basis for the **pickup resonance** (at 2 – 5 kHz). However, much more important than the coil capacitance is the **cable capacitance** (chapter 9) which has the main contribution to the overall capacitance. Several components are involved in the **resonance damping**; of there the loss resistance connected to the coil capacitance (chapter 5.5.4) has the smallest effect.