5.9 Equivalent Circuits

The vibration of a string is always a composite from partials of different frequencies. The conversion of mechanical into electrical vibrations in the pickup includes a frequency-dependent weighting: spectral components in the vicinity of the resonance frequency are emphasized, and those of higher frequency are attenuated. The pickup can therefore be considered as a system with frequency-dependent transfer-function i.e. as a filter. According to the teachings of the theory of electrical system (systems theory, e.g. [7]), the transfer behavior of a linear system is unambiguously described by its transfer function. Linear systems with identical transfer function have an identical filter effect, even if they are differently constructed. The construction of a magnetic pickup seems to be simple (a wound wire) – the frequency-dependent filter effect can, however, not be visualized this way. The telecommunication engineer is more familiar with passive filter networks, i.e. networks consisting of coils, capacitors and resistors. In the case of the pickup we use such networks as replacement for the original system: as long as the network in its transmission parameters is equivalent to the pickup, it represents a replacement (a model) the behavior of which is investigated in place of the pickup.

5.9.1 Models and analogies

The approach in physics is to try to explain natural phenomena and make them accessible by a mathematical description. Influencing factors, states and effects of real processes are, however, so diverse that a complete description is impossible. For this reason, simplified systems (**models**) are developed which are equivalent to the reality in a number of (but not all) characteristics. The model-boundaries define what is to be reproduced and what is not. Famous examples of physical models are the simple law of inertia (Newton) which is not valid for relativistic considerations, or models of atoms. The model is the compromise between the exact mathematical description which cannot be realized, and the variations existing in the real world which are, however, to complex to be describable.

The **analogy** (the analogon) is a model-like description in a related area which is usually better understood. To understand the resonance effects in a spring-mass-system, the electrical engineer finds an illustration via an electrical resonance circuit; the mechanical engineer, on the other hand, will probably prefer the opposite approach and look at an electric resonance circuit using an electro-mechanical analogy. For all models and analogies, the respective ranges of approximations and definitions need to be considered: if, for example, an electro-mechanical analogy models merely the transversal movements in one plane, then torsion-vibrations will remain without consideration. The idealized spring has a single elasticity (Hooke) - and therefore is without any mass, which is in sharp contrast to the real spring. In every model it is necessary to find a compromise between complexity and accuracy. Strong simplifications lead to clear, simple structures; however, these may not be able to reflect the effect under investigation with sufficient accuracy, or even to describe it at all. On the other hand, a highly exact model may result in too great a variety of parameters the calculation of which may take too long, or the complexity of which goes beyond the imagination. Purposeful limits for the accuracy of approximation (and therefore for the complexity) are given by the reachable measurement accuracy, the reproducibility, or whether or not a model-specific inadequacy is in fact audible.

Two approaches are often found when models are put together: **inductive**, concluding generally applicable statements from a single finding (bottom up), and the **deductive** conclusion from the general finding to the single event (top down). Both approaches may be used in parallel. The laws of magnetism and those of electrical networks can be applied to all magnetic pickups. The specific equivalent circuit, which would be not sufficiently accurate to generalize, may be supplemented by additional components and thus be improved in its precision (and general applicability).

Models and analogies have led to an adaptation and broadening of the meaning of familiar terms. For example, the term '**flow**', as it would be used in the context of water circulation, relates to a macroscopically visible matter movement. In an electrical circuit, however, the term is directed to the microscopic movement of electrons, while in a magnetic circuit there is no movement (flow) at all (*panta rhei ?*). Nevertheless, we imagine a magnetic flow including flow-lines, flow-density, turn-offs and junctions - very much in analogy to the electrical <u>circ</u>uit … which in itself is not always happening in a <u>circ</u>le, anyway. ☺

5.9.2 Equivalent Circuits for Electrical Impedances

Regarded from the point of instrumentation, there are two pickup characteristics which are of fundamental significance: its electrical impedance and its transfer behavior. Naturally, in the end only the latter is of interest but the corresponding required system parameters can only be determined with much effort. The impedance, on the other hand, can be determined easily and accurately, and forms a good starting point to arrive at the transfer parameters via calculations.

The **impedance** Z is the complex, frequency-dependent resistance of a two-pole element. The term **two-pole** points to the fact that $\underline{Z}$ is to be determined in relation to two poles (junctions, terminals) in an electrical circuit. If a circuit includes more than two terminals, it is possible to define two-terminal-network impedances between any two of these terminals. Using the complex number terminology (designated by underlining the respective character in a formula), magnitude and phase of the impedance can be described elegantly and economically. For this purpose, two different but each individually complete ways exist: the polar and the Cartesian form. In Cartesian coordinates $\underline{Z}$ is constituted from a real and an imaginary part, while in polar coordinates a radius (= amount, magnitude) and a phase-angle.

The impedance of a purely ohmic resistor is real and independent of frequency: $\underline{Z}_R = R$. The impedance of an ideal inductance (wound up wire, coil) is imaginary and dependent on frequency: $\underline{Z}_L = j\omega L$. j is the imaginary unit $\sqrt{-1}$, which in mathematics also is designated with i. The product $j\omega$ is the **complex frequency**♣, often also termed p or s. The impedance of an ideal capacitance is imaginary and frequency dependent: $\underline{Z}_C = 1 / j\omega C$. If two two-terminal networks are connected in series their impedances add up; if they are connected in parallel their admittances are added. The **Admittance** $\underline{Y}$ is the inverse of the impedance $\underline{Y} = 1 / \underline{Z}$. (This is elaborated on e.g. [7, 18, 20]).

For a real resistor and an inductivity connected in series, their impedances have to be added up: $\underline{Z} = R + pL$. In this example R is the real part of the impedance and L is the imaginary part. The j contained in p is not counted as part of the imaginary part.

♣ $p = \sigma + j\omega$; here: $\sigma = 0$, i.e. $p = j\omega$ (steady state).

The **magnitude** of the impedance is calculated taking the square-root of the sum of the squared real and imaginary parts. To mark the result as magnitude, two vertical lines are added, or the symbol in the formula is written without underline. The latter form is unfortunately somewhat dangerous, because complex values are sometimes found in literature where the underline has conveniently been dispensed with.

$$|\underline{Z}| = Z = \sqrt{\mathrm{Re}^2\{\underline{Z}\} + \mathrm{Im}^2\{\underline{Z}\}}$$ Magnitude of the complex impedance $\underline{Z}$

Equivalent circuits for impedances have the same impedance as the system they replace. Let us take, for **example**, an ohmic resistor: ideally its impedance[♣] should be purely real ($\underline{Z} = R$). However, at high frequencies the contact caps of the resistor act as a small capacitance; the magnitude of the impedance decreases with increasing frequency. This behavior could be reproduced by an equivalent circuit with a capacitor connected in parallel to the resistor. On the other hand, the resistor may be built of a wound-up wire; in that case the resulting inductive component would have to be reproduced by a coil connected in series – possibly in conjunction with the capacitor mentioned above. For DC-considerations neither coil nor capacitor are required; they do not hurt either, though, since the DC-resistance of the coil is zero (i.e. no effect in a series connection), and the one of the capacitor is infinite (i.e. no effect in a parallel connection). At 50 Hz, the contribution of the coil and the capacitor may be so small that it can neglected. Starting from which frequency the consideration is required depends on the desired accuracy.

In a **guitar pickup** the inductive component of the wound-up wire has a significant effect already from 100 Hz. Consequently, the equivalent circuit will require at least an inductance on top of the pure wire-resistance. The calculation of the inductance is in fact not entirely trivial. Even very simple, symmetrical structures will require an extensive integration which can quickly reach an undreamed of scope. Strictly speaking, every one of the 10000 turns?? would have to be broken down into differentially small wire-pieces which all interact with all other wire-pieces and form a complex inductance and capacitance. As a first approximation, however, it is sufficient to supplement the ohmic pickup resistance with an ideal coil and an ideal capacitor. The **quality of the modeling** can easily be checked by measuring the frequency characteristic of both impedance of the pickup and that of the equivalent circuit (put together from a resistor, a coil, and a capacitor). The two measurements should agree within the desired measurement accuracy. Even simpler is *calculating* the impedance of the equivalent circuit using methods of systems theory. This approach removes the obligation to acquire an ideal coil which does not include the inadequacies which are the reason we are making the effort to start with! If we find that the object under measurement and the model differ too much, the model needs to be improved with other parameters or with another – and possible more extensive – structure. Moreover, the purpose of the model must never be forgotten: it is supposed to reproduce the frequency response of the impedance. The model cannot and will not reproduce the transfer behavior or the non-linear distortions.

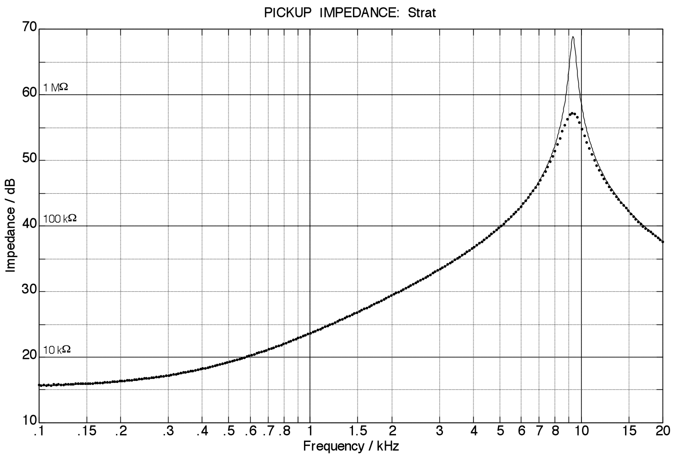
[♣] Purely real numbers are a sub-set of the complex numbers, as are purely imaginary numbers.

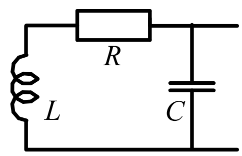
5.9.2.1 Singlecoils with weak dampening of eddy-currents

The typical Stratocaster pickup consist of 6 magnets, two coil flanges, and would-up wire. The impedance of such a pickup can be reproduced in the frequency range up to 20 kHz with good accuracy with few circuit elements. As already discussed, the term „*reproduce*“ means putting together an **impedance-equivalent circuit** consisting of few ideal, concentrated elements (R, L, C). This equivalent circuit, described in the form of an **equivalent circuit diagram**, approximates the pickup impedance in the framework of purposeful accuracy limits, e.g. 5%.

Fig. 5.9.1 shows an electrical equivalent circuit diagram (ECD0) of a Stratocaster pickup as well as the corresponding impedance frequency plots. For the measurements, the pickup was connected via short leads and without further components (i.e. without potentiometers) to an impedance meter. At very low frequencies the impedance is determined by the copper resistance $R;$ at a few kHz there is a resonance maximum and at high frequencies an impedance drop-off occurs which is due to the capacitance. There is a general correspondence between the measurement (of the real pickup) and the calculation (based on the equivalent circuit diagram). The emphasis of the resonant peak is different, however. Obviously, the pickup contains an additional dampening which the simple equivalent circuit ECD0 does not model (eddy currents in the magnet, see chapter 5.9.2.2).

There are several possibilities to extend ECD0 by real dampening restores. Seen from the point of networks theory, the corresponding impedance function is a *second*-order broken rational function, since the network includes *two* independent storage elements (namely L and C). In a fraction, containing (in numerator and denominator) the complex frequency variable p with not more than the power of 2, five polynomial coefficients can be chosen freely – corresponding to five components which may be selected freely. Therefore, apart from L and C, a maximum of three dampening resistors may be determined independently from each other in a 2nd-order system. It is not difficult to draw *more* than two additional resistors into ECD0; the resulting new circuits can, however, be transferred into simpler circuits (with a maximum of three resistors) using equivalence-transformations. There are in fact even several possibilities to extend ECD0 by merely one single resistor – these support various physical interpretations differently, although they are equivalent regarding the impedance modeling.





Equivalent circuit diagram

Fig. 5.9.1: Stratocaster-impedance, ECD0 Measurement (…), ECD-calculation (----**).**

In **Fig. 5.9.2** we see two equivalent-circuit diagrams containing *two* resistors each. To arrive at the left ECD, the overall circuit of Fig. 5.9.1 was supplemented by a resistor connected in parallel, while for the right-hand ECD, the resistor is connected in parallel to the coil. The impedance of both ECDs approximates the measurement curve of Fig. 5.9.1 so perfectly that no difference at all can be seen anymore. Several questions result: do both ECDs have the exact same impedance? Which ECD is correct? How do we arrive at the values of the components?

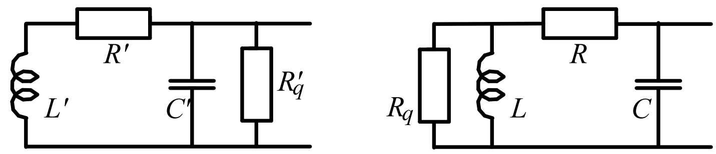


Fig. 5.9.2:
Extended Stratocaster ECD (ECD1)

The following considerations require simple knowledge in network analysis as it is e.g. imparted in [18, 20]. The **impedance** of a resistor is R, that of an inductance is pL, and that of a capacitor is $1 / pC$. For the series connections of two-poles, their impedances are added; for parallel connections the sum of the admittances (inverse) is used. In both the ECDs a section of the circuit consisting of two resistors and a coil is connected in parallel to a capacitor. If both ECDs are supposed to have the same impedance, and if the capacitance is supposed to be of the same value for both ECDs, then these two partial circuits need to have the same impedance, as well. Without compromising the accuracy, it is therefore possible to limit the issue of identifying the ECD-impedances to calculating the impedances of the respective sections of the circuit mentioned above:

Fig. 5.9.3: As above, without C.

$$\underline{Z}' = R'_q \cdot \frac{pL' + R'}{pL' + (R' + R'_q)} \qquad \underline{Z} = \frac{pL(R + R_q) + R \cdot R_q}{pL + R_q}$$

Impedance functions

It is a necessary (but normally not sufficient) requirement for the identity of the impedance function that the impedances for $f = 0$ and for $f = \infty$ must be equal. It follows that:

$$R = \frac{R' \cdot R'_q}{R' + R'_q}; \qquad R + R_q = R'_q .$$

Introducing these requirements into the impedance function described above, we obtain the still missing requirement for the relationship between the inductances:

$$L'/L = \left(1 + R/R_q\right)^2$$

These equations enable us to set up an *impedance-equivalent* ECD from the respective other ECD.

It may be surprising that the two ECDs need different inductances for identical impedances. However, surmising that the imaginary part of an impedance would be determined solely by the reactances (*L* and *C*) is only correct for a series connection. As soon as there is a parallel connection, the imaginary part is determined by the real resistances, as well. Understanding this has strong implications on how the component values need to be interpreted. Let us use a simple concrete example to exemplify this problem:

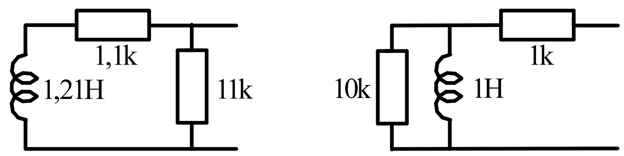


Fig. 5.9.4: Impedance-equivalent circuits

Regarding their impedances, the equivalent-circuit diagrams shown in **Fig. 5.9.4** are fully identical, although the inductances differ by 21%. For the Stratocaster pickup, the differences are significantly smaller, but as soon as additional iron parts are introduced into the magnetic circuit (as e.g. in the P90), considerable differences result. In the end, this means that a precise **pickup-inductivity** can only be given if the corresponding equivalent circuit is specified. It is merely for very simply constructed pickups that the component values differ so little that stating the ECD-topology may be dispensed with.

We can now answer the question posed above: both ECDs shown in Fig. 5.9.2 feature exactly the same impedance, both ECDs are correct, and he values of the components can be derive via a regression-process. If you want avoid deploying the big guns, you may vary the ECD-component values until the measured impedance curve and the ECD-impedance correspond with the desired accuracy. It is not as easy to answer the question which ECD is more purposeful. An additional question could lead the way: which is the purpose of the ECD? Normally, an ECD is put together in order to obtain a clear basis for calculations, and to be able to recognize simple correspondences at a glance. The real use of an equivalent circuit for the impedance only reveals itself once the equivalent circuit for the transmission has been derived from it. Since the theory required for this process is only discussed later (see chapter 5.9.3), we will just take a quick look here: the pickup parameter measured in the easiest way is the DC-resistance R_{DC}. With $R = R_{DC}$, it can be directly included in the equivalent circuit if the right-hand version shown in Fig. 5.9.2 is preferred. It may also be explained using an equivalent circuit diagram of a transformer that resistive losses are considered with a connection in parallel to L and not with a connection in parallel to the series connection $R'L'$. May be … but doesn't have to be. From a network-theory point-of-view both circuits are equal, and the preference is a matter of taste. The following considerations use the $(R_q\,//L)+R$ -structure (**Fig. 5.9.5)**, and the result perfectly approximates the measured curve (Fig. 5.9.1) – to the width of a line.

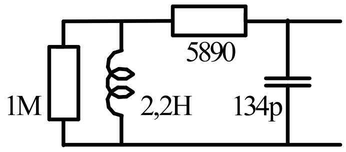


Fig. 5.9.5: Equivalent circuit diagram for the impedance of a Stratocaster pickup

5.9.2.2 Eddy currents in non-magnetic conductors

As a time-variant magnetic flux penetrates a conductor, it induces an electric circulating voltage which results in annular eddy currents. The vibrating string changes the magnetic resistance in the magnetic circuit and modulates the magnetic flux such that an alternating field is superimposed over a constant field. According to the law of induction the flux $d\Phi/dt$ changing over time leads to a voltage U which – depending on the electrical conductivity $\sigma = 1/\rho$ – causes a current I.

Non-magnetic (i.e. non-ferromagnetic) conductors are found in pickups predominantly in the form of assembly and **shielding sheet metals**. Not all pickups are fitted with them: the typical Stratocaster pickup has a plastic cover but the Gibson Humbucker is mounted to a metal plate and shielded with a metal cover. These metal sheets do have an influence on the magnetic field even if they are not ferromagnetic (!), and therefore also on the transfer characteristic of the pickup. The eddy currents flowing within them draw their energy from the magnetic circuit which receives a corresponding dampening effect.

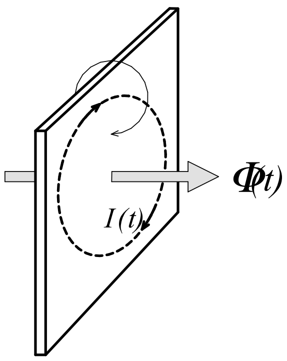


Fig. 5.9.6: The electrically conductive plate is penetrated by a magnetic flux $\Phi(t)$ which increases over time. The result is an annual eddy current $I(t)$ in the direction as drawn. This current flows in the plate as a whole, not only on the indicated circle. It causes a secondary magnetic field (as indicated at the upper edge) which attenuates the primary field.

A time-variant magnetic flux $\Phi(t)$ inducing an eddy current $I(t)$ in a current-carrying plate is shown in **Fig. 5.9.6.** This eddy current again generates itself a magnetic counter-field which attenuates the primary field such that a smaller voltage is generated in the pickup coil (not shown). Since eddy currents depend on the temporal change of the magnetic field, they have an effect particularly at high frequencies (skin effect, see below). Therefore, metal sheets do not only provide shielding against electrical interference but they also deteriorate the **treble response**. This is not necessarily a disadvantage – a full, warm sound may in fact be the objective of pickup design. For a brilliant, treble-laden tone, however, eddy currents must not have too big an effect.

There are several **measures** to get a handle on the eddy-current dampening. Size, thickness and distance of the dampening metal sheet play a role, as does the material used. The eddy current emerges as the quotient of induced voltage and electrical resistance. The induced voltage depends on the magnetic flux; metal sheets in areas of weak magnetic alternating flux attenuate less than sheets in areas of strong alternating flux. Sheet metal bent into a ring-shape (e.g. for covers) may enclose a large surface with strong alternating flux; in such a case it should be checked whether a slot could not interrupt the current flow. Thin sheets offer higher resistance than thick ones; German silver (nickel silver) has a higher resistance than brass. Gold-plated covers have better conductance (dampen more) than chrome plated covers - if the gold layer is thick enough.

The following table offers an overview of the specific resistances of common sheet metal materials. **German silver** is often used in higher quality pickups. This metal of a silvery shine is corrosion-resistance and of relatively high resistance.

Material	ρ in $\Omega mm^2/m$	Material	ρ in $\Omega mm^2/m$
Copper	0.018	Brass (Cu, Zn)	0.08 (0.06 – 0.12)
Gold	0.022	Bronze (Cu, Sn)	0.08 (0.02 – 0.14)
Aluminum	0.029	Steel for strings (ferromagnetic!)	0.20
Nickel	0.070 (ferromagnetic!)	German silver (60 Cu, 17 Ni, 23 Zn)	0.3
Iron	0.098 (ferromagnetic!)	Alnico-Magnet (magnetic source!)	0.6
Chrome	0.12	Chrome-nickel (70 Ni, 30 Cr)	1.2

Table: Specific resistance ρ of metals

Fig. 5.9.7 schematically shows a pickup winding next to which a sheet metal forms a short-circuit winding. The elements of the pickup winding are the DC resistance R (copper resistance), the winding capacitance C, and the winding inductivity L. The Short-circuit winding is characterized by R_K and L_K. Due to the incomplete flux-coupling k we will not find the same flux $\Phi(t)$ penetrating both windings i.e. k < 1. The eddy-current resistance R_K is transformed up as R_w and attenuates a part of the winding (in the transformer-free equivalent circuit on the right-hand side). The eddy currents induced into the sheet metal thus reduce the coil inductance and increase the coil losses. The stronger the coupling (i.e. the closer the sheet is positioned to coil) the larger the part of the coil which is shorted by R_w and the larger R_w itself. In addition, R_w depends on the specific resistance of the sheet metal.

At low frequencies, the parallel-connection of R_w and $(1-\sigma)L$ has the effect of an inductance, at high frequencies it has the effect of a resistance. The cutoff-frequency between inductive and resistive behavior is $f_g = R_w / \left[2\pi(1-\sigma)L\right]$. As a simplification, the eddy-current losses can be neglected below f_g while above f_g the resistance increases from R to $R + R_w$, and the inductivity decreases from L to σL (compare to Fig. 5.9.8).

$R_w \approx R_K \cdot N^2 \cdot (1-\sigma)$ N = number of turns of the winding $\sigma = 1 - k^2$ = degree of scatter

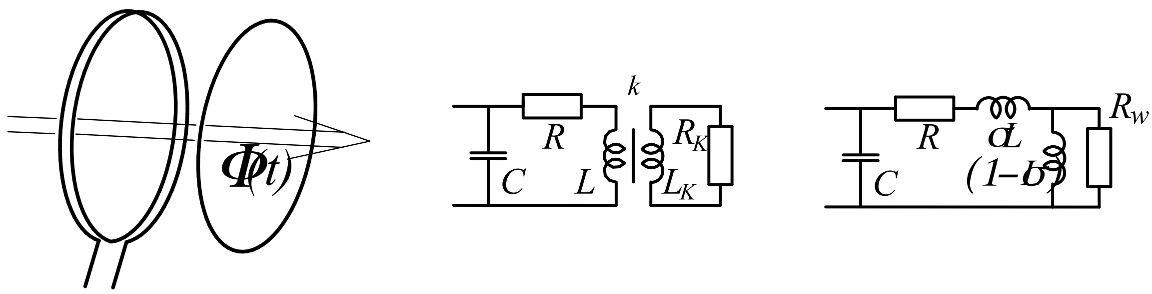


Fig. 5.9.7: Pickup coil with short, equivalent circuit diagrams [4].

Fig. 5.9.8 shows the frequency response of the impedance-magnitude for the equivalent circuit of Fig. 5.9.7 – without capacitance, though (i.e. $C = 0$). For the left section the degree of coupling k was varied. As the short circuit winding is brought closer to the pickup coil, the coupling increases while at the same time the degree of scatter decreases. The treble loss becomes stronger and the impedance level of the circuit is reduced. The left section of the figure shows the effect of the variation of the resistor R_w for fixed coupling, which is equivalent to a change in the thickness of the sheet metal, or of its material type. This measure changes the cutoff frequency f_g: with increasing resistance (thinner sheet metal, higher material resistance) f_g increases.

$$f_g = \frac{N^2 \cdot R_K}{2\pi \cdot L}$$ Cutoff frequency of the parallel connection of R_w and $(1 - \sigma)L$

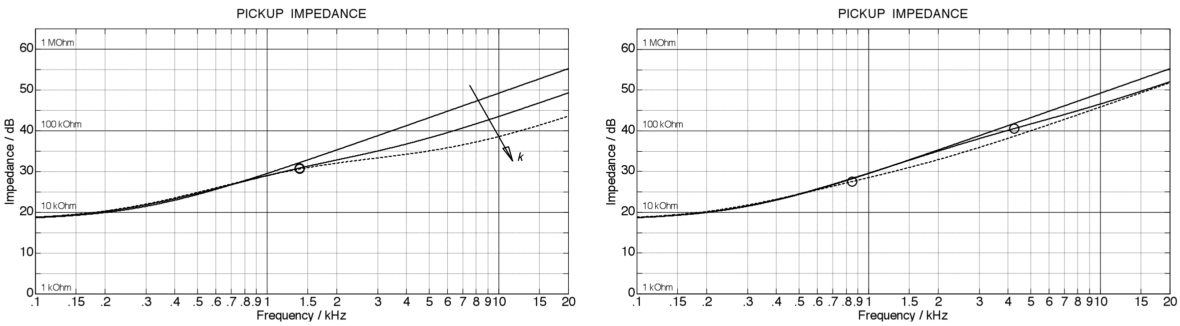


Fig. 5.9.8: Effect of changing the coupling (left) and varying R_w (right). The cutoff frequency is marked by a circle.

Fig. 5.9.9 shows impedance measurements for a Jazzmaster pickup. First a 1mm strong **sheet metal made of brass** was brought in close proximity (2,5 mm) to the pickup and the impedance measurement taken. Subsequently, the brass plate was replaced by an equally strong **copper plate** (positioned at the same distance). The differences in the impedance frequency plot are relatively small but still readily identifiable (at 1 – 3 kHz), and moreover in good agreement with the results from the equivalent circuit diagram (Fig. 5.9.10).

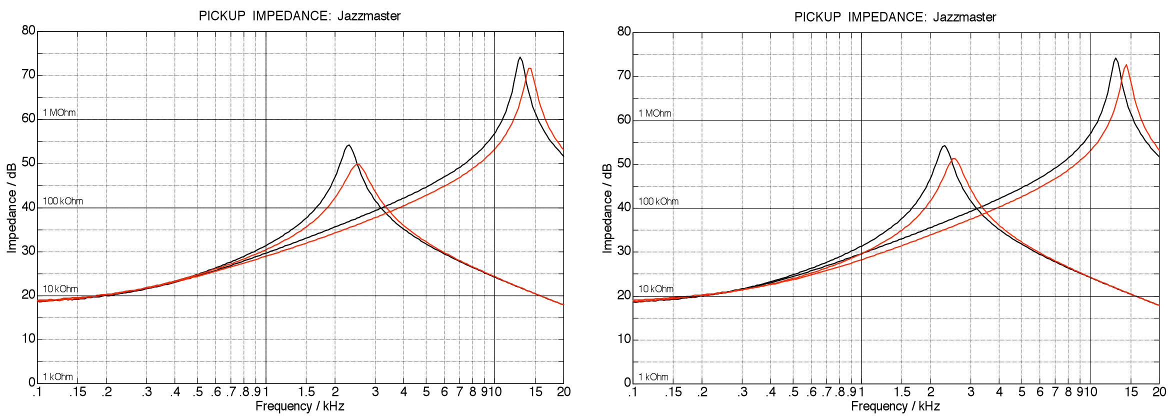


Fig. 5.9.9: Measured impedance chart for a Jazzmaster pickup. The pickup was unloaded (high resonance frequency), or loaded with 1 nF (resonance at 2,5 kHz. With the sheet of brass (left) or copper (right) respectively, the impedance drops and the resonance frequency increases.

The equivalent circuit diagram for the impedance of the Jazzmaster pickup with eddy-current dampening is shown in **Fig. 5.9.10**. The ECD on the left refers to the pickup without any dampening sheet metal. The only eddy-current losses are due to the six alnico magnets; they can be modeled by a 72-kΩ-resistor (see the next chapter). The additional dampening effected by the brass sheet (middle section of the figure) is modeled by the 4-kΩ-resistor shunting about 1/6 of the overall inductivity (0,8 H). As is obvious, *magnetic* losses cannot be always modeled by the same *RL*-element. This is because the magnets and the brass sheet influence each other. Every eddy current changes the field geometry and with it the individual coupling effects. The dampening caused by the copper sheet is modeled via the right hand section of the figure. The conductivity of copper is about four times higher than that of brass, and consequently the 4-kΩ-resistance needs to be decreased to 1 kΩ. The partition of the coil remains since the coupling effects to the brass sheet and the copper sheet are about the same.

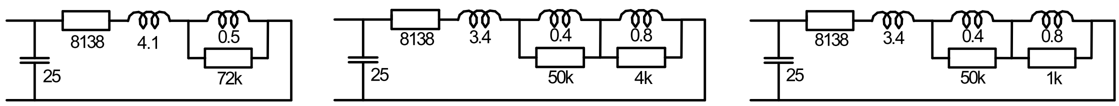


Fig. 5.9.10: Equivalent circuit diagrams for the measurements of Fig. 5.9.9. Left: pickup without sheet metal; middle: with brass sheet; right: with copper sheet. Capacitances are in pF, resistances in Ω, inductances in H.

During the experiments just elaborated sheet metals were brought into proximity of the pickup since their geometry quality could easily be established. Of course, there is no 1-mm-sheet-metal over or under to the Jazzmaster pickup in reality because the pickup is housed in plastic. However, many pickups do have metal bases or metal covers which indeed change the electrical pickup characteristics. The effects of (per se non-magnetic) shielding materials are shown by impedance measurements with a Hoyer-pickup (from the 1960s). The P90-like coil of this pickup is shielded by a metal cover on the surface towards the strings. **Fig. 5.9.11** shows the effects of this shielding on the impedance frequency plot. The eddy currents do not only dampen and attenuate the resonance peak more strongly; the resonance frequency increases, as well.

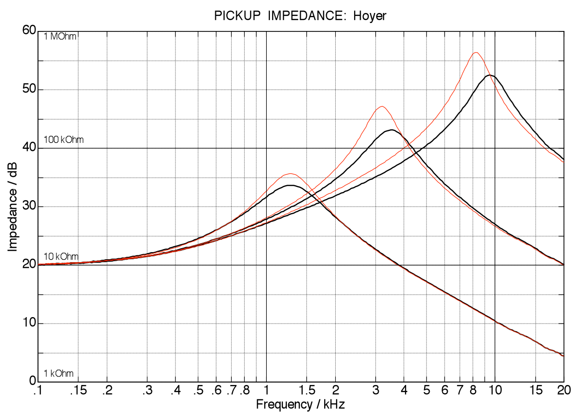


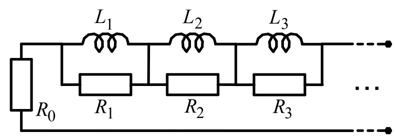
Fig. 5.9.11: Frequency response of the impedance of a Hoyer pickup. The bold lines represent the original condition while the thin lines refer to the cover taken off. The pickup is loaded with 4700pF, 700pF, 0pF, respectively.

The eddy-current dampening effect due to the shielding cover attenuates the high frequencies and reduces the reproduction brilliance. If this is thought to be a disadvantage, the cover may be replaced by one made of plastic.

5.9.2.3 Equivalent two-terminal networks

Magnetic pickups may be represented as two-terminal network or as four-pole network. The basis for the design of a **two-terminal equivalent circuit diagram** is the (frequency dependent, complex) resistance – the **impedance** – measured at the two terminals. In addition, the transfer characteristic may be described by way of this equivalent circuit diagram being extended by two further terminals yielding the four-pole equivalent circuit diagram (chapter 5.9.4). Circuits (networks) are equivalent with respect to impedance if their impedance-functions $\underline{Z}(f)$ correspond; topology and component values may in fact be rather different.

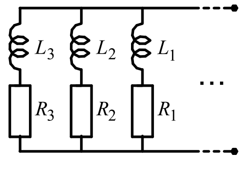
The higher the order n (the number of independent storages) of the network, the larger the number of possible impedance-equivalent but structurally different networks is. In network synthesis three topologies are of particular significance: the resistance partial fraction circuit (**RPFC**), the conductance partial fraction circuit (**CPFC**) and the continued fraction circuit (**CFC**). For the RPFC (**Fig. 5.9.12**), the network analysis is done via series connection of individual impedances, for the CPFC (**Fig. 5.9.13**), this is done via a parallel connection of individual admittances, and via alternating series and parallel connections for the CFC (**Fig. 5.9.14**).



$$\underline{Z} = R_0 + \sum_i \frac{1}{1/R_i + 1/pL_i}$$

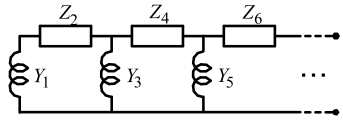
Fig. 5.9.12: Resistance partial fraction circuit (RPFC)

The RPFC is highly suitable to describe magnetic pickups. The DC-resistance is - as R_0 - directly found in the diagram, the values L_i are easily interpreted as components of the overall inductance, and the loss resistances can be attributed via the transformer-equivalent. If all resistances R_i ($i \geq 1$) are finite, the impedance approaches a real, constant value at high frequencies. If one of the resistors is omitted ($R_i=\infty$, $i \geq 1$), the impedance approaches - for high frequencies - a straight line increasing proportionally with the frequency. The CPFC delivers the same impedance, but the large inductance values occurring here are more difficult to interpret, and *R*DC is not immediately evident, either. The CFC shown in Fig. 5.9.14 yields *R*DC in a straightforward manner but is not used due to the high inductivity values.



$$\underline{Z} = \frac{1}{\sum_i \frac{1}{R_i + pL_i}}$$

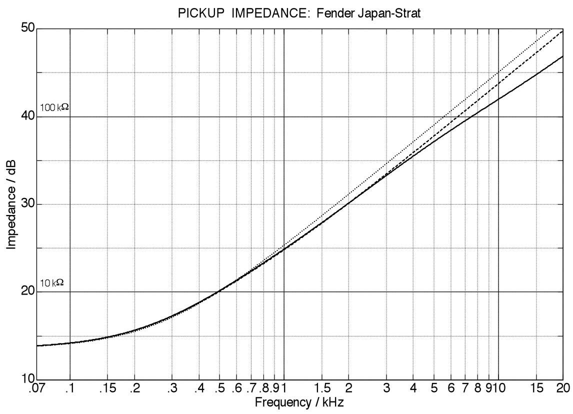
Fig. 5.9.13: Conductance partial fraction circuit (CPFC)



$$\underline{Z} = Z_6 + \cfrac{1}{Y_5 + \cfrac{1}{Z_4 + \cfrac{1}{Y_3 + \cfrac{1}{Z_2 + 1/Y_1}}}}$$

Fig. 5.9.14: Continued fraction circuit (CFC)

Understanding the impedance frequency plot of a resistance partial fraction circuit (**Fig. 5.9.15**) is made easy by - as a first step - assuming all resistances except $R0$ to be infinite. What remains is merely a series-RL-circuit the impedance value of which can be approximated by R_0 at low frequencies and by $\omega(L_1 + L_2 + L_3)$ at high frequencies. For magnetic pickups the impedance growth towards high frequencies is not proportionally to ω but with a shallower slope, and this behavior can be modeled by decoupling the partial inductances using the resistors coupled in parallel. The effective inductance now decreases with increasing frequency and the phase angle does not approach 90° but a smaller value.



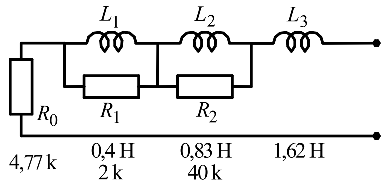


Fig. 5.9.15: resistance partial fraction circuit, magnitude of frequency response. For the upper curve, $R_1 = R_2 = \infty$ was assumed, for the middle curve $R1$ = 2 kΩ, $R2 = \infty$, and for the lowest curve $R1$ = 2 kΩ, $R2$ = 40 kΩ.

The differences between the curves shown in Fig. 5.9.15 may seem rather small. However, the magnitude by itself is not adequate to unambiguously describe a network. As soon as a **capacitor** is connected (capacitance of the coil, or of a cable), real and imaginary part change in different manner. It is therefore necessary to precisely model both real and imaginary part and not only their magnitude. Depicted in **Fig. 9.5.16** are impedance frequency plots as they result from a capacitor of 1 nF being connected to the terminals of the CPFC according to Fig. 9.5.15. For the dashed line, again R_1 = 2 kΩ, $R_2 = \infty$ was set, the solid lines refer to the unchanged circuit (R_1 = 2 kΩ, R_2 = 40 kΩ). Although the magnitudes of the impedances of the circuits without capacitor are almost identical at 3 kHz, there are large differences with a capacitive load. This clearly demonstrates that a high precision is necessary when putting together an impedance-equivalent circuit diagram.

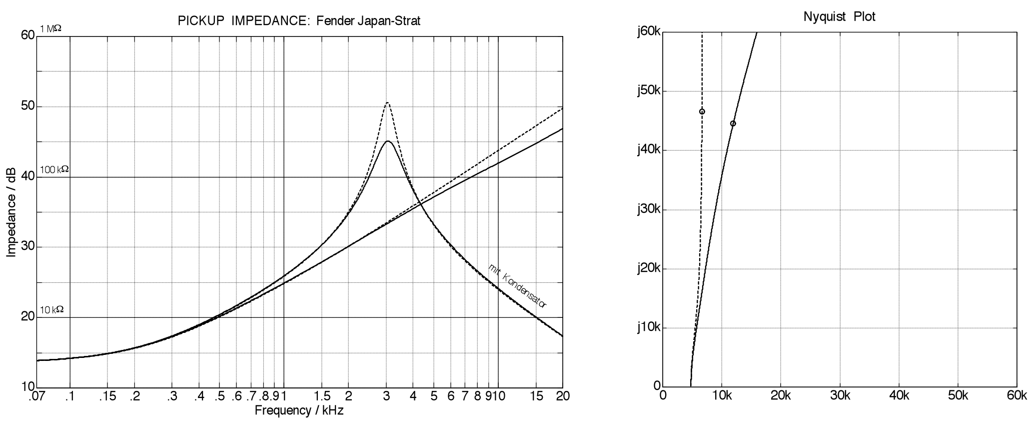
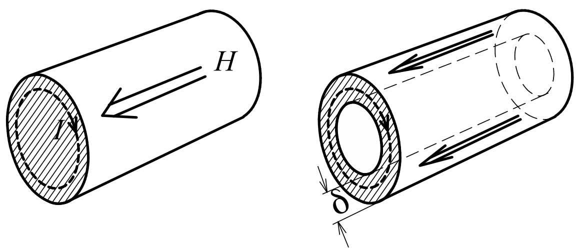


Fig. 5.9.16: Frequency responses of the impedance magnitude of the circuit according to Fig. 5.9.15, with and without capacitive load; Nyquist plot of the impedance without capacitive load (right, marking at 3 kHz.

5.9.2.4 Eddy currents in the magnetic conductor

As a ferromagnetic, *electrically conductive* material is brought into a time-variant magnetic field, there are two effects: the (relative to air) higher permeability of the material increases the magnetic flux density, and at the same time eddy currents diminish this permeability. Since eddy currents are proportional to the temporal changes of the magnetic field, the effective permeability (and thus also the inductance) decreases with increasing frequency. As we have already shown for the non-magnetic conductor, the eddy currents generate active power drawn from the primary field – the pickup receives a dampening♣.

Fig. 5.9.17 depicts a cylindrical magnetic conductor axially permeated by a magnetic field H. Examples for such a scenario are the magnets as they are found in typical Fender pickups under each string, or the pole-pieces (slugs) of a pickup with a bar magnet. If the field is flowing in the direction as indicated and increases over time, it induces a clockwise flowing eddy current I. This eddy current weakens the primary (generating) field, especially close to the axis. As a simplification we can imagine that the axial area is left without any field at all, and a magnetic flux remains only in a thin border layer with a **depth penetration** δ (**skin effect**). δ depends on the electrical conductivity ρ, on the frequency f, and on the permeability μ. Permanent magnets show practically no skin effect in the audio range due to their small reversible permeability (μ_r = 1.1 – 5) and their relatively bad conductivity (≈ 0.6 Ωmm²/m). Steel behaves less favorably: at 2 kHz we get merely $\delta \approx 0.4$ mm (with $\mu_r = 100$). The magnetically effective cross-section is thus reduced to 1/7th!



$$\delta = \sqrt{\frac{\rho}{f\,\pi\,\mu_r\,\mu_0}}\,; \quad [5]$$

Fig. 5.9.17: Cylinder with axial magnetic field H, eddy current I, and border layer with penetration depth δ.

The penetration depth δ (also called conductive-layer thickness) determines both the cross-sectional area for the eddy current and the magnetically effective cross-section area. Both areas are reciprocal to the *square-root* of the frequency, and since the square-root is an irrational function, the impedance cannot be described with a rational function (i.e. a function with a finite number of polynomial sections) – and therefore cannot be modeled with a finite number of components. An equivalent circuit diagram as given in Fig. 5.9.7 is only possible below the approximated cutoff frequency. For **magnets** (with small μ_r), this cutoff lies above the relevant frequency range, and consequently a single loss resistor is sufficient. The common pole pieces (with a larger μ_r) require a more elaborate modeling including several R//L-two-terminal networks. Of course, the desired accuracy plays a role, as well: the circuit behavior can always be reproduced in principle with one coil, one capacitor and two resistors – however depending on the situation there may be considerable differences to the original.

♣ See the theoretical derivation in chapter 4.10.4

Apart from eddy currents there is a further source for losses: the **remagnetization** of the iron and magnetic parts requires energy which is taken from the magnetic field, as well, and thus requires a load resistor in the equivalent circuit diagram. Since the magnetic field changes direction twice with every period of the signal, the remagnetization losses increase with rising frequency. Other lossy mechanisms do exist – however, these are of minor importance.

The following measurements were taken from the „screw-coil“ of a **Gibson** humbucker (PU490). The pickup was disassembled and the screw-coil removed. Unscrewing the 6 screws leaves a coil without ferromagnetic parts. Its impedance can be described rather perfectly by a resistor (4379 Ω), an inductance (1125 mH) and a capacitor (43 pF). **Fig. 5.9.18** shows the magnitude frequency response of the impedance with and without coil capacitance. In conjunction with the inductance, the capacitance causes a resonance maximum at 23 kHz.

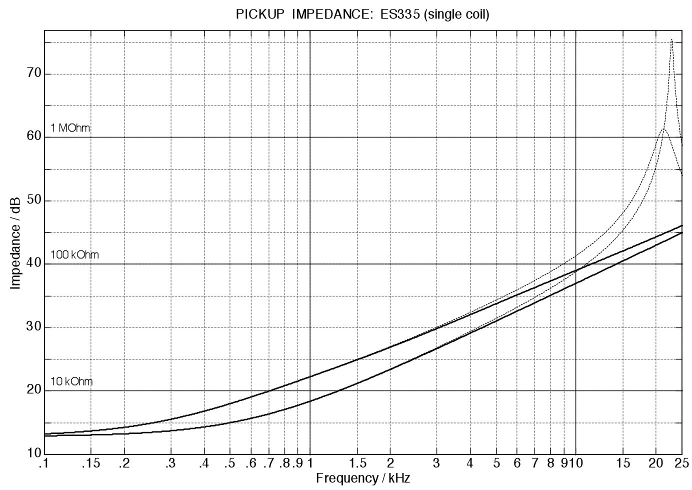
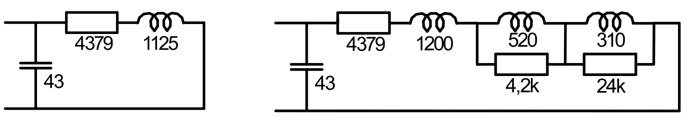


Fig. 5.9.18: Calculated magnitude of the impedance of a coil, with (---) and without (—) coil capacitance. The curve with smaller impedance belongs to the coil with all iron removed, the curves above it refer to the coil with mounting block and 6 screws.

Adding in the mounting block located beneath the coil does not change the impedance frequency plot much. Not until the 6 screws are moved in place does the inductance increase significantly: the impedance curve slides upwards. However, it does not run in parallel with the original course. The reason is the appearance of the eddy current which – with rising frequency – increasingly displace the magnetic field out of the screws and partially undo the inductivity gain. Since the impedance increase of the upper curve (with iron) is not anymore proportional to the frequency, and approximation with several *RL*-sections is required (**Fig. 5.9.19**). The influence of the iron screws is considerable, as the transfer frequency responses shown below in Chapter 5.9.3 also show. If eddy-current losses are undesirable, it is possible to moderate their effects by lamination of the sheet metals or the ferrite materials.



For a high degree of accuracy of the approximation, more than two R//L-two-terminal networks are required.

Fig. 5.9.19: Equivalent circuit diagrams for a PU-490 coil without (left) and with (right) 6 pole-screws and mounting block. Capacities given in pF, resistances in Ω, inductances in mH. The frequency response of the impedance is shown in Fig. 5.9.18.

As already mentioned, the ECD of Fig. 5.9.19 is only one of several equivalent options. The more components an ECD includes, the more topologies are possible. It is purposeful to divide the total inductance into a series connection of *R//L*-two-terminal networks (RPFC, Chapter 5.9.2.3). Alternatively, a parallel connection of *RL*-series circuits could also be used (CPFC), but here the inductances require very large values, e.g. 100 H. Although the total impedance can be perfectly approximated that way, it is difficult to interpret such a circuit. The series circuit above mentioned series makes more sense: at first glance the inductivity decreasing with increasing frequency is evident.

While the *magnetic* losses were already extensively discussed, the **dielectric losses** may be looked at in a more concise manner. Non-conductors cannot carry any current – and thus no eddy currents, either. In the case of pickups, non-magnetic non-conductors are all insulators, i.e. coil bobbins and the wire insulation. These materials are in fact the source of dielectric losses – this effect is rather indistinct, however (Chapter 5.5).

Magnetizable non-conductors ($\mu >> 1$) are, for example, **ferrites** i.e. ferrimagnetic materials. They may be (but don't have to be) used for field-guiding parts (polepieces) and/or magnetically hard ferrite magnets. The electric conductance of ferrite magnets (e.g. barium ferrite) is very small which makes for almost no eddy currents at audio frequencies. The resonance dampening consequently is less compared to alnico magnets. Since the reversible permeability μ_{rev} of alnico magnets is – by a factor of 3 to 4 – larger that that of ferrite magnets, it is possible to create a larger coil inductance with alnicos ... but: the much higher conductance of alnico leads to eddy currents and thus again to a reduction of the inductivity (see Fig. 5.9.9). How large the differences individually are depends on where the magnets are mounted and which alternating flux penetrates through them. For example, the alnico magnets mounted underneath the coil of the P-90 increase (!) the resonance frequency by as little as 5%. Therefore no big differences could be expected if the alnicos were to be replaced by ferrite-magnets. A stronger effect would occur, on the other hand, from exchanging the cylindrical alnico magnets (penetrated by an alternating field) of a Stratocaster pickup for ferrite magnets: the resonance frequency would rise by about 10 – 15%. However, for many pickups of this type, these considerations have to remain a pure thought experiment, because pushing the magnets out of the coil is dangerous, and the pickup may be irreversibly destroyed.

Material	ρ in $\Omega mm^2/m$		Material	ρ in $\Omega mm^2/m$
Steel for strings	0.20	(ferromagnetic)	Hard ferrite (oxide magnet)	about 10^{12}
Nickel	0.070	(ferromagnetic)	Alnico-Magnets	about 0.4 – 0.7
Iron	0.098	(ferromagnetic)	Magnetically soft ferrites	about 10^6 (up to 10^{12})

Table: Specific resistance ρ of magnet materials.

5.9.2.5 Singlecoils with strong eddy-current dampening

As soon as pickups contain other metal parts in addition to the magnets it is necessary to check whether the equivalent circuit diagrams introduced in Chapter 5.9.2.1 are still of sufficient accuracy. The magnetic alternating flux does not only induce a voltage into the coil but into all other metal parts as well, and this leads to **eddy currents**. In this process, the metal parts act like a shorted secondary coils. The resistance of this short (a few milliohm) is transformed upward with the squared winding transmission ratio (e.g. 5500^2) and results in a non-negligible cross-resistance in the equivalent circuit diagram. **Fig. 5.9.20** shows an impedance measurement for a pickup from Hoyer guitar (made in the 1960s). Underneath the coils there are two bar magnets held by the base-sheet, and in addition there is a shielding cap put over the pickup. In **Fig. 5.9.21** we find a simple ECD containing the winding resistance *R*, the winding inductance *L*, the winding capacitance *C* as well as an additional dampening resistor R_q. Using this diagram, the measured curve can be approximated at 0 Hz and around the resonance; the agreement at 1 kHz is merely moderate, however.

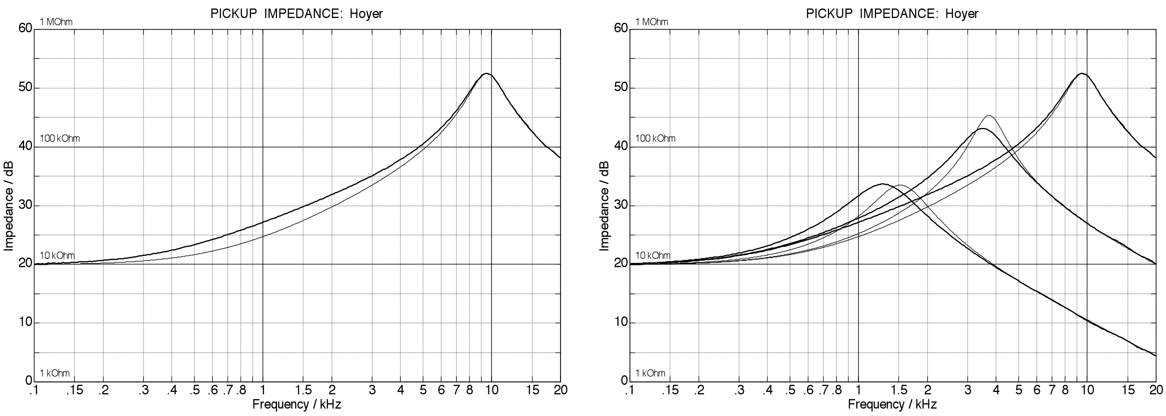
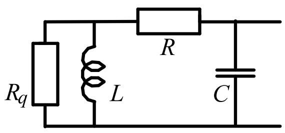


Fig. 5.9.20: Hoyer-pickup, impedanc frequency response. Measurement (——), ECD1-calculation (––––). On the left readings for the unloaded pickup are shown, on the right loads are connected: 4700pF, 707pF, 0pF.



L = 2,17 H R = 9850 Ω

R_q = 540 kΩ C = 125 pF

Fig. 5.9.21: Hoyer-pickup with metal cover. Equivalent circuit diagram ESB1.

The differences grow more noticeable as a customary **guitar cable** is connected to the pickup. Its effect is purely capacitive in the audible frequency range; depending on the length there will be a cross-capacitance of 300 – 1000 pF. The instrument used for the measurement allows for a connection of 0 pF, 707 pF, 4700 pF. The larger the capacitance, the lower the resonance frequency is. In the right part of the figure curves for different capacitive loads are given: the eddy-current losses lead to clear deviations between measurement and calculations. The equivalent circuit diagram presented in Fig. 5.9.21 (with a topology designated **ECD1**) needs to be extended by additional components in order to achieve better agreement.

To obtain a better approximation it is necessary to model the eddy-current losses with the equivalent circuit diagram of a loosely coupled transformer (Chapter 5.9.2.2). There are several equivalent possibilities for this. As already shown in Chapter 5.9.2.3, the series connection of *R//L*-two-terminal networks is particularly easy to interpret; it is used again here. **Fig. 5.9.22** shows the extended equivalent circuit diagram (**ECD2**).

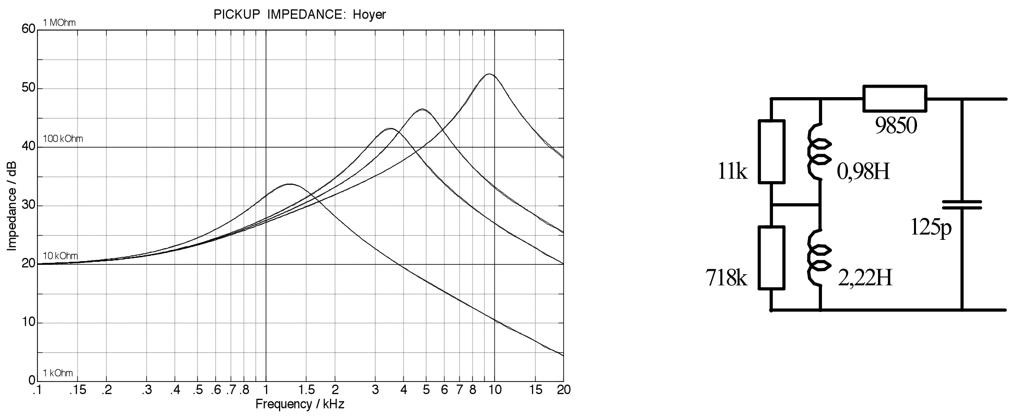


Fig. 5.9.22: Hoyerpickup, impedance frequency plot; 4700, 700, 330, 0 pF load; measurment and calculation are not distinguzishable anymore. ECD2 on the right.

Whether the eddy-current losses indeed need to be modeled depends on the construction of the pickup and the desired accuracy. In many cases (such as for the Stratocaster pickup) already ECD1 delivers very good results. On the other hand, for pickups with additional metal parts more or less significant discrepancies between measurement and calculation should be expected. **Fig. 5.9.23** shows the impedance frequency plots of a P-90 pickup with a capacitive load (0pF, 330pF & 1000pF): with ECD1 there are clearly visible deviations, while for models of higher order a perfect agreement between measurement and calculation can be achieved for the p-90 as well.

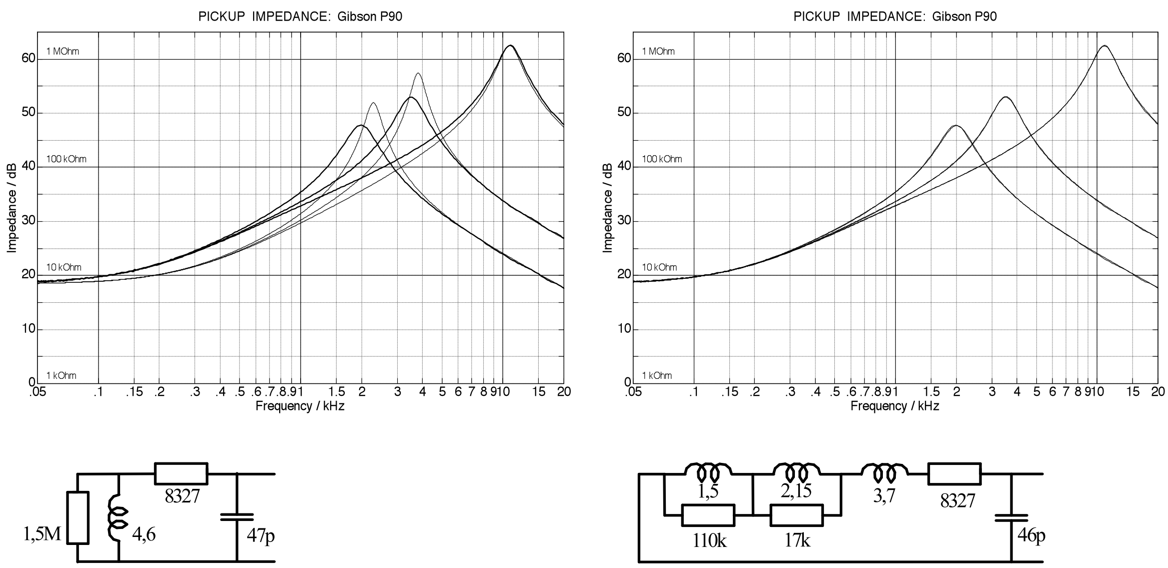
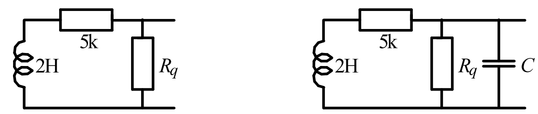


Fig. 5.9.23: Gibson P90, impedance. Measurement (▬▬), ECD-calculation (______). Left: ESB1; right: ESB2. Pickup without coaxial cable. Component values in H, Ω, F.

At this point we will take another look at the effect of load capacities. Every pickup is loaded with a capacitance – in fact only this way it receives its charachteristic resonance. When putting together the equivalent circuit diagram and during the approximnation process, it is necessary to keep this load in mind. The following example includes a simple circuit: a coil of 2 H having a copper resistance of 5 kΩ has a dampening resistor connected in parallel to it. In one case, this resistor has 3 MΩ, in the other it has 300 kΩ. The corresponding frequency response of the impedance magnitude (**Fig. 5.9.24**) shows a difference only at higher frequencies; at 3,5 kHz both magnitudes are almost identical. However, connecting a 1-nF-capacitor in parallel causes considerable deviations between the two magnitude curves.

Without the parallel-connected load capacitor, the impedance magnitude at 3,5 kHz is mainly formed by the imaginary coil-impedance – compared to it the parallel dampening resistor is of high impedance and negligible. A load capacitor connected in parallel compensates the imaginary part created by the coil, and the real part becomes dominant. In the Nyquist curve on the right (showing the real part of the impedance on the abscissa and the imaginary part of the impedance on the ordinate – with the frequency as parameter) sections of two clock-wise curved circles are shown; due to the different coordinate scaling they are distorted to ellipses. For 0 Hz both circles start at about 5 kΩ (more precisely at 5//300 and 5/3000, respectively) and turn upwards in a clock-wise manner. In both curves, f = 3,5 kHz is marked as a dot. The points for which the distance to the origin is constant are indicated with a *dashed* line (this is in fact a circle, but again the different scaling on the coordinates distorts it to an ellipse). As is clearly evident, both indicated 3,5-kHz-points have an almost equal distance to the origin – the magnitude of their impedances therefore is almost identical, but the real parts of their impedance differ by almost a factor of two. This shows that the magnitude of the impedance alone does not give a complete description.



R_q = 300kΩ and 3 MΩ. C = 1 nF.

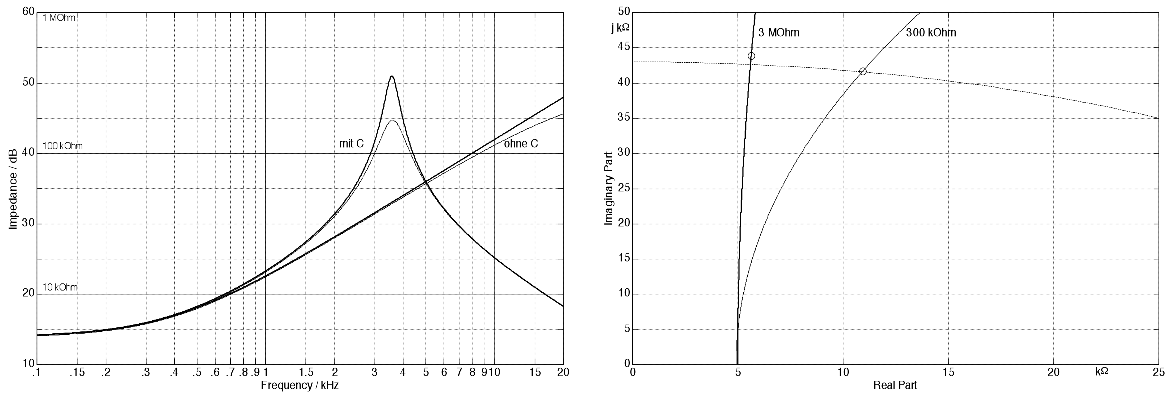


Fig. 5.9.24: circuit (above), frequency response of the impedance magnitude)left); Nyquist curve of the impedance (right). Thin line: 300 kΩ, bold line: 3 MΩ.

5.9.2.6 Gibson Humbucker: coil with screws

Seth Lover, developer at Gibson, reports in [13] that every pickup coil had 4500 turns of thin enameled copper wire. He states an AWG-#42-diameter which equals 64 µm. On the other hand, he also notes that the wire diameter was subject to tolerances dictated by manufacture: "*The DC resistance varied, because the diameter of the wire was not constant. As the diameter decreases the resistance increases, but if the inductance remains within certain tolerances, then it's OK"* [13]. A further manufacturing issue with early Gibson pickups were shorts within the winding (*short turns*) which apparently were due to insufficient insulation; these reduced the resonance emphasis and thus the treble content in the signal. Regarding the magnet material, Set Lover remarks [13]: "*We also used Alnico II and III, and the reason is, that you couldn't always buy Alnico V, but whatever was available we would buy as they were all good magnets*". ISO 9000 hadn't arrived yet.

We can assume that the early Gibson pickups were subject to substantial manufacturing tolerances, and that therefore their transmission characteristics (i.e. their sound) included inter-individual variances. Tom Wheeler writes in his Guitar Book [14]: "*Later Humbuckers have slightly smaller magnets and other minor differences in construction*", and he does add about the color (!) of the cosmetics "*The color of the bobbin has no direct bearing of the tone*" [14]. Seth Lover, however, does not remember any changes [13]: "*No, we kept the pickups pretty much the same, they were all identical. ... Actually the PAFs weren't any better than the later pickups that were built right.*" Seems to be all a question of the point of view. Tom Wheeler [15]: *"The PAF's popularity, which is unsurpassed, is a blend of performance and snob appeal*".

The patent for the Gibson-Humbucker talks about two corresponding coils. A magnetic field emanating from an interference source induces the same interfering voltage into both coils, and the out-of-phase (reverse poled) connection of the two coils causes the two interference voltages to cancel each other out. Production units of the pickup, however, sported two different coils: they included a "slug"-coil and a "screw"-coil. The slug-coil contains 6 cylindrical pins (pole pieces) of a diameter of 4,8 mm. The pins are positioned such that their upper surface is flush with the string-facing surface of the coil while their lower surface extends 3 mm out of the bobbin. The screw-coil holds, instead of the pins, 6 round-head screws of a length of 21 mm and a diameter of 3,2 mm = 1/8". The sensitivity of the pickup can be adjusted for each string individually by rotating the screws.

The following measurements and calculations refer to a bridge pickup of a 1968 Gibson ES 335 TD – it was taken out of the guitar and disassembled (with rather mixed feelings!). The screw-coil (**Fig. 5.9.25**) is penetrated by 6 screws which are screwed into an iron block at the lower side of the coil. Unscrewing all 6 screws allows for taking off the iron block such that a coils without any ferromagnetic parts remains. The impedance frequency plot of this coil is shown in **Fig. 5.9.26**. In the low frequency range, the impedance is determined mainly by the copper resistance, and in the middle frequency range by the reactance of the coil ωL; in the high frequency range the reactance of the capacitance $1/(\omega C)$ is the main factor. At the upper end of the frequency range a pronounced resonance is clearly visible.

$$\omega_{\text{resonance}} = 1/\sqrt{L \cdot C}; \qquad f = \omega/2\pi$$ Resonance frequency

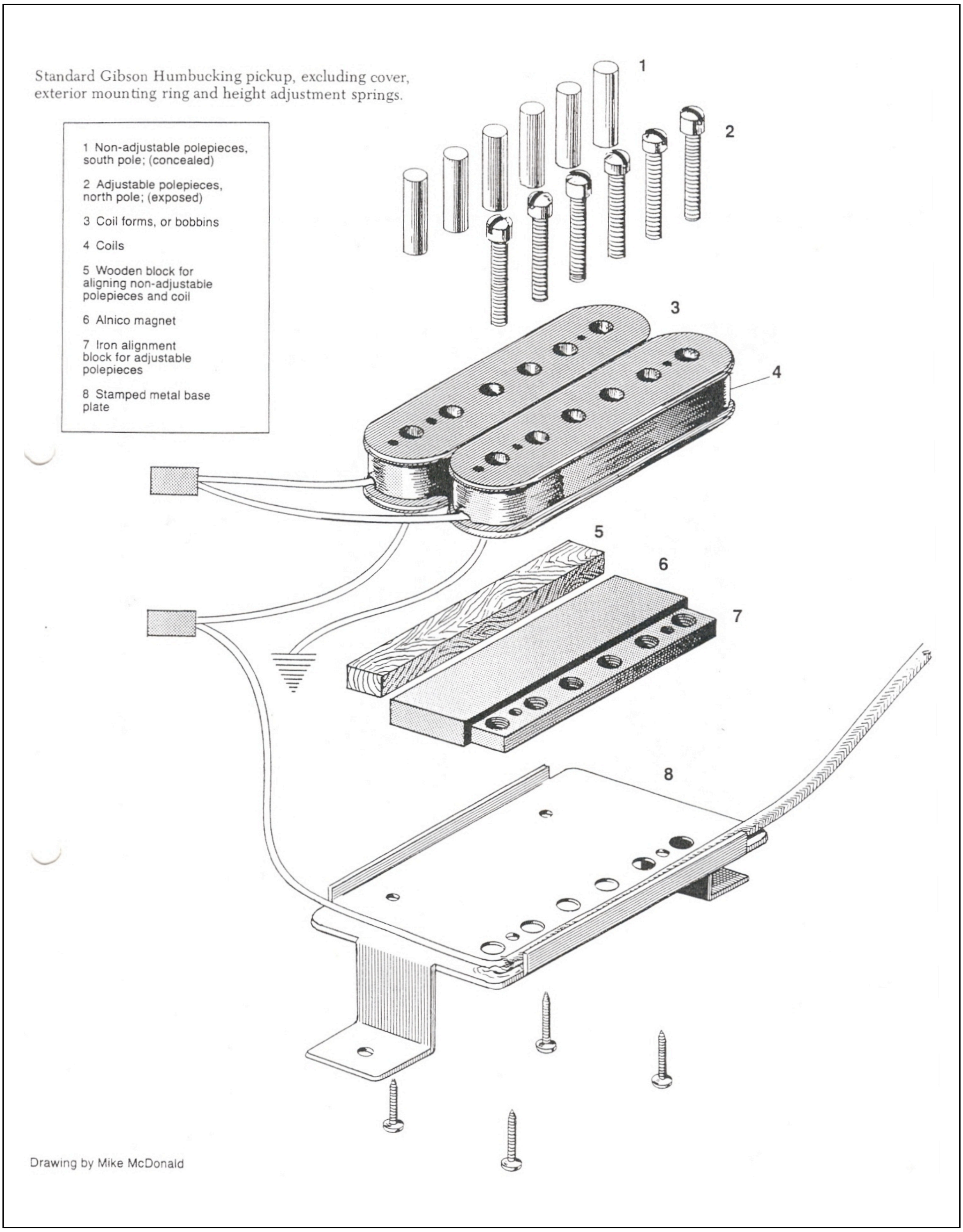


Fig. 5.9.25: Exploded view of a Gibson-Humbuckers (according to Mike McDonald)
1 = fixed pole pin, south pole (not accessible); 2 = adjustable pole screw, north pole;
3 = bobbin; 4 = coil; 5 = wooden spacer; 6 = alnico bar-magnet;
7 = block with threads for the pole screws; 8 = metal base plate.

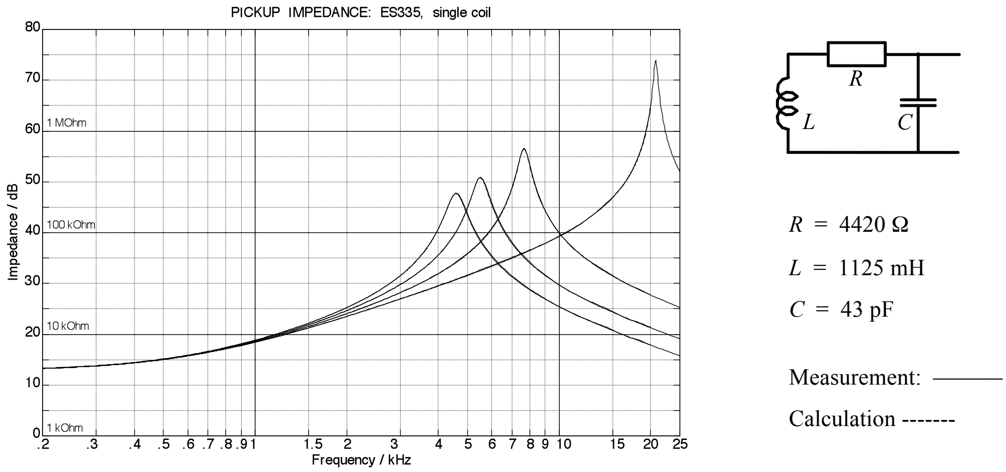
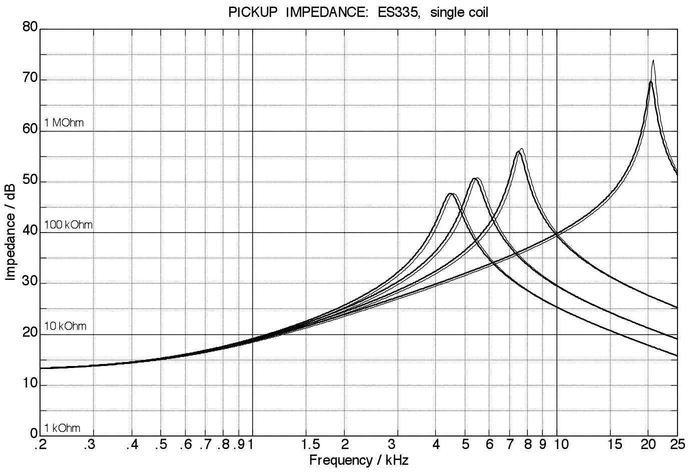


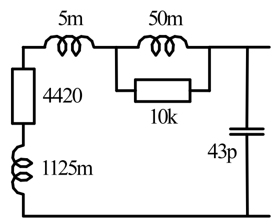
Fig. 5.9.26: Magnitude of pickup-impedance frequency plot. Screw-coil without metal parts. Measurement and calculation (equivalent circuit diagram) are practically identical. The pickup was loaded with 0/330/700/1030 pF.

The measurement gives a DC resistance (copper resistance) of 4420 Ohm. The inductance of the winding determines the impedance increase at middle frequencies. The cutoff-frequency at which the inductive reactance corresponds to the ohmic copper resistance is f_g = 625 Hz. Below f_g, R dominates, above, L dominates. The wound-up coil wire does not only show inductive but also capacitive behavior (winding capacitance), due to the neighboring coils. Inductance L and capacitance C result in a resonance maximum in the frequency response at the resonance frequency $f \approx 1 / \left(2\pi\sqrt{LC}\right)$. The larger the capacitance is, the lower the resonance frequency is located. The measurements were taken without and with a load (in the form of an external additional capacitor) connected to the pickup; this way the resonance frequency of 21 kHz at 0 pF could be lowered to 7.7 kHz (330 pF), 5.5 kHz (700 pF), and 4.6 kHz (1030 pF). The resonance shift gives additional information about the quality of the modeling. As can be seen from Fig. 5.9.26, the measurement and the calculation agree to the line width. The shown equivalent circuit diagram (**ECD**) is therefore well suitable to model the impedance behavior of the pickup coil.

In the real pickup coil, resistance, inductance and capacitance are of course not concentrated into one single point but differentially distributed. Every little piece of wire of the length *dl* contains a partial resistance *dR* and a partial inductance *dL*, and forms a partial capacitance *dC* with all other pieces of wire. Measurement and simulation (calculation) do however show that a modeling of the impedance by concentrated elements (*R, L, C*) is fully sufficient. Whether the transmission characteristic of the pickup can equally well be described this way needs to be investigated separately (see below).

Next, the screw-coil is to be looked at in conjunction with the iron **block**, but still without screws. For the measurements, the block was fixed in its normal position underneath the coil using sticky tape. The ferromagnetic behavior of the block reduces the magnetic resistance in the magnetic circuit; permeability and inductivity are increase that way.





ECD: Screw-coil with iron block (compare to. Abb. 5.9.26).

Fig. 5.9.27: Frequency response of the pickup-impedance magnitude. Screw coil with/without iron block. ECD.

Fig. 5.9.27 shows a comparison of the impedance frequency plots with/without the **block**. The inductivity is increased by the presence of the block, which leads to a lowering of the resonance frequency. This is not a pronounced effect, though, and could be ignored for a simple model. A precise model requires that on top of the inductance increase, the **iron losses** are reproduced, as well. To re-magnetize the iron, energy is necessary which is taken out of the electrical circuit (re-magnetization losses). In addition, the time-variant magnetic field causes **eddy currents** to be induced in the iron – again these are fed energy from the resonant circuit. Coil and block may be though of as a transformer coil: the block represents a short-circuit winding withdrawing energy from the pickup coil. In the end, the block is made warmer; this effect is however so minute that the temperature-rise will not be noticeable. Still, the dampening effect in the resonance circuit can be seen in the frequency response of the impedance: the impedance maximum is slightly reduced.

Modeling the iron losses is complicated. In the above example it could be dispensed with, as well, since the effect is so weak. However, when inserting the screws into the coil, substantial iron losses occur which may not be ignored anymore. A fundamental discussion is therefore necessary. In every electrically conductive material a time-variant magnetic field causes eddy-currents; these flow on a circular path within the conductor. Since the cross section of the block is relatively large, the eddy currents meet merely a rather small resistance, and the load is of relatively low impedance. With increasing frequency, however, a displacement of the current happens which is called '**skin effect**'. The eddy currents do not flow in the whole block anymore, but only along the surface of the block. The cross-section available to the current flow is reduced and the resistance increased. This resistance increase has a dependency on frequency described with the square root. Since the square-root, however, is not a rational but an irrational function, it is not possible to model the corresponding behavior with a finite number of RLC-elements. An infinite number of elements are not a practicable solution, though. Again, an approximation presents itself as a way out: the skin effect may be modeled by a special RL-two-terminal network. The higher the requirements regarding accuracy, the more components need to be put into his two-terminal network. For most pickups, however, 3 – 7 elements will suffice (Chapter 5.9.2.2).

The block is modeled in **Fig. 5.9.27** by three additional components (5 mH, 50 mH, 10 kΩ). The two coils increase the overall inductivity (ferro-magnetic effect of the block), at high frequencies, however, the 50-mH-coil does not have its full impact anymore due to the resistor connected in parallel (modeling of the skin effect). With this ECD, the reproduction of the impedance is of such accuracy that it agrees with the measurement up to line-width.

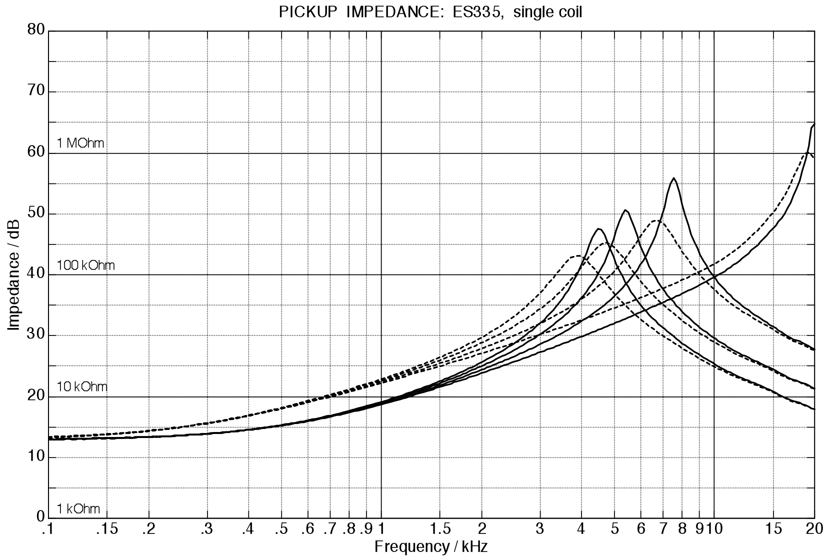


Fig. 5.9.28: Comparison: screw-coil with block, without (——) nd with **screws** (-----).

The impedance changes clearly once the 6 **pole-screws** are inserted into the bobbin **(Abb. 5.9.28)**. The ferromagnetic screws are positioned in the area of strong magnetic alternating flux; they substantially reduce the magnetic resistance and thus increase the coil inductivity. On the other hand, the screws also produce losses, which is why the ECD requires additional (ohmic) loss resistors.

Further difficulties arise from combining coil, screws, and block: the screws change the distribution of the magnetic field in space. With the screws, the magnetic field permeates the block in a different manner than without the screws. Consequently, the block-ECD, which is valid for the setup without screws, cannot be used once the screws are added. Let us be reminded at this point that the equivalent circuit diagrams present here do not model the distribution of the field in space but are equivalents for the impedance. It is only possible to explicitly identify the resistor *R* effective for DC. All other elements of the ECD are the result of an approximation without physical correspondence.

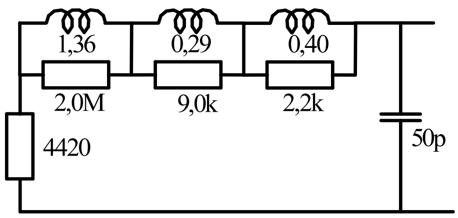


Fig. 5.9.29: Equivalent circuit diagram (ECD) for "screw-coil with blocjk and screws", Gibson-Humbucker. Compare to Fig. 5.9.30.

In **Fig. 5.9.29** we see an equivalent circuit for the screw-coil with block and screwed-in screws. The impedance frequency plot is very well approximated with it, as shown by measurement (——) and calculation (-----) in **Fig. 5.9.30**; both families of curves are practically identical.

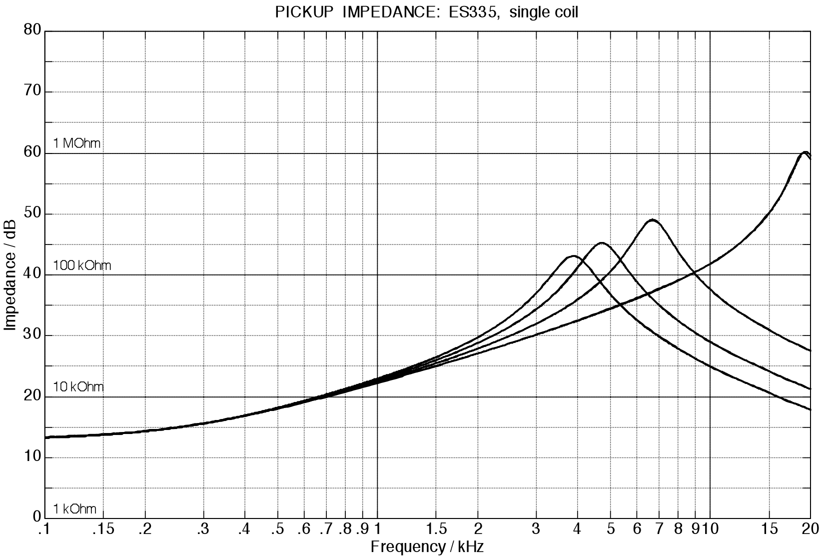
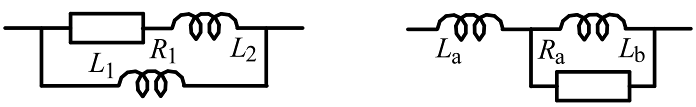


Fig. 5.9.30: Frequency response of impedance: "screw-coil + block+ screws".
Measurement (—) = calculation (----).

The circuit according to Fig. 5.9.29 is not the only one possible – there are several equivalent replacements with the same high quality of approximation. **Fig. 5.9.31** shows two simple equivalent circuit diagrams, both including a resistor and a two independent coils each. Consequently, the impedance functions are of 2nd order and include the frequency to the power of 2, 1 and 0. Equating the corresponding polynomial coefficients (comparison of coefficients), we obtain 3 requirements for the 3 components; this enables the conversion of the component values from one circuit into the components of the other. For circuits of higher order (i.e. additional coils), there are still more different topologies with the same impedance frequency plot. Which equivalent circuit is used in the end remains a matter of taste. The supplement via series connection of RL-parallel-circuits as proposed in Fig. 5.9.29 appears more purposeful than the parallel connection of RL-series circuits, though (see also Chapter 5.9.2.3).



$$L_1 = L_a + L_b$$
$$L_2 = L_a \cdot (1 + L_a / L_b)$$
$$R_1 = R_a \cdot (1 + L_a / L_b)^2$$

Fig. 5.9.31: Two equivalent circuit diagrams wth equal imepdance frequency response.

The only purpose of the equivalent circuit diagrams present here is to deliver impedance frequency plots equal to those obtained in measurements, and to create the basis for equivalent circuit diagrams of the transmission. As soon as a purposeful compromise between component-complexity and accuracy was found, the approximation was successfully applied and not optimized further.

The consideration about impedance related to the coil with screws and block. Now we add the bar-**magnet**, finally transforming the coil which by itself is insensitive to string vibrations into a pickup. The effects of the magnet on the transmission behavior are of existential importance; its influence on the impedance is, however, very small. It does belong to the group of ferro-magnetic materials but its permeability at the operating point is relatively small, and moreover it is not within the coil but positioned to the side of it. **Fig. 5.9.32** shows the frequency response of the inductivity with and without magnet. Adding the magnet makes for a slightly larger inductivity (compare to Fig. 5.9.33), for a quality factor which is a bit reduced, due to the eddy currents induced in the magnet, and in the end for a slightly better coupling of the two coils.

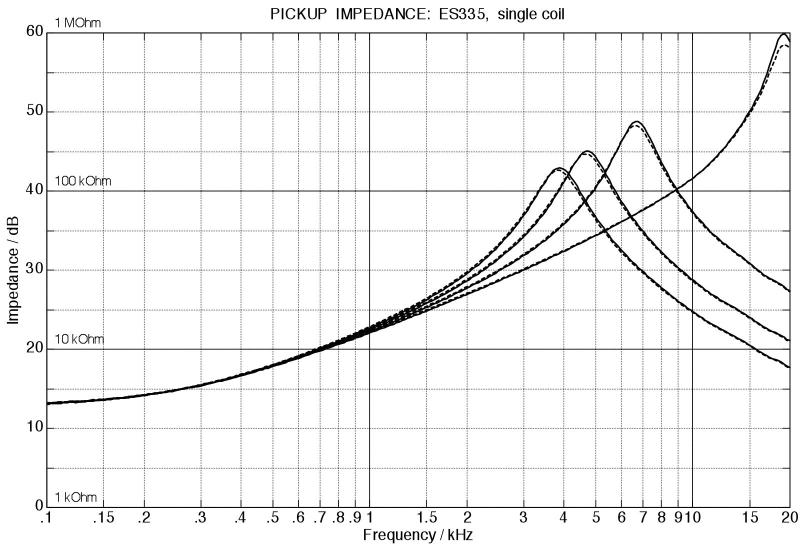


Fig. 5.9.32: Frequency response of the impedance: "screw-coil + block+ screws";
without (——) / with (-----) **magnet**

The effects of the magnet on the inductivity show most at mid-range frequencies. **Fig. 5.9.33** depicts an enlarged part from Fig. 5.9.32. The inductivity increase amounts to merely 2 – 3 %.

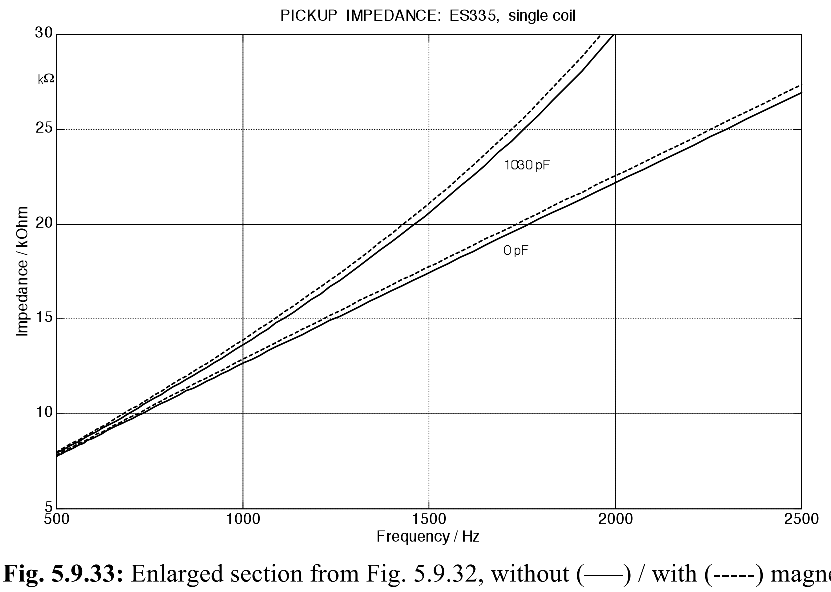


Fig. 5.9.33: Enlarged section from Fig. 5.9.32, without (——) / with (-----) magnet.

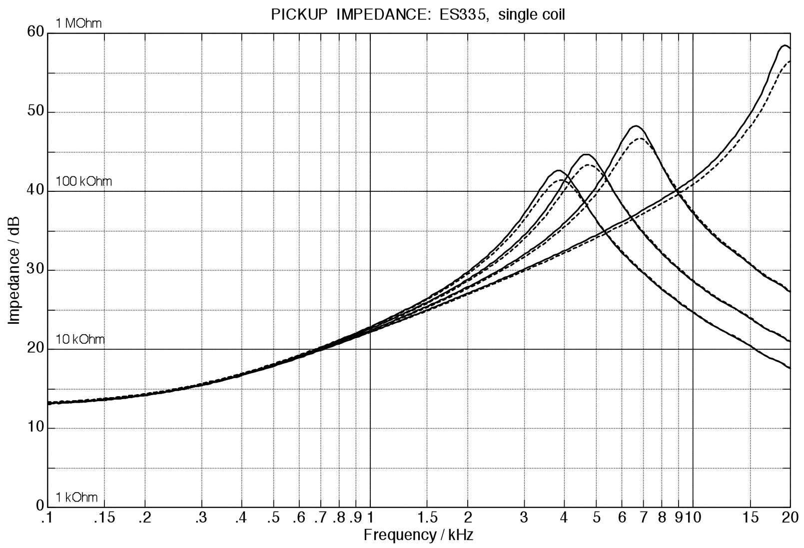


Fig. 5.9.34: Impedance: screw-coil + block+ magnet; without (——) / with (-----) **sheet metal**

As a next step, the sheet metal forming the base plate of the pickup is added in. It consists of German silver, which is a non-ferro-magnetic material. Its specific resistance is 0.3 $\Omega mm^2/m$, compared to 0.018 $\Omega mm^2/m$ for copper. Still, even with this higher resistance eddy-current losses cannot be completely avoided, as **Fig. 5.9.34** shows. The resonance quality factor has again gone down compared to that of Fig. 5.9.32.

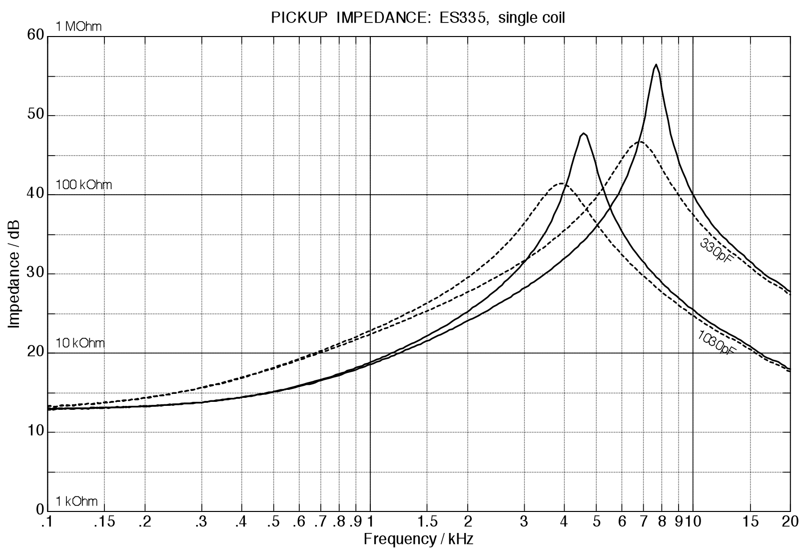


Fig. 5.9.35: Impedance: screw-coil without any metal parts (——);
screw-coil + block + screw + mgnet + base plate (-----); loaded with 330 pF and 1030 pF.

Fig. 5.9.35 indicates a summary of the influences of the metal parts on the impedance. The increase of the inductance is due to the screws, the decrease of the emphasis is caused by the screws and the base plate. The metal pickup cover had been removed in the past – it would have further reduced the emphasis.

Now, the fully assembled **slug-coil** is mounted next to the screw-coil, but it is not yet electrically connected. The measurements are still directed to the screw-coil; the objective is to clarify whether the magnetically soft slugs increase the inductivity of the screw-coil. As **Fig. 5.9.36** shows, this is not really the case; the two measurements differ merely by the line width. The ferromagnetic slugs are positioned so far away from the screw-coil that they have little disturbing effect on the latter's magnetic field. The magnetic coupling of both coils is, however, not zero (Chapter 5.9.2.8)!

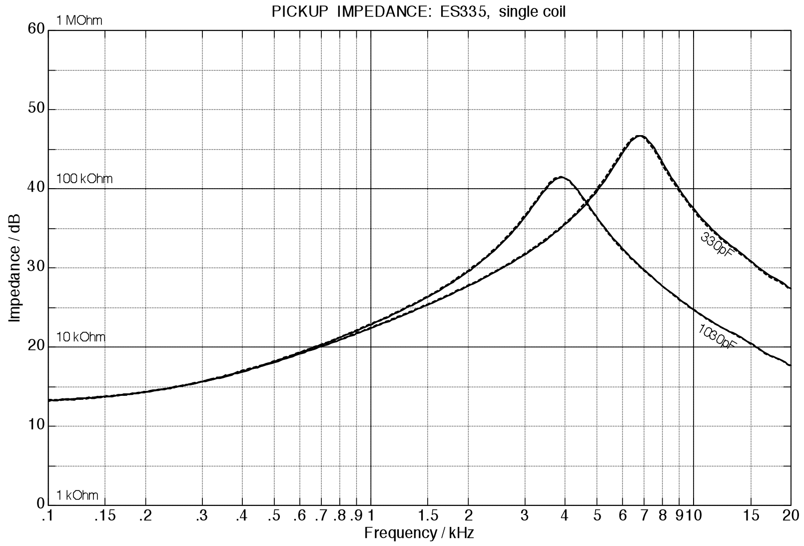


Fig. 5.9.36: Impedance: complete screw-coil without (——) and with slug-coil next to it (-----); loaded with 330 pF and 1030 pF.

Finally, **Fig. 5.9.37** shows the equivalent circuit diagram for the impedance of the complete screw-coil. We thus have arrived at an impedance model for an individual specimen of a humbucker taken from a ES-335-TD made in 1968. On the basis of this pickup we could exemplify the influences of various pickup components; however, we must not expect that every Gibsom Humbucker can find its exact description in this equivalent circuit diagram. Manufacturing tolerances and modifications have led to different versions and thus to different data.

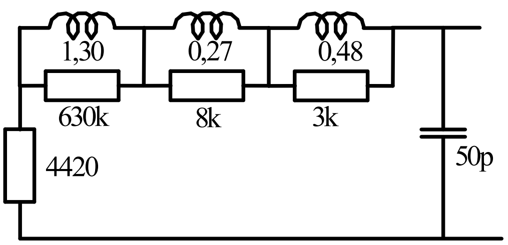
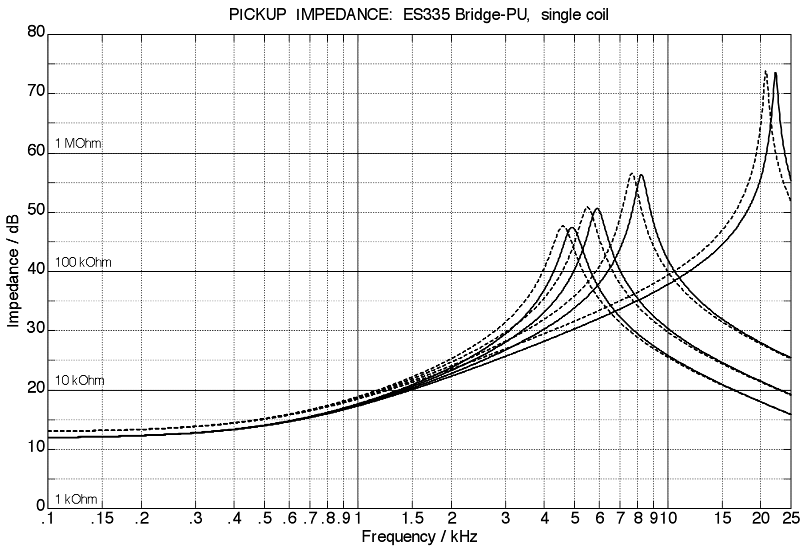


Fig. 5.9.37: Equivalent circuit diagram for the impedance of the screw-coil of the fully assembled pickup.

5.9.2.7 Gibson Humbucker: coil with slugs

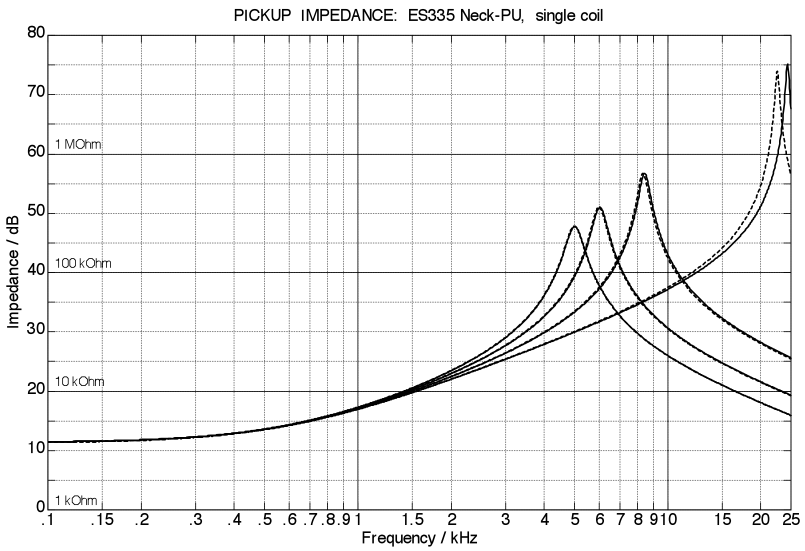
The Gibson Humbucker contains *two* coils: a coil with screws, and a coil with slugs. In order to accomplish the hum-suppression, both coils should feature the same electrical characteristics. Comparative measurements yield an inconsistent picture (**Fig. 5.9.38**): the neck pickup of the ES-335 under investigation indeed had two equivalent coils, while for the bridge pickup (of the same guitar) there were differences. The slug-coil featured an 11% smaller DC resistance and a 13% smaller inductivity compared to the screw-coil; most probably the coils differ in the number of turns. We cannot determine anymore whether this **lack of symmetry** was on purpose, or happened by accident during manufacture (in 1968). In any case it seems not entirely undesirable; otherwise Gibson would not offer, in the form of the new "Burstbucker", a pickup which replicates the uneven number of turns found in old humbuckers.



Slug-coil:
0,98 H, 3928 Ω, 47 pF.

Screw-coil:
1,13 H, 4420 Ω, 47 pF.

Fig. 5.9.38a: Comparison screw-coil (---) vs. slug-coil (——). Only bobbin and wire for a ES335-**bridge**-pickup.
1000pF, 700pF, 330pF, 0pF.



Slug-coil:
0,95 H, 3693 Ω, 40 pF.

Screw-coil:
0,96 H, 3660 Ω, 47 pF.

Fig. 5.9.38b: Comparison screw-coil (---) vs. slug-coil (——). Only bobbin and wire for a ES335-**neck**-pickup.
1000pF, 700pF, 330pF, 0pF

The impedance frequency plots shown in Fig. 5.9.38 were measured with the coils taken off the pickup assembly, i.e. there were only the bobbins wound with the wire, and no metal parts were included. In the assembly process, the inductances increase due to the effect of the ferro-magnetic metal components (Fig. 5.9.39).

6 ferro-magnetic metal cylinders (“slugs”) are inserted into the **slug-coil** (∅ = 3/16" = 4,8 mm), and 6 ferro-magnetic metal screws are screwed into the **screw-coil** (thread-∅ = 1/8" = 3,2 mm, head-∅ = 3/16" = 4,8 mm); in both coils the inductance is increased by these metal parts, while the resonance emphasis is decreased – but not in the same way. With the neck pickup (having rather similar “empty” coils, Fig. 5.9.38b) there is a larger inductivity increase at low frequencies in the screw-coil; from 1 kHz, however, the slug-coil features the larger inductivity. The resonance emphasis of the screw-coil decreases more strongly that of the slug-coil (**Fig. 5.9.39b**). These effects also appear for the coils of the bridge pickup (**Fig. 5.9.39a)** but the different numbers of turns hamper any analysis.

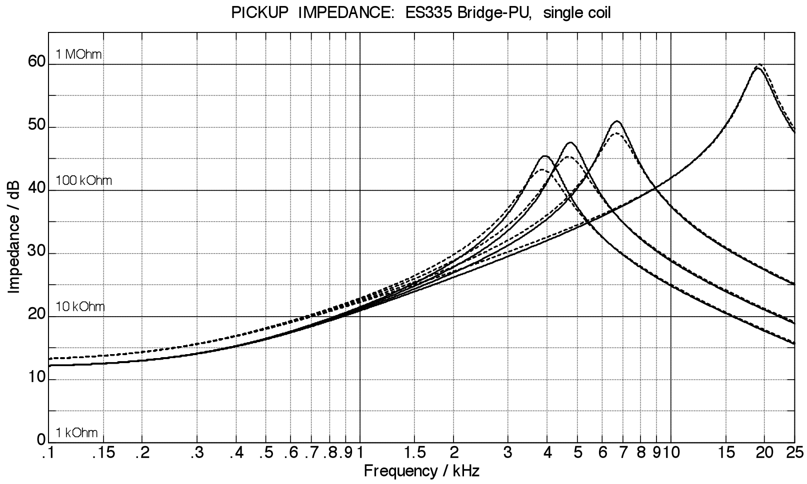


Fig. 5.9.39a: Comparison: screw-coil with screws, with block (---); slug-coil with slugs (——). Without bar magnet and base plate ES335-**Bridge** pickup. 1000pF, 700pF, 330pF, 0pF.

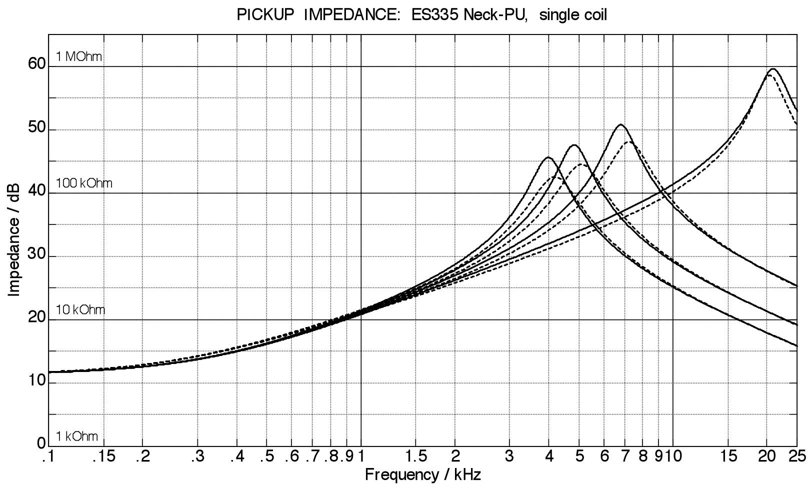


Fig. 5.9.39b: Comparison: screw-coil with screws, with block (---); slug-coil with slugs (——). Without bar magnet and base plate ES335-**Necl** pickup. 1000pF, 700pF, 330pF, 0pF.

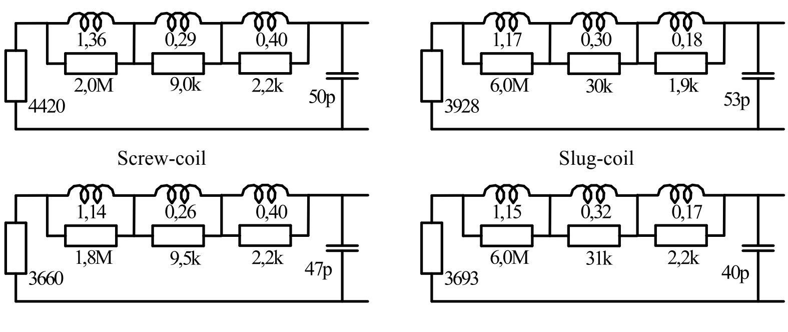


Fig. 5.9.40: Equivalent circuit diagrams for the impedances in Fig. 5.9.39 Bridge pickup (top) Neck pickup (bottom)

5.9.2.8 Gibson-Humbucker: coil-coupling

The two coils of a typical Gibson Humbucker (e.g. 490R; but not P-100) are connected in series. Therefore, it could be expected that their impedances add. For the DC resistance this assumption is correct; for frequencies which are not zero, however, there are deviations. The measured impedance of the series connection is larger than the sum of the individual impedances. The reason for this is the magnetic coupling of the two coils, and a different provision for the addition results (compare Chapter 5.5.2).

Two series-connected inductivities L_1, L_2 carry the same current $\underline{I}$. If there is no magnetic coupling, a voltage $j\omega\cdot\underline{I}\cdot L_1$ and $j\omega\cdot\underline{I}\cdot L_2$ across them is created, respectively. However, if the magnetic flux generated in one of the coils partially or completely penetrates the other coil, it will induce an additional voltage there, depending on the coupling factor [20, Band II]. The coupling factor k is zero for non-coupled coils; it is +1 for coils ideally coupled in the same sense and -1 for coils ideally couple in the inverse sense. Ideal coupling is not possible in reality; therefore, the magnitude of the coupling factor needs to be always smaller than 1. Equal-sense coupling implies that the coil voltage is increased by the coupling – this is the case for humbuckers ($0 < k < 1$). As two coils coupled by a joint magnetic field are connected in series, the overall inductivity of this series connection is

$$L_\Sigma = L_1 + L_2 + 2k \cdot \sqrt{L_1 \cdot L_2} \approx (L_1 + L_2) \cdot (1 + k)$$ Summed inductivity

In **Fig. 5.9.41** we see the frequency responses of the impedances of the (separately measured) coils of a Gibson Humbucker. The pickup was fully assembled and the coils isolated against each other. A distinct non-symmetry is recognizable with the screw coil having a higher impedance. Connecting the two coils with the customary polarity in series results in an overall impedance which is not the sum of the individual impedances but one which has an about 20% larger value. Only at very low frequencies the overall impedance corresponds to the sum of the sum of the individual impedances (**Fig. 5.9.42**). The coupling factor of the two coils thus amounts to 20%, i.e. a fifth of the magnetic flux created by one coil penetrates the other coil as well and induces a voltage there. If one coil would be connected with reverse polarity, the overall voltage would decrease by 20% – no measurements for this are shown here.

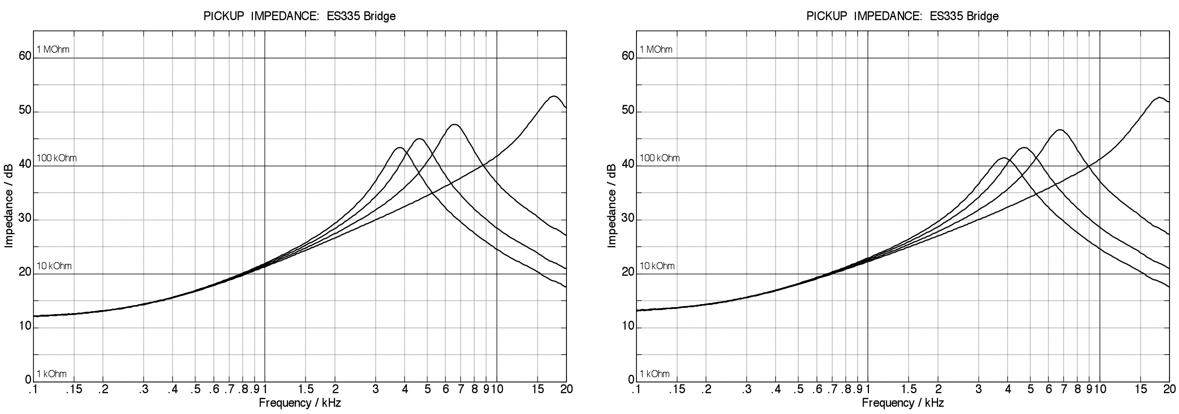


Fig. 5.9.41: Frequency repsonses of impedances: ES-335-bridge-pickup, slug-coil (left), screw-coil (right). Capacitive load: 1030 pF, 700 pF, 330 pF, 0 pF. For the measurement, the coils were insulated against each other.

In the left part of **Fig. 5.9.42**. the calculated sum of the individual impedances is shown; the solid curve depicts the measurement results. In the right-hand section of the figure, the equivalent circuit diagrams of the individual coils formed the basis of the calculated curve for the sum (Chapter 5.9.2.6 and 5.9.2.7), with all inductances and loss resistances being increased by k = 20%. With this, the agreement with the measurement results is much better – the only discrepancy shows for the connected coils without capacitive load at the 15-kHz-resonance. Apparently there is also a capacitive coupling of the two coils which, however, can be ignored in the framework of practical operation.

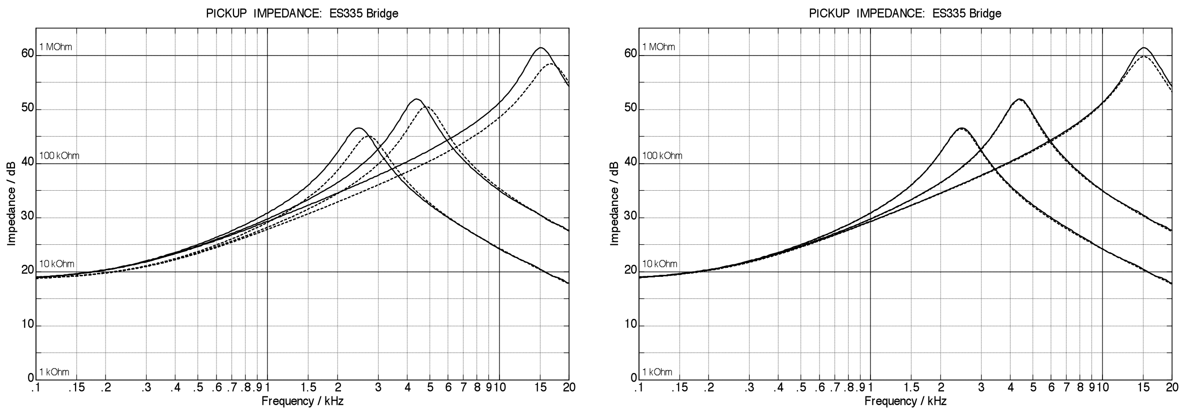


Fig. 5.9.42: Frequency response of the impedances: ES-335-bridge-pickup. Model wthout coupling of magnetic fields (left), with 20%-coupling of the magnetic fields (right). External capacitve load: 1030 pF, 330 pF, 0 pF. Replacing the bar magnet (Alnico 2 vs. Alnico 5) may change the coupling factor; no measurements shown here.

In principle all pickup coils of a guitar are magnetically coupled. However, since the coupling factor decreases rapidly with increasing coil distance, the magnetic field coupling has any significance only for the neighboring humbucker coils. Just to be safe, a Stratocaster (3x singlecoil) was also analyzed: indeed, the coupling factor could be measured but at k = 0,5% has no significance at all.

5.9.3 Equivalent circuit diagram for the transmission

In the previous sections we presented two-terminal equivalent circuits modeling the frequency response of a pickup. The actual aim is, however, to describe the sound of the pickup – or, more precisely, its transfer characteristics. Using the two-terminal-network theory discussed in chapter 5.9.2, the electrical circuit (the network) is investigated regarding two *terminals* (network nodes); for a pickup, these are the two connecting terminals at which a complex impedance is measured. In contrast, for the **quadripole theory** two of the network nodes are defined as input port and the other two as the output port; therefore occasionally the term **two-port-theory** is found instead of quadripole theory. At each of the two ports two-terminal impedances may be defined, but more important is how the signals at one port depend on the signals at the other. For an electrical network, the signals at the ports are voltage and current. Between the nodes of every port we find a voltage $\underline{U}$ that as a special case may be zero, while the currents $\underline{I}$ flow in the connecting wires to external systems; again $\underline{I}$ may be zero as a special case.

In order to make the complicated transfer behavior of a pickup describable, it needs to be simplified. This is achieved by defining the pickup as a linear, time-invariant system of finite order. **Linearity** implies among other things lack of sources and proportionality between input and output signals. "Lack of sources" means that the pickup does not contain any signal source – which is a matter of course in the framework of the usual approach, save for noise disturbances (Chapter 5.12). "Proportionality" stands for an output signal multiplied by k if the input signal is changed by the same factor of k. This is only approximately true for a pickup, as distortion measurements show (Chapter. 5.8). For small levels, a pickup is a linear system but for strong string vibrations a non-linear model is required. **Time-invariant** means that the pickup always behaves in the same way: a condition generally met with good approximation. The **order** n of the model marks the number of free energy storages, in other words the number of independent capacitances and inductances within the equivalent circuit. The more precise the model is supposed to be, the higher the order will be (it does – in this context – not indicate the contrary of disorder). For usual equivalent circuits of pickup an order of $n = 2 \dots 5$ is to be expected.

Using the simplifications mentioned above we can determine a broken rational transfer function of n-th order, which maps the input signal onto the output signal. The output signal is the voltage at the output terminals – but what is the input signal? If one does not want to immediately go back to the action potentials of the guitarist, we could define the **string vibration** as the input signal. This, however, is a spatially distributed vector field difficult to describe. Again, several simplifications are required: measurements of the aperture (Chapter 5.4.4) suggest that only the string section vibrating directly in front of the magnet causes the significant change in the field, and within this again predominantly the magnet-axial (fretboard-normal) component. Movement of the string changes the magnetic resistance and modulates the flux generated by the permanent magnet. A DC-source and a time-variant impedance may model this process in the equivalent circuit, or one can imagine the magnetic AC-flux as being generated by an AC-driven **transmitter coil**. This transmitter current is then imagined to be proportional to the string movement; the latter in turn can be presented in several ways: as deflection, velocity or acceleration.

The **transfer function** describes the projection of the string movement (the input quantity) on to the pickup voltage (output quantity) as a complex frequency function $\underline{H}(j\omega)$. The **spectrum** of the pickup voltage $\underline{U}(j\omega)$ is a complex frequency function, as well, in contrast to $\underline{H}(j\omega)$; however, it is not a system quantity but a signal quantity. $\underline{U}(j\omega)$ is dependent on the input signal but $\underline{H}(j\omega)$ is not. For an excitation with a known input spectrum $\underline{E}(j\omega)$, the corresponding output spectrum $\underline{U}(j\omega)$ can be calculated in the linear model:

$$\underline{U}(j\omega) = \underline{E}(j\omega) \cdot \underline{H}(j\omega)$$

From a systems theory perspective, each one of the three motional quantities of the string (deflection, velocity, acceleration) could be defined as input quantity, but using the **string velocity** is particularly purposeful and can be interpreted well. The pickup-transfer-function used in the following is thus the velocity→voltage-transfer-function $\underline{H} = \underline{H}_{\mathrm{Uv}}$, the first index (U) of which points to the generated output-quantity while v yields the input quantity which causes the effect. For the magnetic pickup, $\underline{H}_{\mathrm{Uv}}$ is a **low-pass function** which sometimes raises the question whether such a pickup can actually generate a “0 Hz”-signal. In this respect, we need to consider that – as pointed out above – the output spectrum is not only dependent on $\underline{H}_{\mathrm{Uv}}$, but also on the input spectrum. At 0 Hz, the string is without movement, its velocity is thus zero and therefore the output voltage is zero as well – although $\underline{H}_{\mathrm{Uv}}$ is not zero.

In order to now put together a purely electrical transfer equivalent circuit we need to find an electrical input quantity matching the string velocity. The cause for the induced pickup voltage is the changing magnetic flux, the instantaneous value of which depends on the distance of the string to the magnet: the closer the string to the magnet, the larger the flux. Since the distance is equivalent to the integral of the string velocity over time, and since the spectral operation corresponding to this is a division by $j\omega$ [6], it is possible to use as equivalent model a **transmitter coil** positioned on the pickup. This coil – excited by a current source with a $1/f$-characteristic - generates a magnetic alternating field.

$$\underline{v} \rightarrow \underline{I} \rightarrow \underline{U}; \qquad \underline{H}_{\mathrm{Uv}} = \underline{H}_{\mathrm{Iv}} \cdot \underline{H}_{\mathrm{UI}}; \qquad \left.\underline{H}_{\mathrm{Iv}} = \frac{\underline{I}}{\underline{v}} = \frac{const \cdot \Phi}{j\omega \cdot \xi} = \frac{const}{j\omega} \right\} \underline{I} = \frac{const \cdot \underline{v}}{j\omega}$$

In other words: to obtain a frequency-independent velocity-amplitude, the amplitude of the deflection is reciprocal to the frequency, and so is the amount of the magnetic AC-flux. Whether this AC-flux is generated by a moving string or instead by a current-excited transmitter coil is – in the framework of this model – equivalent.

In the following the velocity→voltage-transfer behavior of the pickup is presented with a low-pass-model. The input quantity is generated by an ideal current source with frequency-reciprocal amplitude $I \sim 1/f$ while the output quantity is the pickup voltage.

Special emphasis is put on the fact that per pickup in the two-terminal- and in the quadripole-equivalent-circuits one and the same components are used. If the basic models are viable it needs to be possible to derive a two-terminal-equivalent circuit from an impedance measurement (doable with little effort), and to further determine – with the components calculated for the two-terminal-equivalent – also the quadripole-equivalent-circuit and the transfer function $\underline{H}_{\mathrm{Uv}}$. As a precaution it is mentioned again that $\underline{H}_{\mathrm{Uv}}$ is not identical to the spectrum of the pickup voltage; for the latter the velocity spectrum is required on addition (Chapter 1 – 3).

Fig. **5.9.43** shows the quadripole-equivalent circuit for a **Stratocaster** pickup. Structure and component values are taken from the two-terminal equivalent circuit (Fig. 5.9.5). The alternating field caused by the string may be imagined to be generated by a transmitter coil driven by an impressed current $\underline{I}$. In the left-hand picture, the transmitter coil is the primary winding of the transformer with the current source marked by the broken circle. The current source may be transformed over to the right-hand side of the transformer (right-hand picture); this merely changes the amount of the current, and the primary winding becomes redundant and is dropped. From the transformer, only the pickup inductance (2,2 H) remains.

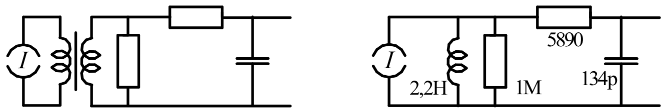


Fig. 5.9.43: quadripole-equivalent-circuit of a Stratocaster pickup.

We can now define the quotient of output voltage and input current as the transfer function: $\underline{H}(f) = \underline{U}_2(f) \,/\, \underline{I}_1(f)$. If the current amplitude is equal at all frequencies, the result is a band-pass-characteristic ($\underline{H}_{U\xi}$, **Fig. 5.9.44,** left-hand side). However, an excitation with frequency-reciprocal current amplitude is more easily interpreted since it yields a **low-pass-characteristic** ($\underline{H}_{Uv}$). The left-hand section of Fig. 9.5.44 shows the results of measurements taken for field-coupling with a small transmitter coil (constant current amplitude) wound around the 6 magnets extending from the Stratocaster pickup; the pickup was loaded with 4700, 1000, 330, and 0pF resp. In addition, calculations based on the equivalent circuit shown in Fig. 5.9.43 are included. The two sets of curves are practically identical; any differences are hardly noticeable. Using a frequency-reciprocal current amplitude $I \sim 1/f$ instead of the frequency independent current brings us to the right-hand section of the figure ($\underline{H}_{Uv}$, low-pass ECD).

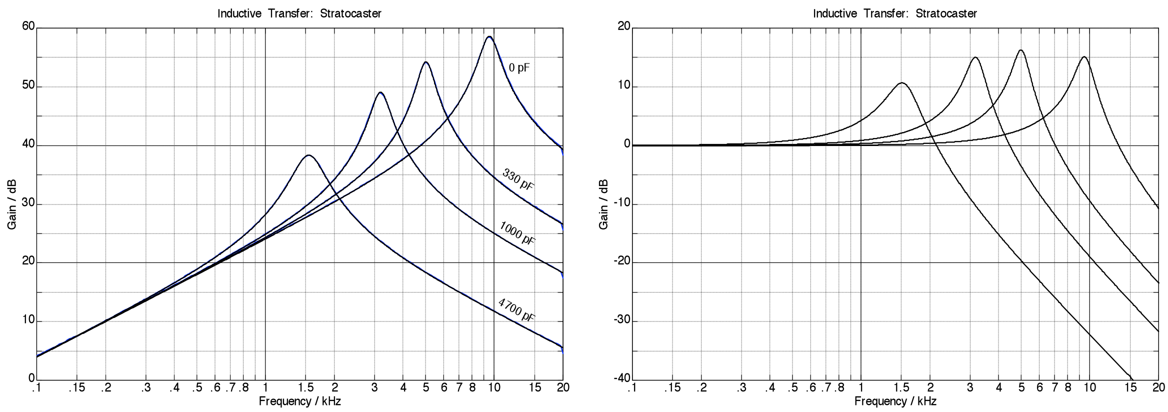


Fig. 5.9.44: Transfer function for current impression (= field impression); left: band-pass, right: low-pass

The low-pass transfer function shown on the right are **normalized** such that for low frequencies we obtain a transfer factor of 0 dB. Using the field coupling, only *relative* frequency responses can be measured since the strength of the static magnetic flux is not captured. The absolute scaling (i.e. the vertical position of the plots) may be determined on the shaker test stand (Chapter 5.4.5). Given a capacitive load, every magnetic pickup shows a low-pass characteristic (LP-ECD) including a resonance arising from the pickup inductance and the capacitance of the pickup plus cable. The resonance emphasis is high for a purely capacitive loading; it drops off with the resistances of typically connected potentiometers (Chapter 9) coming in.

As already shown, various transfer functions ($\underline{H}_{Uv}$, $\underline{H}_{U\xi}$, ...) may be defined from a systems-theory point-of-view. However, besides the analytical description also a visual recognition and evaluation of the shape of the frequency dependency is desirable, and here we find a similarity between the third-octave spectra often used in acoustics on the one hand, and the velocity→voltage-transfer-function $\underline{H}_{Uv}$.

In **Fig. 5.9.45** we see the third-octave spectrum of the pickup voltage of a Stratocaster. The guitar was in its original condition and externally loaded with 700 pF (cable). All strings were plucked repeatedly in quick sequence while being slightly dampened at different neck positions with the left hand. The left hand at the same time fingered various bar chords without pushing the strings fully down onto the frets. This generated a wide-band noise-type signal without too many tonal components (which could have disturbed the spectrum). The **third-octave spectrum** obtained via main- and auxiliary-third-octave analysis (according to DIN) is shown as a **polyline**. Between 100 Hz and 4 kHz there is a nearly horizontal characteristic while below 100 levels drop off – the fundamental frequencies of the strings do not cause us to expect anything else. Above the pickup/cable resonance (3 kHz) we find a treble attenuation. Except for the low frequency range, this analysis matches a low-pass model quite well while no similarities are recognizable to a 3-kHz-band-pass.

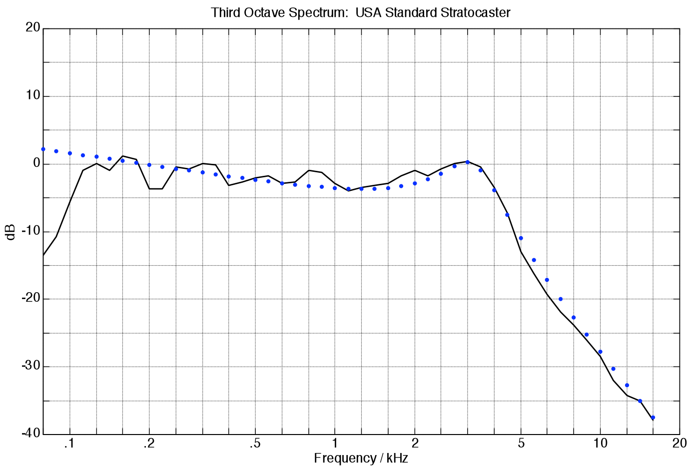
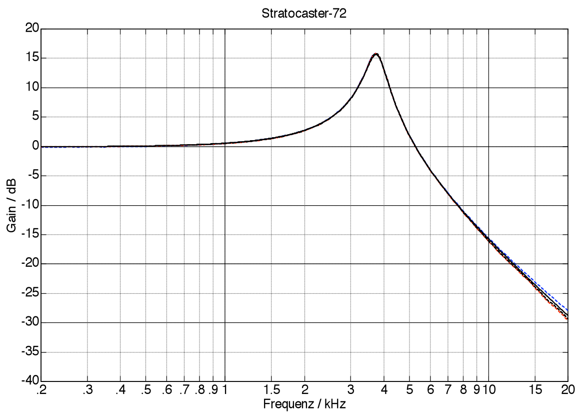


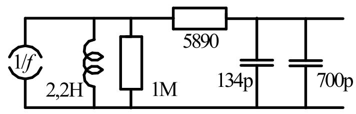
Fig. 5.9.45: Fender Stratocaster. Measured levels of third-octave spectrum (line); calculated LP-transfer function (dots). The pickup was loaded with an external capacitance (700 pF).

As a supplement, the $\underline{H}_{Uv}$-transfer function is indicated **as dots** in Fig. 5.9.45. It was calculated on the basis of the quadripole equivalent circuit (Fig. 5.9.45) to which a slight treble attenuation of 1,8 dB/Oct. was added. Both plots globally show strong similarities; moreover the two do not have to correspond in detail since these are two fundamentally different quantities. Of course, the third-octave-level spectrum depends on the strings and the way the playing style while the transfer function does not. The latter is a system quantity and as such gives the transfer characteristic in a time-invariant and signal-independent manner: the pickup resonance is, for example, independent of which music the guitarist is playing at the time. The voltage spectrum, however, is dependent on the string excitation and the transfer characteristic.

That the correspondence in Fig. 5.9.45 nevertheless is so pronounced indicates that the choice of the low-pass transfer function is a good one. Although the other choice for the band-pass transfer function would be scientifically also possible, a visual analysis is much more difficult.

The quadripole equivalent-circuit-diagram describes the transmission from the magnetic field excitation (input port) to the connectors (output port); it can serve to determine – with little effort – the transfer function ("frequency response") of the pickup. The components of the quadripole-ECD **are the same** as those of the two-terminal equivalent circuit and may be determined via impedance measurement and curve fitting, or via special methods of network analysis. The magnetic alternating-field-source corresponds, in the electrical quadripole-ECD, to an AC current source fitted in parallel to the inductance dampening the eddy-currents (**Fig. 5.9.46**, right hand section).





Quadripole equivalent circuit diagram.

Fig. 5.9.46: Calculated and measured low-pass transfer-function. Stratocaster pickup, 700pF.
Measurement: using coupling of themagentic field; calculation: using the quadripole ECD shown on the right.

Comparing the measurement and the calculation shows how accurate the quadripole-ECD is. To achieve the coupling of the magnetic alternating field there is a choice from several possibilities:

1. Using a **pair of Helmholtz coils** we can generate a homogeneous, quantitatively well defined field which however is in its shape very different from the locally limited AC-field caused by a string.
2. A small **coaxial coil** positioned on a magnet (e.g. Stratocaster) or pole-screw (e.g. P-90) yields a locally limited field, which however does not yet correspond to the field generated by a string.
3. Much closer to reality is the excitation with a tripole-coil. For this, we wind …..

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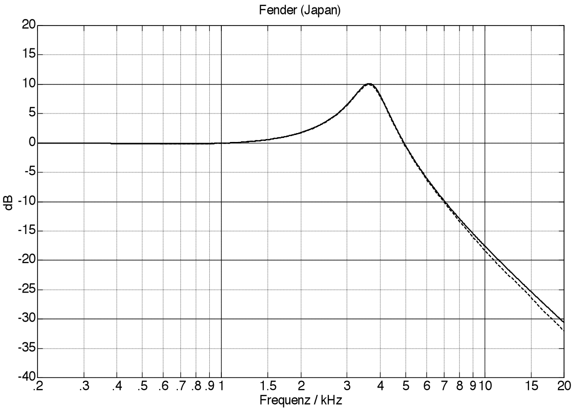
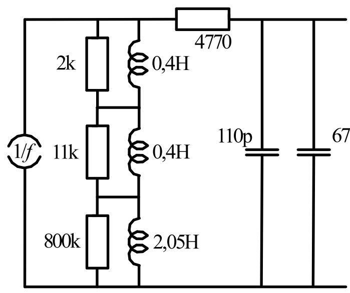
Fig. 5.9.46 contains 4 plots: one each for the measurement with Helmholtz-, coax- und tripole-coil plus one calculated from the quadripole equivalent circuit diagram. Up to 10 kHz all four correspond perfectly ($\Delta L < 0{,}5$ dB). Only in the highest octave we find differences: the excitation with the Helmholtz-coil (dashed line) yields the highest levels while the lowest levels are given by the other two excitation coils with the calculation (solid line) positioned in between. The pickup ware loaded merely with a capacitance; further loading (controls, amplifier) would reduce the resonance emphasis for all plots in a similar manner.

The transfer functions depicted in Fig. 5.9.46 are normalized in such a way that their position accommodates a level of 0 dB at low frequencies. While the shape of the curves is thus set, the **absolute scaling** remains undetermined. To know the vertical position of the transfer curves, measurements with a real vibrating string are required – these need to be taken at merely one single frequency, though. It is purposeful to choose a frequency at which few artifacts can be expected.

As has been shown, the results of measurement and the calculation agree very well for the original Stratocaster pickup, which features only little eddy-current losses. This agreement is not as good for the pickup variant manufactured in Japan (**Fig. 5.9.47**) the construction of which is based on a bar magnet and 6 iron slugs rather than on 6 cylindrical alnico magnets. In the low and middle frequency ranges a perfect agreement does remain between the measurements with a tripole excitation (----) and the calculation while for higher frequencies there is a larger difference although this range is less important for electric guitars.

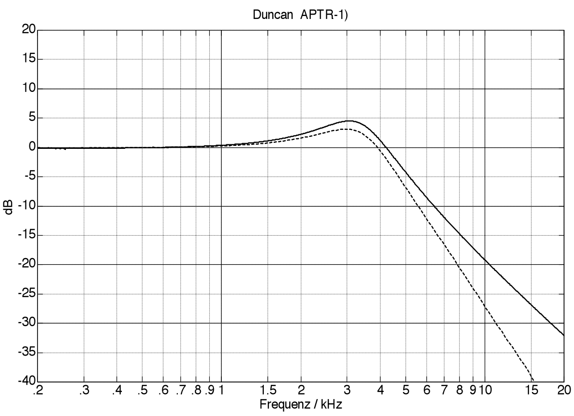


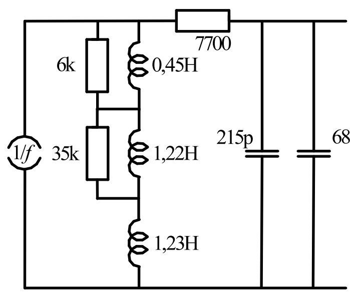


Quadripole equivalent circuit diagram.

Fig. 5.9.47: Calculated and measured lowpass-transfer function. Fender-Japan-Strat, 670pF load. Measurement with tripole-coupling (---), calculation with the quadripole ECD shown on the right.

The situation turns out differently for the Telecaster neck pickup (**Fig. 5.9.48)**: there is a clear divergence between measurement (----) and model calculation. We can pinpoint the reason in the metal cover the eddy-currents of which necessitate a modified equivalent circuit diagram (see Chapter 5.10).





Quadripole equivalent circuit diagram.

Fig. 5.9.48: Calculated and measured lowpass-transfer function. Telecaster (S. Duncan), 680pF load. Measurement with tripole-coupling (---), calculation with the quadripole ECD shown on the right.

The equivalent circuit diagrams shown in Fig. 5.9.47 and 5.9.48 contain *three* different inductances. Whether this effort is justified can be determined on the basis of the desired accuracy. Using the Gibson screw-coil discussed in Chapter 5.9.2.6 as an example, **Fig. 5.9.49** compares the transfer function derived from a 4th-order model (n = 4, Fig. 5.9.37) with the transfer function calculated on the basis of a simple 2nd-order low pass (n = 2). Whichever way we approach the alignment (whether going for identical maxima – left-hand section – or for equal high-frequency asymptote – right-hand section): the resonance of the 2nd-order plot comes out too broadband. In other words: if we are looking for more than just a coarse approximation, the more exact modes should be preferred – the effort is manageable.

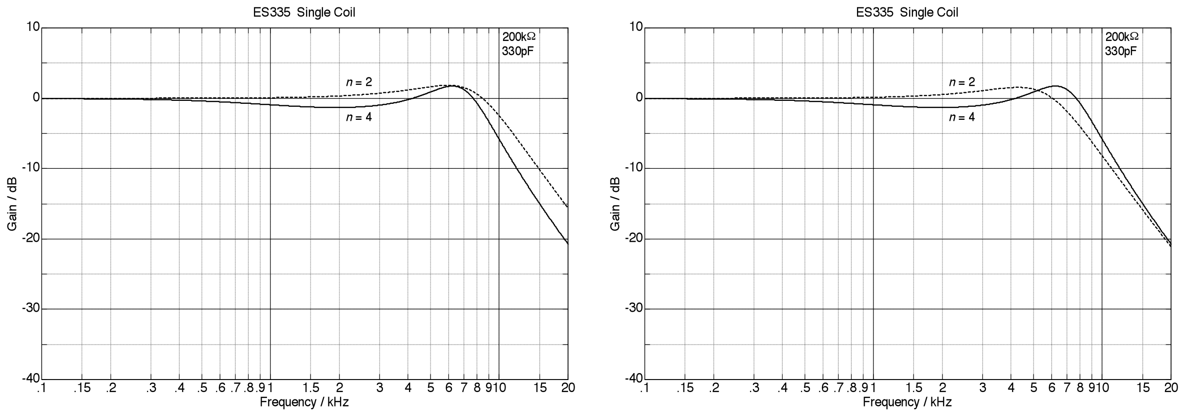


Fig. 5.9.49: Transfer functions calculated on the basis of models of different complexity.

5.9.4 Pickups connected in combination

Connecting two pickups in parallel changes the sound in two ways: due to the halving of the inductance the resonance frequency rises by 40%, and in addition we get an interference filter similar to a humbucker – although with a larger distance of the coils and negligible coil coupling. **Fig. 5.9.50** shows the frequency responses for a coil distance of 6 cm; this interference filter is shifted further towards lower frequencies for the combination of neck- and bridge-pickups (d = 12 cm). The resonance emphasis grows because the source impedance is cut in half.

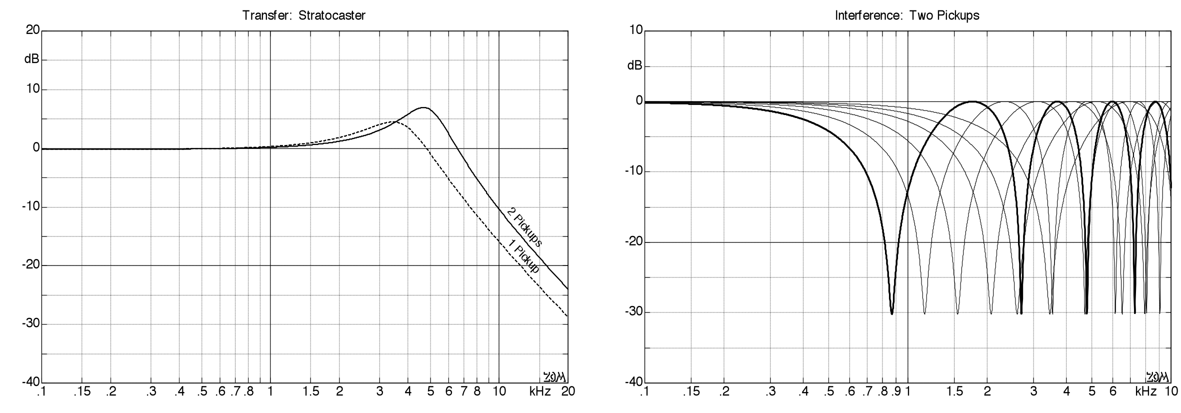


Fig. 5.9.50: Transfer frequency response and interference filter (d = 6 cm), pickups cinnected in parallel. The interference effect is specific to the string; the bold line holds for the E2-string.