A.3 Transverse waves

In the transverse wave, all particles of the medium move transversely to the propagation direction of the wave. To calculate the wave, the medium is divided into thin discs, as it was for the longitudinal wave (see above). However, these slices now do not shift in parallel to the direction of propagation, but (with an additional change in shape) perpendicular to it. In the simplest case, the center of gravity of each disc oscillates in the same plane of vibration (plane or linear **polarization**). In the general case, the center of gravity of each disc oscillates on a space curve (e.g., z = constant, circular or elliptical polarization). In contrast to the bending wave (see A.4), however, the flat separating surfaces of the discs always remain in parallel. (**Fig. A.3.2**, **Fig. A.4.2**).

A.3.1 Pure transverse waves

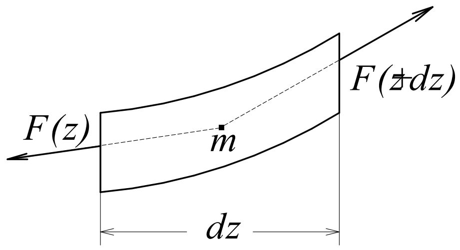
If all particles of the medium move at the same velocity in the same direction, no wave propagation will occur – this case is not interesting in the context of the present considerations. In the case of a transverse wave, the transverse movement is place-dependent and time-dependent, with non-constant dependencies in each case. This requires a change in shape in the medium, which in the case of the pure transverse wave will deform a cuboid into a parallelepiped (oblique prism) via the acting shear stresses. The system quantities are the material data **density** $\boldsymbol{\rho}$ and **shear modulus** $\boldsymbol{G}$**.** For a string, this state of tension would at best be taken into account if the string were moved without the presence of a tensioning force. Since this operating case is untypical, it will not be pursued any further.

A.3.2 Transverse waves in strings

Newton's axiom of inertia states that the velocity of a mass can only be changed by the action of a force. Each of the sections of string (cut into slices – see above) has a mass resulting from its density and its volume. At the disc-separating surfaces, mechanical stresses are engaging that (averaged and multiplied by the cross-sectional area) result in *one single* external force per separating surface. The separation surfaces are perpendicular to the string axis (z-axis); each external force can be decomposed – with respect to the separation surface – into a normal and tangential component. In the resting state of the string, the tangential component is zero, while the normal component corresponds to the **tensioning force** $\boldsymbol{\Psi}$. In a transverse oscillation, the sections of the string move only in the transverse direction, and the normal force thus contains no alternating component – it remains at a constant value Ψ. The tangential forces acting on both sides of the sections result – as a vector sum – in the **lateral force** which is responsible for the lateral acceleration (with any losses being neglected here).

The mass of each piece of string is thought to be concentrated in the respective center of gravity, and the directions of the external forces follow the connecting lines between the centers of gravity (**Fig. A.3.1**). If $\xi(z,t)$ describes the place- and time-dependent string displacement (plane polarization), then $\partial\xi / \partial z$ represents the slope of the string, or the forces. For a non-zero shear force to arise, the left and right slopes must be different; otherwise – because of the same normal forces – the tangential forces would be the same and the resultant would be zero. Consequently, a non-zero transverse force can only occur at locations where the *change* in the slope (i.e. $\partial^2\xi / \partial z^2$) is non-zero, i.e. at locations featuring a non-zero **curvature**.

For small amplitudes, the *length-specific* lateral force difference $F_q(z+dz) - F_q(z)$ is approximately: $\Psi \cdot \partial^2\xi / \partial z^2$; it corresponds to the product of length-specific mass $m' = \rho S$ and lateral acceleration $\partial^2\xi / \partial t^2$. The effects of shear stress are neglected here. The resulting 2nd-order partial DE is solved – like the DE for the longitudinal wave was solved – with the Bernoulli approach:



$$\Psi \cdot \partial^2\xi \big/ \partial z^2 = m' \cdot \partial^2\xi \big/ \partial t^2 ; \qquad m' = \rho S$$

$$c_P = \sqrt{\Psi / m'} \;\; ; \; k = \omega \sqrt{m'/\Psi}$$

$$\underline{\xi} = \hat{\xi} \cdot e^{j\varphi} \cdot e^{j\omega t} \cdot e^{-jkz} = \underline{\hat{\xi}} \cdot e^{j(\omega t - kz)}$$

Fig. A.3.1: Curved section of string with the two engaging external forces. The surface-normal *z-component* of the external force corresponds to the constant (!) tensional force Ψ. The shear forces are tangential to the interfaces (that are oriented normal to the *z*-direction). The transverse wave has two wave quantities (also called signal quantities): the velocity, and the lateral force. The shear-force *difference* corresponds to the inertial force.

For both the longitudinal and the transverse wave, the solution contains a complex *e*-function, with the time-dependent term ωt and the place-dependent term kz. The signal quantities that can be described for the transversal wave (*longitudinal wave*) are: lateral force (*longitudinal force*), transverse velocity (*longitudinal velocity*), and their respective temporal integral/differential. Regarding the forced oscillation, the time-dependency of the string oscillation (ωt) is given by the external excitation; for the free oscillation it is determined by the geometry and the phase velocity (see Chapter A.1). The location-dependency (kz) is given by the phase velocity c_P in both cases.

It should in particular be noted that the rigidity of the transverse movement described here is not caused by the material properties (Young's modulus *E*), but solely by the string tensioning force Ψ. For simple wave propagation over short distances, as well as for low-frequency considerations, this description is sufficient. However, a closer analysis reveals that in addition to the tensioning rigidity, the (material- and geometry-dependent) **bending stiffness *B*** must also be taken into account. It causes the propagation speed of the signal to be not constant: it rather increases with increasing frequency. In the formal description, we would have to consider not only the forces, but also the torques and, in addition to the translational motion quantities, the rotational motion quantities (Chapter A.4).

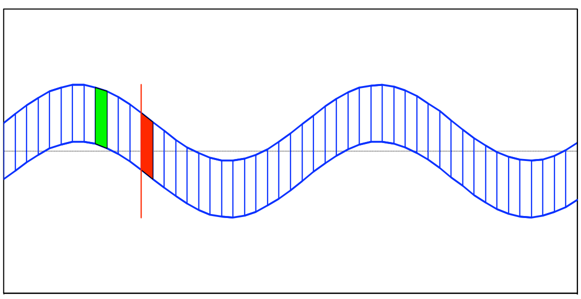


Fig. A.3.2: Mono-frequency transverse wave

