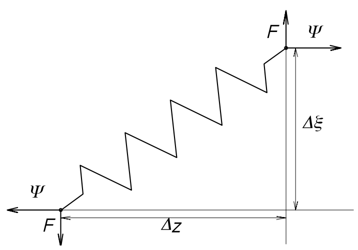
## A.6 Stiffnesses

The rigid string is described by the tension-force-dependent transverse stiffness $s_Q$ and the bending stiffness $B$. In the case of static displacement in the transverse direction, the string acts approximately like a spring with the spring stiffness $s_Q$, whereas for transverse-wave propagation (without dispersion) the length-related compliance $n'$ applies:

$$s_Q = \frac{F}{\xi} = \frac{\Psi \cdot M}{R \cdot L} \qquad n' = \frac{\Delta\xi / F}{\Delta z} = \frac{1}{\Psi} \qquad \text{Transverse load}$$

Herein: $F$ = transverse force, $\xi$ = transverse displacement, $\Psi$ = tensioning force, $M$ = scale length, $R$ = distance between force-engagement point and bridge, $L$ = distance between force-engagement point and nut. **Fig. A.6.1** offers an alternative to the string model as shown in Fig. 2.5 - advantageously of being able to process a tension force running in the $z$-direction, but with slight deficiencies with respect to the algebraic sign. Both $s_Q$ and $n'$ are a function of the tensioning force - they are independent of the elastic modulus $E$ !




$$\frac{\Delta\xi}{\Delta z} = \frac{F}{\Psi}; \qquad \Delta n = \frac{\Delta\xi}{F} = \frac{\Delta z}{\Psi}; \qquad n' = \frac{\Delta n}{\Delta z} = \frac{1}{\Psi}.$$

**Fig. A.6.1:** Decomposition of forces in the string model. The length-specific compliance $n'$ is reciprocal to the tensioning force $\Psi$.

**Fig. A.6.2** shows typical transverse stiffnesses of the strings (compare to Fig. A.1.2).

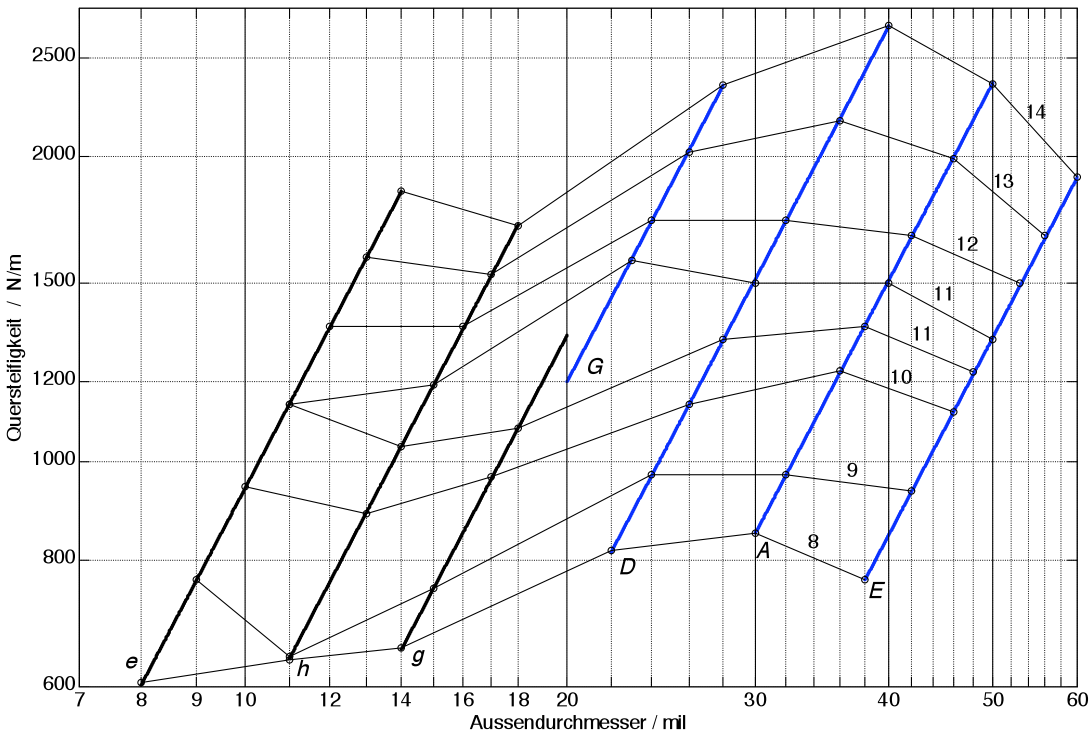



**Fig. A.6.2:** Transverse stiffness; Scale $M$ = 64.8 cm, the plucking-engagement point is 9 cm from the bridge. “h” (string) = “B” (-string).

While flexural stiffness may be neglected for static loading ($f = 0$), it should be considered for thicker strings and high frequencies. As the string diameter increases, $B$ grows in proportion to $D^4$, which would have audible effects on solid strings (chap. 1.3.1). To reduce the bending stiffness, the heavy strings are not solid, but wound; for them $B$ is determined practically only by the core.

$$B = E \cdot I = E \cdot D^4 \pi / 64$$ Flexural (bending) stiffness

$I$ = area moment of inertia, $E$ = modulus of elasticity, $D$ = string diameter (solid strings) or core diameter (wound strings). The **bending stiffness** $B = E{\cdot}I$ can be thought of as a length-specific torsional rigidity, similar to how the product of modulus of elasticity $E$ and cross-sectional area $S$ can be interpreted as a length-specific spring stiffness $s'$ in the uni-axial strain state. However, for the analogy, the reciprocal (the compliance $n$) is of advantage:

$$n = \frac{\Delta z}{E \cdot S} \quad \Rightarrow \quad n' = \frac{n}{\Delta z} = \frac{1}{E \cdot S}$$
uniaxial strain state

$$n_D = \frac{\Delta z}{E \cdot I} \quad \Rightarrow \quad n'_D = \frac{n_D}{\Delta z} = \frac{1}{E \cdot I}$$
bending load♣

The length-specific compliance $n'$ is compliance $n$ per length $\Delta z$. For the reciprocal of the compliance (i.e. the stiffness $s$), the reference to the length is unfamiliar at first: $s = 1/n$, and $s' = 1/n' = s{\cdot}\Delta z$. To get from stiffness to length-specific stiffness, we have to multiply by the length $\Delta z$! Unfamiliar, but need be – because with the length approaching zero, the compliance converges to zero, while the rigidity approaches infinity.

**Fig. A.6.3** shows the bending stiffness for customary guitar strings. The modulus of elasticity of all strings was assumed to be $E = 2{\cdot}10^{11}$ N/m$^2$; for the wound strings, only the bending stiffness of the core was taken into account.

E4: $D_A =$ 8 ... 13 mils, solid.
B3: $D_A =$ 11 ... 17 mil, solid.
G3: $D_A =$ 14 ... 20 mil, solid.
G3: $D_A =$ 20 ... 26 mil, $\kappa = 0.48$ and 0.60.
D3: $D_A =$ 22 ... 36 mil, $\kappa = 0.40$ and 0.60.
A2: $D_A =$ 30 ... 46 mil, $\kappa = 0.33$ and 0.50.
E2: $D_A =$ 38 ... 52 mil, $\kappa = 0.33$ and 0.42.

$D_A =$ outer diameter, $\kappa$ = core-diameter/outer-diameter.

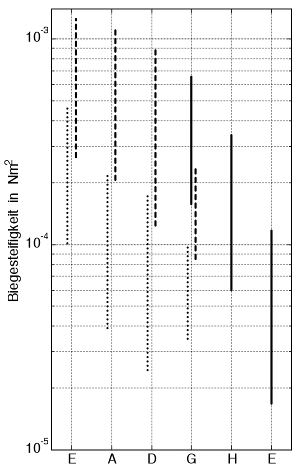


**Fig. A.6.3:** Bending stiffness (= "Biegesteifigkeit")
"H" (string) = B (-string)

The bending stiffness $B$ connects the local change of the string direction (i.e. $d\beta/dz$) to the bending moment $M$ (Fig. A 4.1). The minus-sign corresponds to a sign convention.

$$-M = B \cdot \frac{\partial^2 \xi}{\partial z^2} = B \cdot \frac{\partial \beta}{\partial z}$$

♣ $n_D$ is also called rotational compliance.

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