7.2 The Frets

7.2.1 Position of Frets

The right-handed guitar player changes the length of the vibrating string by ‘fingering’ (i.e. fretting) with the left hand. (Other fretting and playing techniques are also possible.) With the finger, he or she presses the string to be sounded towards the neck so that contact is made with the **fret**♣. Frets are metal wires with a T-shaped profile (Fig. 7.3). They are set into grooves in the fretboard cut transversely to the direction of the strings. The upper part of the fret that protrudes from the fretboard is rounded, and the lower part that is set into the fretboard is designed as a barbed hook to ensure that the fret remains fixed in place. Frets make it easier for the guitar player to achieve clean intonation (achieve correct pitch). The length of the vibrating string is "fretted" at discrete intervals only rather than continuously. As a first approximation, it does not matter where between the two neighboring frets the finger is pressed. For the string, the important contact occurs on the fret. Upon closer inspection, though, we observe that, particularly in the case of tall frets (protruding more), the strength and position of the fingering can have a small effect on the pitch (see also Fig.7.5).

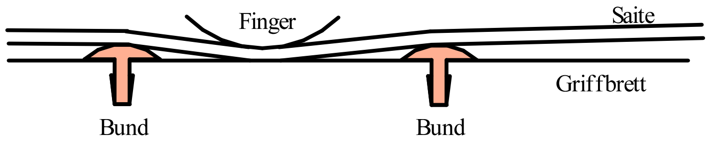


Fig. 7.3: *Longitudinal cut along the neck. Usually, the finger does not press the string („Saite“) all the way down to the fretboard („Griffbrett“). „Bund“ = Fret.*

The *open string* is supported at bridge and at nut. The distance between the latter two, the **scale**, is 24” – 25.5” i.e. 61-65 cm. However, guitars with a longer scale (baritone guitar, LONG NECK GUITAR) are also in use, as are short-scale guitars (3/4-guitar). Electric guitars generally have 21-24 frets, not counting bridge and nut. The length of the fretted neck (i.e. the length of the fretboard) amounts to approximately ¾ of the scale. In some guitars, the strings do not run directly from fretboard to nut but pass over a **zero fret**. In that case the string is always in contact with the same material, regardless of whether it is played open or fretted. The resulting higher friction has noticeable disadvantages, though: for easy tuning, the string should be able to longitudinally move over the nut or the zero fret with as little friction as possible. With too much friction, undesirable hysteresis may be the result.

The difference between simple theory and reality can be found in the **distance between frets**. To simplify the calculation, the string and its supports are assumed to be ideal. The guitar is tuned in equal temperament, with the semi-tone intervals being uniformly approximately 6%.

$$I_H = \sqrt[12]{2} = 1.05946 \qquad \text{Semi-tone interval}$$

Already the choice of this approach will invite and define fundamental deviations from just intonation, amounting e.g. to -0.9% for the minor third, and +0.8% for the major third. Just intonation is not the preferred ideal (Chapter 8.1): (equally) tempered tuning is the standard used today. The reciprocity between string length and fundamental frequency results in a geometric progression for the distances between frets. If the distance between the nut and the first fret amounts to ΔB, then the distance between the nth fret and (n+1)-th fret is $\Delta B \,/\, I_H^n$. The distance between frets thus diminishes from nut towards bridge, while at the same time the neck width (and thus the length of the frets) increases (**Fig. 7.4**).

♣ Sometimes the area between two fret wires is referred to as “fret”, as well.

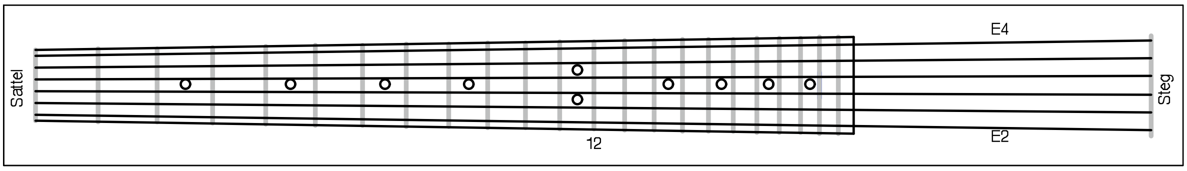


Fig. 7.4: Fretted neck (22 frets) with strings and bridge. E_2= low E string (at the thumb), E_4 = high E string. The octave relative to the open string is fretted at the 12th fret – at the mid-point of the string. The full length of the strings across the nut on the headstock is not shown. ("Sattel" = nut, "Steg" = bridge)

The above calculation does not take into account that the string tension is increased when the string is pressed down; this results in a further change in the pitch. For instance, if a string is fingered at the 12th fret, its fundamental frequency should actually be doubled. However, pressing down on the string causes a minimal lengthening of the string, causing a further increase in the frequency. The fact that the frequency of the lengthened string is higher rather than lower is due to the tension change that is dominant here (compared to the change in length).

In the following calculations, it is important to distinguish between the string length L and the change in length ΔL. The change in length ΔL is designated the **strain** ξ. A string that has the length L in its <u>un</u>fretted state (scale + residual lengths♣ to the tailpiece and to the tuners) is stretched by the tension Ψ to the new length of $L+\xi$. The more the string is stretched, the higher the fundamental frequency f_G (given a fixed scale M).

$$\xi = \frac{4\overline{\rho}L}{E} \cdot \left(\frac{f_G \cdot M}{\kappa} \right)^2 \qquad \text{Strain } \xi$$

$\overline{\rho}$ is the mean density (see annex), E is Young's modulus, κ is the ratio of core diameter to outer diameter for wound strings (for solid strings, $\kappa = 1$) . With the latter, the E-modulus of the core should be used; κ is between 0.3 and 0.6. The E_4 string must be stretched by 5.3 mm (standard tuning, L= 77 cm), die E_2 string by 1.7 mm ($\kappa = 0.42$). We observe that the strain depends on the square of the fundamental frequency f_G, and that the fundamental frequency is proportional to the square root of the strain. The formula for the relative changes is derived from the differential quotient of the curve.

$$\frac{\Delta f_G}{f_G} = \frac{1}{2} \cdot \frac{\Delta \xi}{\xi} \qquad \text{Relative change in frequency}$$

The relative change in frequency is half the relative change in <u>strain</u>. **Note**: This is not about the relative changes in length! If, for example, a string fretted at the 12th fret is extended by 0.02 mm, its *length* changes by 0.026‰, but this is not what is meant here. The *strain* changes by 3.8‰ (E_4), or by 11.8‰ (E_2) – it is this difference that causes problems. Even if the E_2- and E_4-strings are pressed down an equal distance towards the fretboard, the E_2-string **goes out of tune** by a much greater amount. In practice, however, the E_2- string is given an even greater distance to the fretboard (2 – 3 mm inside width at the 12th fret) than the E_4-string (1 – 1.5 mm). The frequency-increase for the E_4-string therefore is negligible in practical terms, while for the E_2-string it is quite large: 0,5·11.8‰, corresponding to 10 cent.

♣ Here, the friction that occurs in the nut and bridge needs be considered. Given high friction, $L = M$ applies.

To correct this frequency error, it would be possible to mount the frets in a slanted fashion, but except for a few exotic constructions, this is not done. Rather, the bridge is positioned with a slight slant such that the bearing (bridge saddle) of the E_4-string is exactly at double the distance from the nut compared to the 12th fret, but the bearing for the E_2-sring is moved back a few millimeters (= longer string). The exact amount necessary for this correction depends on the strings, the bearings, and the **string action** (inner distance of string to fret). For nylon-string guitars, almost no correction is required due to the smaller Young's modulus. In steel-string acoustics we often find around 3 mm (E_2); a slant of up to 6 mm (E_2) may be necessary for typical electric guitars.

As shown above, this **shift of the bridge** does not only depend on the fundamental frequency but also on the type of string winding. The E_2-, A- and D-strings are of the wound type while the B- und E_4-Saite are plain; the G-string may or may not be wound. The individual string data require a string-specific shift in the bridge. Therefore, many electric guitars feature a bridge with individual bridge saddles that are **adjustable** via small screws. After the guitar is restrung, the natural harmonic of the respective string is played (by very lightly touching the string – as it is being picked – exactly at its half-way point), and the bridge saddle is adjusted such that fretting the string at the 12th fret generates the same pitch as that harmonic. In some cases, two adjacent strings share a common bridge saddle – requiring a compromise in terms of the intonation.

A special example will show the influence of the **overall length** of the string: on some guitars, the string runs a considerable additional length on the other side of the nut and bridge – up to 25 cm in extreme cases. Conversely, on guitars with a string-clamping system, freely moveable string length and scale are practically identical. If all other parameters are kept the same, the string-strains differ by a factor of 88/63 = 1.4 between these two conditions. However, the (absolute) *change* in strain due to pressing the string to the fretboard depends solely on the scale and on the inner distance between (open) string and fret, and not on the overall length. This means that the longer the string is run outside of nut and bridge, the smaller is the detuning due to fretting♣. In the example, the relative change in strain (and thus the detuning) is a factor of 1.4 in the clamped string compared to the unclamped string. This needs to be considered if a guitar is to be retrofitted with a clamping system.

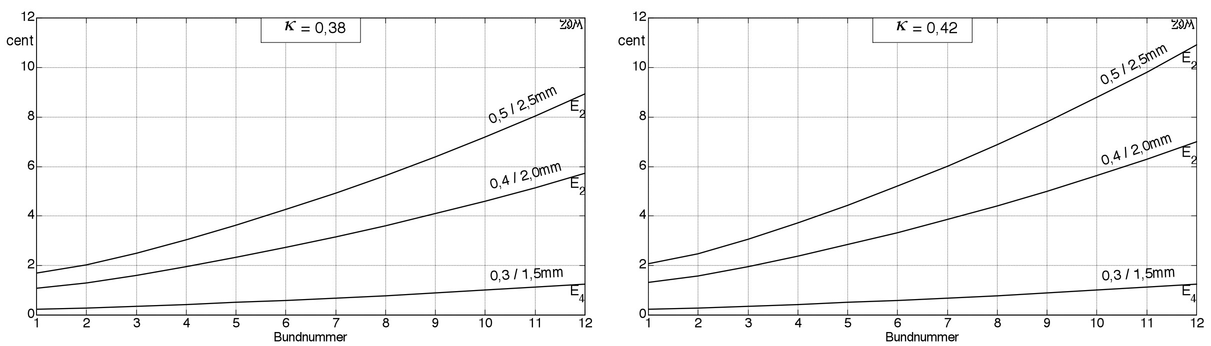


Fig. 7.5: Relative detuning due to pressing the string to the fret. Scale: M = 0.625m; L = 0.72m.
String-to-fret distance (1. fret / 12. fret) = 0.3 / 1,5mm (E_4); 0.4 / 2.0mm (E_2); 0,5 / 2,5mm (E_2).
Left: core-/outer diameter = κ = 0.38. Right: κ = 0.42. Position of bridge not compensated.
"Bundnummer" = number of fret.

♣ If the string were tensioned via a weight, the tensile force would not change at all when pressing down the string; the detuning would be negligible (merely a minimum change in length).

By slanting the bridge, the problems mentioned above can be taken care of – such that the octave (12th fret) can be played fully in tune. That does not imply, however, that all other frets offer correct intonation. **Fig. 7.5** shows the relative detuning occurring as the string is fretted – at first without slanted bridge position. The distance of string to fret (the "action") was assumed to grow linearly between the 1st and the 12th fret; for the low E-string, two different cases are calculated. In the left-hand graph, the ratio between core diameter and outer diameter equals $\kappa = 0.38$, on the right it amounts to 0.42. A smaller core diameter results in smaller detuning but also increases the danger of string-breakage.

For the left-hand section of **Fig. 7.6**, the bridge received a slant. The change for the E_4-string is very small (0.5mm); for the two E_2-cases, 2.5 and 3.9 mm are necessary, respectively. This already offers a decent solution. A detuning of 1 cent does not really require correction, anyway. Shifting not only the bridge but also the nut (in the direction towards the bridge), a further improvement is possible (right-hand graph), although a precision of 0.1 cent (0.0006%) is merely of theoretical interest. Basis for the calculation was that the string runs in a straight line from the nut to the tip of the fret, and continues to the bridge from there. Since the finger fretting the string during play does not actually provide an ideal line-contact but presses down the string behind♣ behind the fret, an additional strain of the string results and the required shift of the bridge increases.

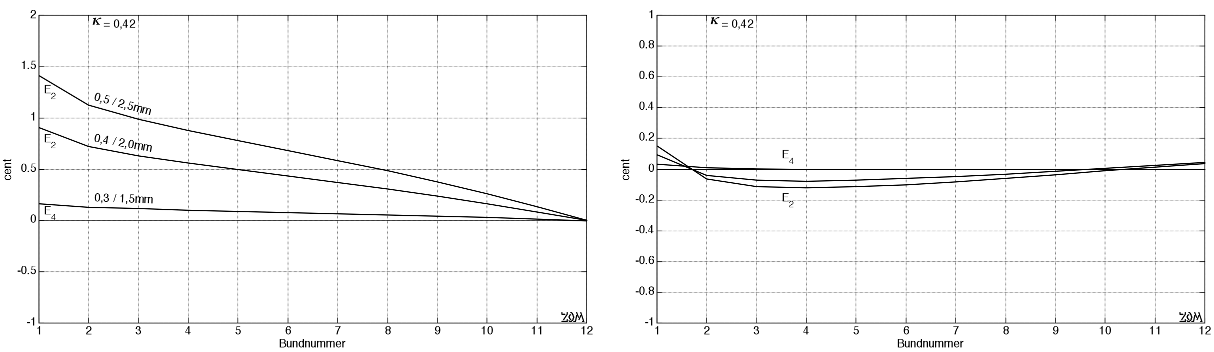


Fig. 7.6: Data as in Fig. 7.5 but $\kappa = 0.42$ (unchanged). In the graph on the left, only the bridge is shifted; on the right-hand graph additionally also the nut. For the E_4-string, the calculated shift of the bridge by 0.05 mm is not relevant, for the E_2-string, 0.3 and 0.5 mm, respectively, are calculated (line-contact at the tip of the fret only). "Bundnummer" = number of fret.

In summary, we find the following rule for guitar construction: first, the theoretical scale *M* is set, e.g. at 625 mm. Then the calculation of the distance between nut (or zero fret) and the 1st fret results in $M \cdot \left(1 - \sqrt[12]{0.5}\right) = M / 17.817$. We obtain the distance between the *n*-th and the *n+1*-th fret by dividing the distance between *n*-th fret and bridge saddle by 17.817. <u>Alternative:</u> the *n*-th fret is at a distance of $M / 1.05946^n$ from the bridge. As a next step, the nut is slightly slanted: its position remains unchanged for the E_4-string, while at the E_2-string it is shifted in the direction of the bridge by about 1 mm. Now the bridge saddle is shifted such that we can play the exact octave at the 12th fret. Given these adjustments, every string should now be tuned with equal temperament. An additional check using a measurement device, or our hearing, is advised – possibly small modifications are necessary.

♣ "behind" means: in the direction of the headstock.

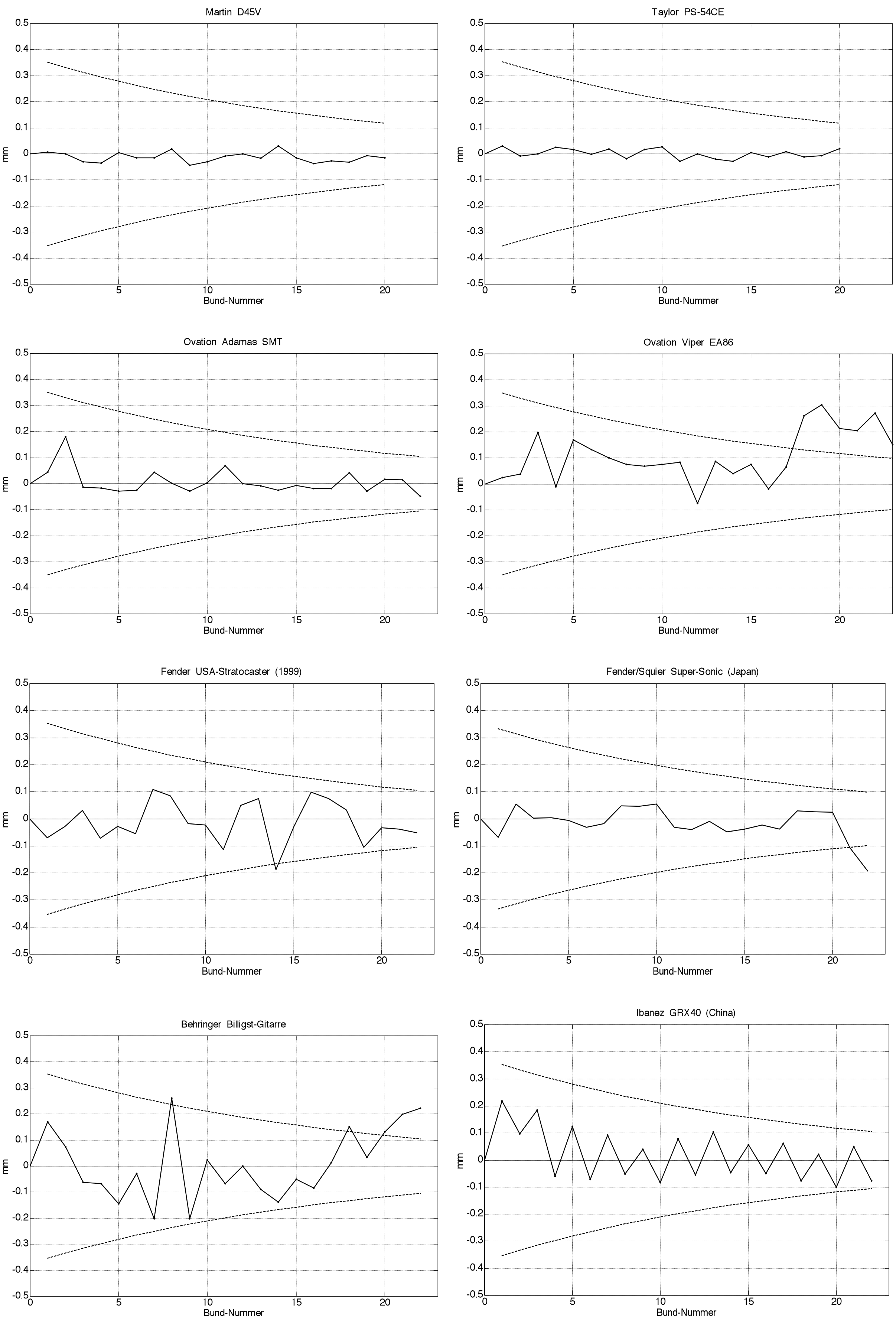
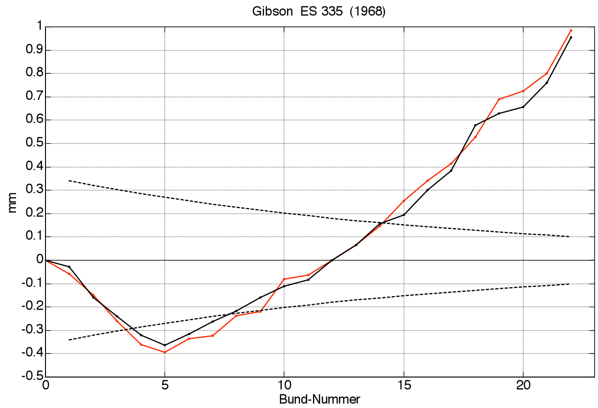
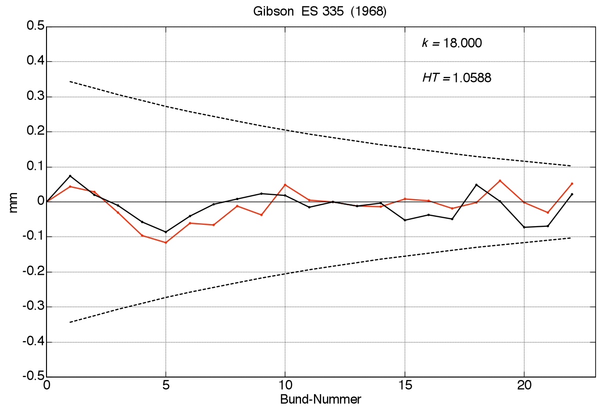
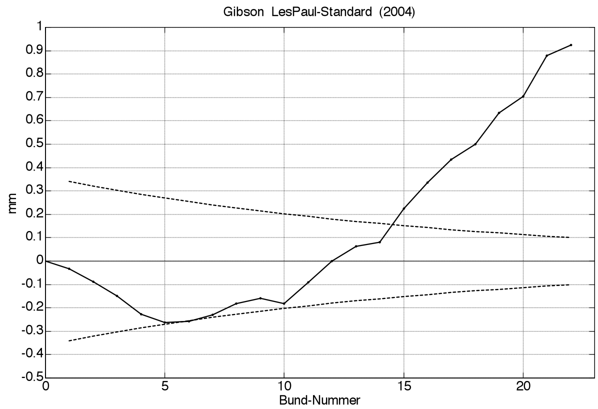
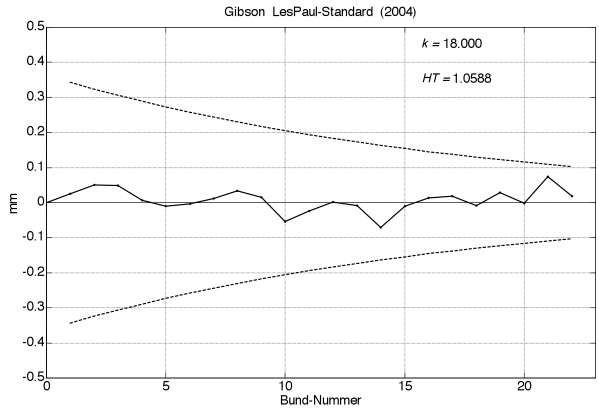


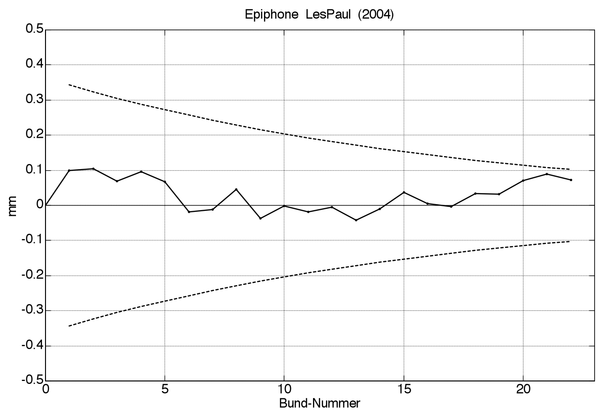
Fig. 7.7a: Deviation of the measured position of a fret from the theoretical position. The dashed limit lines show pitch deviations of ± 1 cent. Measurement tolerance: ± 0.05mm. "Bund-Nummer" = number of fret.











Relative to the exact equal-temperament tuning (i.e. HT = 1.05946, k = 17.817), both Gibson guitars deviate significantly from nominal value in the higher positions. With the (inaccurate) rule-of-18, however, (column on the right), the tolerances are o.k. Could it be that Gibson would still use outdated rules for manufacture even in the year 2004? Epiphone, Gibson's budget line produced in Asia, on the other hand, shows much higher precision.

Fig. 7.7b: Deviation of the measured position of a fret from the theoretical position. The dashed limit lines show pitch deviations of ± 1 cent. Measurement tolerance: ± 0.05mm. "Bund-Nummer" = number of fret.

Fig. 7.7 depicts the measured fret positions for a number of guitars. Since frets are rounded-off metal wires and do not provide sharp delimitations, the exact position is only available in approximation, with a measurement tolerance of about ± 0,05 mm. We can see exemplary precision for Taylor and Martin; for the other guitars the deviations are larger but still acceptable. Only Gibson shows to be the odd one out. Interesting in the two Fender guitars: Japanese "budget" production by no means shows worse results compared to fabrication in the US – rather, the contrary is the case. For all graphs, scale and position of the nut were set for an optimal curve. This is because the actual effective position of the string bearing is difficult to determine (rounding off of notches, bending stiffness of the strings).

 Translation by Tilmann Zwicker