7.4 Dynamics of the Strings

With *dynamic*, what is meant here is not a contrast to *static*, but a reference to the forces acting on the string. In Chapters 1 and 2, this topic was already addressed, albeit in a rather theoretical fashion, and without connection to a specific string bearing. In Chapter 7.4.1 now follows an analysis of the real playing situation: the fingers of the fretting hand need to exert a playing force F on the string to press it downward or to "bend" it (push and pull the string parallel to the frets). Since the string is deflected to the fret, two themes join up: the string dynamic and the guitar geometry. In Chapter 7.4.2, the forces transmitted by the string to its bearings (nut and bridge) are investigated both in static condition, and for the vibrating string.

7.4.1 Playing forces

When pressing down the strings, the fingers need to muster the force F that runs transversely to the direction of the strings. If we assume, for example, that a string is to be deflected at the 12th fret transversely by a distance η, then the transverse force F required for this amounts to:

$$F = 4 \cdot \Psi \cdot \eta / M$$ Ψ = tension force, M = scale

From the differential equation of the oscillation, the tension force Ψ results in:

$$\Psi = \pi \overline{\rho} \cdot \left(A \cdot M \cdot f_G \right)^2$$ A = overall diameter, $\overline{\rho}$ = mean density

From this, we can calculate the force necessary to fret the string when playing:

$$F = 4\pi \overline{\rho} \cdot \eta \cdot M \cdot A^2 \cdot f_G^2$$ Force required to press down the string

The playing force is proportional to the **scale length**: electric guitars with a scale of 25,5" (648 mm) require forces higher by 6,25% compared to guitars with a 24"-scale (610 mm). The playing force is also proportional to the **action** of the guitar: if the distance between string and fret is increased by 10%, the necessary playing force also rises by 10%. Moreover, the playing force is proportional to the square of the **string diameter**: using a 10-mil-E-string rather than a 9-mil-E-string increases the required playing force by 23%. The playing force is of the same value for all strings if the string-diameter relates inversely to the fundamental frequency. For wound strings, we need to consider that their effective density is smaller by about 10% compared to the fully solid string (see appendix A1). Light strings require playing forces between 0.5 and 1.5 N; for heavy strings, the playing forces are about double.

When executing **string bending**, the string is not only pressed against the fret but simultaneously stretched <u>in the transversal direction</u> – this leads to a clearly noticeable pitch increase. Applying a fast transversal movement creates a vibrato, slower movements result in glissandi that often amount to a pitch-change by a **whole step**. Since the string-tension force Ψ is proportional to the square of the frequency, a pitch increase by a whole step implies an increase in the string-tension force by 26%. The tension force is proportional to the elongation (strain) ξ that depends on the **overall** string length – i.e. also the (residual) sections of the string that extend beyond the nut and the bridge (that is assuming there is only small friction at the nut and at the bridge).

From the strain ξ we can deduce the required transversal displacement η. The latter may be recalculated into the required **transversal tension force F_Z**:

$$F_Z = 11{,}4 \cdot \sqrt{\frac{\overline{\rho}^3}{E}} \cdot M^3 \cdot \sqrt{1 + R/M} \cdot f_G^3 \cdot \frac{A^2}{\kappa} \qquad \text{Force for string-bending (12}^{\text{th fret}}\text{, +1 whole step)}$$

The "string-bending force" depends on the scale length with a power of three! Changing from a 24"-guitar to a 25.5"-guitar increases the required pulling/pushing force by 20%. The **remaining residual string *R*** (i.e. the part of the string behind the bridge and behind the nut) enters the section of the equation below the square root as relative difference. On a Gretsch 'Tennessean', for example, the G-string extends by 16 cm from the zero-fret to the machine head, and another 11 cm add themselves behind the bridge towards the Bigsby-vibrato. This extends the 24.5"-scale (M = 62 cm) by R = 27 cm to an overall length of 89 cm – 44% more than what would correspond to the actual scale length. The string-bending force rises by 20% due to this setup. In the bending force, the ratio κ of core-diameter to overall diameter of the string makes itself felt, as well; κ is 1 for solid strings, and 0.3 … 0.6 for wound stings.

If small string-bending forces are desired, the string should end as close as possible behind nut and bridge. Since with the gauges as they are typically included in string-sets, the bending force for the E_4-string is about 50 – 60% higher than that for the B-string, a short remaining string length would be particularly desirable for the E_4-string. The Stratocaster (and most other Fender guitars), however, feature an E_4-string that is the longest of all … well, Leo was not a guitar player *(Translator's note: also, in the early 1950's, bending strings was only starting to become fashionable)*. Those who would like to experiment can restring a left-hand guitar to be played right handed – bending will be easier on it. Or, conversely, a right-hand guitar may be restrung for left–hand use … oh – hi there, Jimi! A clamping nut (Floyd Rose, Schaller, Steinberger) will also bring improvements.

Bending by a whole step at the 12$^{\text{th}}$ fret will typically require string-bending forces in the range of 5 – 10 N for light strings. Since there is a square dependence on the string diameter, heavier strings will easily demand (up to) double the bending forces, requiring quite strong muscles in the hand and lower arms.

Easier on the muscles is changing the pitch via the **vibrato arm** (tremolo). The latter engages at the spring-loaded tailpiece and ensures a comfortable lever-transmission for changing the string tension and thus the pitch. In this construction, the tailpiece is not rigidly mounted to the guitar body but remains moveable by via a rotatable shaft, or a knife-edge bearing. One to five springs counteract the pull of the six strings. The effective spring stiffness related to operating the vibrato results from the sum of the stiffnesses of strings and tailpiece-spring. Soft springs are necessary if the vibrato arm is to be operated with little force. Such a setup, however, will increase the forces required for pushing/pulling the strings. A simple *thought experiment* may elucidate this: let us assume that the guitar is bolted down, and the string tension is provided by a weight – e.g. 6 kg for the G-string. (**a**) Pulling that string will, despite the exertion of force, not change the pitch since the force of the weight is not changed (given that the bearings have no effect), after all. (**b**) If now all 6 strings receive their tension combined from a single weight (e.g. 60 kg for a 009-string-set), the stiffness of the bearing of the G-string is given by the sum of the stiffnesses of the remaining 5 strings. (**c**) If the tension force is not generated by a weight but by a tension spring, we get, for the bearing stiffness of the G-string, the sum from 5 times the stiffnesses of the strings plus that of the tension spring. (**d**) If the tailpiece is fixed to the guitar body in a non-moveable fashion, the stiffness of the string bearing is infinite. Only this latter case provides for easiest string bending.

 Translation by Tilmann Zwicker

Case (a) is unusable for string bending: despite applying a bending force we do not get any pitch change. The last case (d) is ideal: due to the immobile tailpiece, the whole of the string bending force is used to change the tension force (via the force parallelogram). The practical situation with a vibrato-tailpiece remains in between: the stiffer the vibrato-strings, the easier the string bending gets. That's why some guitar players opt for the stiffest of all variants, and block the vibrato mechanism with a piece of wood.

Most vibrato-systems operate based on a simple principle: as the vibrato arm is pressed down, the pitch is lowered, if it is pulled up, the pitch increases. As a rule, the different strings are not equally de-tuned, and therefore the notes in a sounded chord will not remain relatively in tune when using the vibrato arm.

In the **Bigsby vibrato**, the tailpiece (string retainer) is constituted by a metal cylinder around which the strings are run. As the vibrato arm is pressed down, the cylinder rotates: all strings are shortened by the corresponding (equal) distance. To obtain equal de-tuning for all strings, not the absolute strain ξ, but the relative strain $\Delta\xi/\xi$, would have to change by the same amount. Since the elongations for correct tuning amounts to, for example, 1.2 mm (E_2) and 4,8 mm (E_4), respectively, an absolute constant change of the elongation will give 4 times the detuning for the E_2-string relative to the E_4-string!

For the **Stratocaster**, the situation is mildly better: the bridge/tailpiece-contraption as a whole tilts around a straight line, with this line – defined by 6 (or 2) set screws – located just ahead of the string-anchoring points. Since the bearing point of the E_2-string is set back (Chapter 7.2), the effect of the operation of the arm is a bit weaker for this string, and the relative detuning is not as strong compared to the Bigsby vibrato. Not that much is gained, though: instead of detuning the E_2-string by a factor of 4 relative to the E_4-string, the detuning is by a factor of 3 in the Strat. If indeed depressing the vibrato arm was to detune all strings by the same interval, an individual lever-transmission would be required for each string, and it would need to depend on the string data (κ). Luthiers did apparently not see much of a need for this, and neither did most guitarists.

As a typical example, we measured the **force** required on the vibrato arm of a Stratocaster to change the pitch by a ¼-note (2.93%): E_2: 2,9 N, A: 4,9 N, D: 5,2 N, G: 3,3 N, H: 3,9 N, E_4: 8,5 N; (009 string set, 3 vibrato springs installed). The guitar was fitted with all 6 strings and the vibrato arm was depressed until a detuning of ¼-note was reached for the respective string. A rough estimate shows that the main part of the spring stiffness results from the strings; the vibrato only provides about 1/3rd; it is relatively soft. However, the guitar player wishing for higher stiffness may increase the number of the vibrato springs up to 5 ... with corresponding provisions and suitable space already provided by the manufacturer.

A vibrato system is **detrimental** to the tuning process: as one string is tuned to pitch and the next string is tensioned, the spring-loaded tailpiece slackens, and the string already tuned up will be out of tune. Tuning up all 6 strings thus becomes an iterative process. If a string breaks during playing, the pitch of the remaining strings rises because the pull previously provided by the (now) broken string is lost. Last not least, worse **tuning stability** should be noted: due to unavoidable bearing friction, hysteresis appears. Depending on whether the vibrato arm is let go from the push- or the pull-position, different tuning will result. Resulting frequency deviations are inaudible only with high-grade vibrato systems. The **sound** may be influenced, as well: the spring-loading of the mass of the tailpiece may lead to low-frequency resonances, and spring vibrations may be transformed into electrical signals by magnetic pickups.

