7.4.2 Bearing forces at the nut

Unless fretted, the middle, freely vibrating section of the string is delimited by two "supporting bars". The upper support bar (at the headstock) is called nut, while the lower one is termed bridge (sometimes specified by constructional details such as roller bridge, bar bridge, etc.). For the following considerations of the bearing forces, nut and bridge are not actually distinguished, and for this reason both shall be called saddle here.

In classic guitar design, the string experiences a (sharp) bend at each saddle, changing direction by the bend angle α (**Fig. 7.10**). Most electric guitars have adopted this bearing-principle. The clamping nut would represent an alternative – but this will not be considered here. The change in direction of the string leads to a spatial (vectorial) force-decomposition: besides the tension-force Ψ of the string, there is also the saddle-force F_S that rises as the bend angle increases. For our first considerations, let's assume that the string can slip across the saddle **without any friction**: the tension forces on both sides of the saddle are therefore equal in strength. The saddle force can be calculated from:

$$F_S = 2 \cdot \Psi \cdot \sin(\alpha / 2) \approx \Psi \cdot \alpha / 57°$$ Saddle bearing-force

Given a tension force of 850 N (013 set of strings, all 6 strings tuned to pitch), and a bend angle of 10°, a saddle force of 148 N results – corresponding to the weight of a mass of 15 kg. Back in the day when guitars did not have electric pickups, the guitar top needed to be made of vibration-happy wood as thin as possible – this in order to achieve a decent sound volume. The bridge (-saddle) therefore could not absorb strong forces, and the bend angle thus had to be small. For the above example, small bend angles involve a proportionality between bend angle and saddle-force: halving α to 5° will halve the saddle-force, as well – to 74 N. A high saddle-force is nevertheless desirable for the string to solidly lie on top of the saddle and not start any secondary motions that would kill off vibration-energy. Guitars with thin tops require a compromise between stability and sound: a large bend angle guarantees safe and solid bearing, but the resulting strenuous loading of the top may lead to fracture. Solid-body guitars do not have that problem; any bend angle is possible. However, for large bend angles, friction effects increasingly need to be considered. If the tension-force changes on one side of the saddle, the string slides length-wise across the saddle; in the case of considerable frictional forces, the string may be out-of-tune. Specifying the bend angle of the strings therefore is an important step in the design of a guitar.

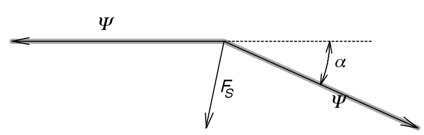

Fig. 7.10: Force-decomposition at the saddle.
Ψ = tension-force of the string, F_S = bearing-force.

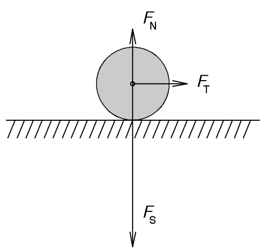

Fig. 7.11: Static and dynamic saddle-forces.
F_T = tangential force, F_N = normal force.

 Translation by Tilmann Zwicker

At rest, two static forces act onto the string: the tension force and the saddle force. However, as soon as the string is plucked, a **dynamic**♣ load is added. In the general case, the string vibrates with a combination of a fretboard-parallel and a fretboard-normal vibration: the **normal force** F_N, with its direction perpendicular to fretboard and saddle, changes the bearing force (**Fig. 7.11**). If the normal forces get larger than the saddle-force, the string lifts off the saddle – it is imperative to avoid this. Undesirable string movements may however occur already when the normal force is not quite as big as the saddle force: the fretboard-parallel **tangential force** F_T acts against the static friction force F_R. As soon as F_T becomes larger than F_R, lateral movement of the string is possible if the notches in the saddle are too wide. If this movement happens, vibration energy is irreversibly transformed into caloric energy (heat), and the duration of the string vibration is shortened. Additionally, interfering noise may become audible.

As an example we can check out the B-string of a 010-string-set (13-mil). Its tension force Ψ amounts to 70 N; for a bend angle of 10°, the bearing force at the saddle will be 12.3 N. Let us further assume that, at a distance of 9 cm from the saddle, the string is **moved away from the fretboard** using a perpendicular force that rises from 0 to 5 N. As this plucking force increases, the saddle mounting force decreases at the same time by 4.3 N from 12.8 to 8 N (**Fig. 7.12**). At the moment when the plucking force jumps back to zero, wave movements start: given a dispersion-free model, the saddle force jumps back and forth between 8 and 13 N; if there is dispersion, additional oscillations occur (Chapter 1.3.1). In this example, we do not see a negative bearing force, though: the flow of force is not interrupted. However, if we recalculate using a bend angle of merely 4° (as it is the case for the *Gretsch Tennessean*), the static bearing force is merely 4.9 N – now a 5-N-plucking-force will already lead to short occurrences of lift-off of the string, resulting in a buzzing, less-than-clean sound.

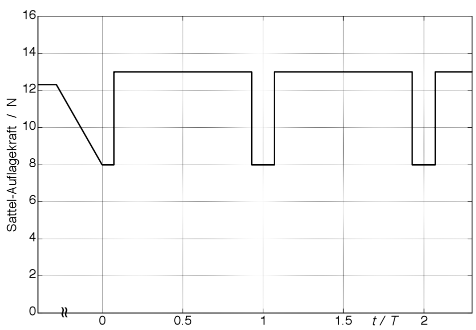

Fig. 7.12: Time function (modeled) of the saddle bearing force ("Sattel-Auflagekraft"). At $t < 0$, the string is pulled away from the guitar; the saddle force thus decreases to 8 N. At $t = 0$, the tension force jumps to 0; it takes 1/14 period (9cm / 126 cm) until this change arrives at the saddle.

If the direction of the plucking force is not perpendicular but parallel to the fretboard, shifts of the string to the side can occur. Given bearing at the rim of the saddle (Fig. 7.11), a tangential movement starts as soon as the static friction force is surmounted. In the individual case, the static friction coefficient depends on the specific pairing of materials – for a rough estimate we may assume μ = 0,1 ... 0,5. Ideal would be a small friction along the direction of the string, and a high friction in the perpendicular direction – but this only works if a groove is employed. Choosing μ = 0,1 and a bearing force of 12.3 N (as we calculated it in the above example), it shows that already a tangential force of merely 0.1 x 12.3 N = 1.2 N acting along the rim of the saddle may push the string back and forth on the saddle. A precisely machined V-shaped notch may prevent this back-and-forth sliding – however not every string bearing acts as a V-shaped notch!

♣ "Dynamic" is used here in contrast to "static", and not in the sense of "dynamis = force".

In the **V-shaped bearing** (**Fig 7.13**), the string rests in a defined fashion at two points. The sideways sliding is prohibited that way, however the static friction in the direction of the string is about double of what it would be using a rim bearing. The bearing in a **threaded groove** – as it is found in old Fender guitars – may result in two different states depending on whether the radius of the string is larger or smaller than the rounding-off of the groove of the thread. A bearing within a slot either has a clamping effect or a clearance – there are no special fits. A rounded-off bearing (fabricated using a round file) is similar to the bearing in a wide threaded groove – for heavy strings, there may however be clamping effects, as well.

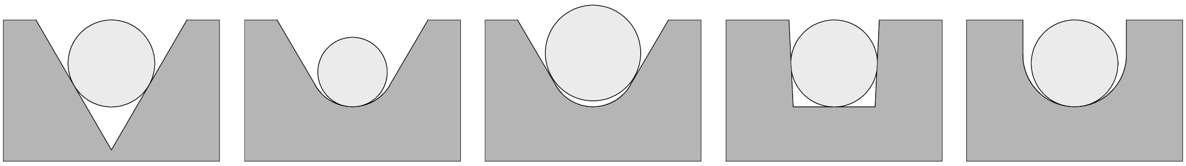

Fig. 7.13: String bearing: V-shape, threaded groove (w/light & heavy string), notch bearing, rounded bearing.

The ideal bearing should prohibit the string to move sideways. At the same time, it should have the least amount of friction in the direction of the string in order to allow for an unambiguous tension force. The tension force changes in particular during tuning of the guitar, and when bending strings. In case a vibrato-system is installed, operating it will also provide a change of the tension force. To illustrate the effects of bearing friction (in the direction of the string), we investigate a string with two fixed end-points and a saddle (**Fig. 7.14)**; the main section of the string has a scale length *M*.

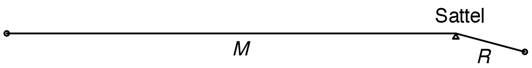

Fig. 7.14: String with scale length *M* and remaining section *R*. “Sattel” = saddle.

The maximum force *F* that can be applied to the saddle in horizontal direction[♣] is the **static frictional force** F_R (= friction at the bearing). If this limit-value is surpassed, the string will slide across the saddle. Assuming that we are, force-wise, just below this occurrence, F_R acts onto two spring stiffnesses working in parallel: the longitudinal stiffness of the main section of the string (with the length *M*), and that of the residual *R*. Given smallish bend angles, we obtain (with *S* = area of the cross-section) the effective overall stiffness *s*:

$$s \approx S \cdot E \cdot \left(1/M + 1/R\right)$$ Effective longitudinal stiffness

The change in length of the main section of the string (due to the frictional force) amounts to $\Delta x = s \cdot F_R$. As we move the saddle to the right (in the figure) such that no string slippage across the saddle occurs, the main section of the string is elongated: its frequency rises. Moving the saddle to the left, the frequency drops. However, the relative frequency change does not depend on the relative change in length but on the relative change in *strain*. A string of the length *M* needs to be stretched by *x* to generate the frequency f_G. The strain for a B-string (247 Hz), for example, is 2.6 mm. Since the frequency is proportional to the square root of the strain, the relative frequency change corresponds to half the change in relative strain (differential for small changes).

♣ parallel to the longitudinal axis of the string.

Given strain x, Young's modulus E, scale length M, and density ρ, the fundamental frequency f_G is:

$$f_G = \sqrt{\frac{x \cdot E}{4\rho \cdot M^3}}; \qquad \frac{df_G}{f_G} = \frac{1}{2} \cdot \frac{dx}{x}.$$ Dependency of the frequency on the strain

The change in strain Δx results from the frictional force♣ that in turn depends, via the frictional coefficient μ, on the tension force Ψ. The relative changes are:

$$\frac{\Delta x}{x} = \frac{\mu}{1 + M/R}; \qquad \frac{\Delta f_G}{f_G} = \frac{1}{2} \cdot \frac{\mu}{1 + M/R}$$ Relative changes

To stick to our B-string example: given M = 64 cm, R = 12 cm and μ = 0,15, we calculate a **relative detuning** of ± 1,2 %, corresponding to ± 21 cent. Therefore, the actual fundamental frequency of the string is in fact indeterminate for a saddle having friction, with $\pm \Delta f_G$ as the frequency incertitude. There are three ways to decrease this incertitude: either the friction is made so strong that changes in the tension force will always remain smaller than the frictional force – this results in the **clamping saddle** (clamping nut, clamping bridge) in which the string cannot budge at all. Or the friction is reduced as far as possible by using low-friction saddle materials and/or small bend angles – this will increase the danger of relative movements between saddle and string, though. Or, the remaining string section between saddle and tailpiece is shortened to just a few centimeters (e.g. Les Paul with Stopbar tailpiece), and the longitudinal stiffness is thus increased. Besides these approaches, there have been experiments making the saddle (or the bridge as a whole) moveable in its bearings: examples are the Fender Jazzmaster and Jaguar … not very successful, getting little love, and resurfacing more as a cult object. For further data see Table 7.1.

At this point, however, we need to remind ourselves of a fundamental law of sound generation: *individuality is imperfection*. Whether a bridge has little attenuation effect on the string vibration (i.e. long sustain), or much attenuation (i.e. percussive sound) – in the end that remains a matter of taste. The **sitar bridge** that can be retrofitted to Telecasters may serve as particularly succinct example: for it, a special string bearing intentionally generates "interfering noise", kind of a "boiiiinnnggg" … not to be mistaken with a "bonk" … you hear me, Mr. Clinton?

	Bend angle at the nut	Bend angle at the bridge
Stratocaster	7° – 9°	up to 90°
Telecaster	7° – 9°	9° (top loading), 34° (through-body)
Jazzmaster	7° – 9°	6° – 7°
Gretsch Tennessean	5° – 15°	4°
Rickenbacker 335	7° – 12°	4,5°
Gibson ES 335	15°	9° – 10°
Les Paul '59 reissue	17°	19° – 26°
Taylor PS-54-CE	13°	12° / 27°
Ovation SMT	11°	25° – 30°
Ovation EA68	11°	33°
Martin D45V	15°	25° – 30°

Table 7.1: Typical bend angles of strings

♣ The formula for wrap-around friction exp($\mu\alpha$) lends itself for a more precise calculation.

