

7.4 Dynamics of the Strings

With *dynamic*, what is meant here is not a contrast to *static*, but a reference to the forces acting on the string. In Chapters 1 and 2, this topic was already addressed, albeit in a rather theoretical fashion, and without connection to a specific string bearing. In Chapter 7.4.1 now follows an analysis of the real playing situation: the fingers of the fretting hand need to exert a playing force F on the string to press it downward or to “bend” it (push and pull the string parallel to the frets). Since the string is deflected to the fret, two themes join up: the string dynamic and the guitar geometry. In Chapter 7.4.2, the forces transmitted by the string to its bearings (nut and bridge) are investigated both in static condition, and for the vibrating string.

7.4.1 Playing forces

When pressing down the strings, the fingers need to muster the force F that runs transversely to the direction of the strings. If we assume, for example, that a string is to be deflected at the 12th fret transversely by a distance η , then the transverse force F required for this amounts to:

$$F = 4 \cdot \Psi \cdot \eta / M \qquad \Psi = \text{tension force, } M = \text{scale}$$

From the differential equation of the oscillation, the tension force Ψ results in:

$$\Psi = \pi \bar{\rho} \cdot (A \cdot M \cdot f_G)^2 \qquad A = \text{overall diameter, } \bar{\rho} = \text{mean density}$$

From this, we can calculate the force necessary to fret the string when playing:

$$F = 4\pi \bar{\rho} \cdot \eta \cdot M \cdot A^2 \cdot f_G^2 \qquad \text{Force required to press down the string}$$

The playing force is proportional to the **scale length**: electric guitars with a scale of 25,5" (648 mm) require forces higher by 6,25% compared to guitars with a 24"-scale (610 mm). The playing force is also proportional to the **action** of the guitar: if the distance between string and fret is increased by 10%, the necessary playing force also rises by 10%. Moreover, the playing force is proportional to the square of the **string diameter**: using a 10-mil-E-string rather than a 9-mil-E-string increases the required playing force by 23%. The playing force is of the same value for all strings if the string-diameter relates inversely to the fundamental frequency. For wound strings, we need to consider that their effective density is smaller by about 10% compared to the fully solid string (see appendix A1). Light strings require playing forces between 0.5 and 1.5 N; for heavy strings, the playing forces are about double.

When executing **string bending**, the string is not only pressed against the fret but simultaneously stretched in the transversal direction – this leads to a clearly noticeable pitch increase. Applying a fast transversal movement creates a vibrato, slower movements result in glissandi that often amount to a pitch-change by a **whole step**. Since the string-tension force Ψ is proportional to the square of the frequency, a pitch increase by a whole step implies an increase in the string-tension force by 26%. The tension force is proportional to the elongation (strain) ξ that depends on the **overall** string length – i.e. also the (residual) sections of the string that extend beyond the nut and the bridge (that is assuming there is only small friction at the nut and at the bridge).

From the strain ξ we can deduce the required transversal displacement η . The latter may be recalculated into the required **transversal tension force** F_Z :

$$F_Z = 11,4 \cdot \sqrt{\frac{\rho^3}{E}} \cdot M^3 \cdot \sqrt{1 + R/M} \cdot f_G^3 \cdot \frac{A^2}{\kappa} \quad \text{Force for string-bending (12th fret, +1 whole step)}$$

The “string-bending force” depends on the scale length with a power of three! Changing from a 24”-guitar to a 25.5”-guitar increases the required pulling/pushing force by 20%. The **remaining residual string** R (i.e. the part of the string behind the bridge and behind the nut) enters the section of the equation below the square root as relative difference. On a Gretsch ‘Tennessean’, for example, the G-string extends by 16 cm from the zero-fret to the machine head, and another 11 cm add themselves behind the bridge towards the Bigsby-vibrato. This extends the 24.5”-scale ($M = 62$ cm) by $R = 27$ cm to an overall length of 89 cm – 44% more than what would correspond to the actual scale length. The string-bending force rises by 20% due to this setup. In the bending force, the ratio κ of core-diameter to overall diameter of the string makes itself felt, as well; κ is 1 for solid strings, and 0.3 ... 0.6 for wound strings.

If small string-bending forces are desired, the string should end as close as possible behind nut and bridge. Since with the gauges as they are typically included in string-sets, the bending force for the E_4 -string is about 50 – 60% higher than that for the B-string, a short remaining string length would be particularly desirable for the E_4 -string. The Stratocaster (and most other Fender guitars), however, feature an E_4 -string that is the longest of all ... well, Leo was not a guitar player (*Translator’s note: also, in the early 1950’s, bending strings was only starting to become fashionable*). Those who would like to experiment can restring a left-hand guitar to be played right handed – bending will be easier on it. Or, conversely, a right-hand guitar may be restringed for left-hand use ... oh – hi there, Jimi! A clamping nut (Floyd Rose, Schaller, Steinberger) will also bring improvements.

Bending by a whole step at the 12th fret will typically require string-bending forces in the range of 5 – 10 N for light strings. Since there is a square dependence on the string diameter, heavier strings will easily demand (up to) double the bending forces, requiring quite strong muscles in the hand and lower arms.

Easier on the muscles is changing the pitch via the **vibrato arm** (tremolo). The latter engages at the spring-loaded tailpiece and ensures a comfortable lever-transmission for changing the string tension and thus the pitch. In this construction, the tailpiece is not rigidly mounted to the guitar body but remains moveable by via a rotatable shaft, or a knife-edge bearing. One to five springs counteract the pull of the six strings. The effective spring stiffness related to operating the vibrato results from the sum of the stiffnesses of strings and tailpiece-spring. Soft springs are necessary if the vibrato arm is to be operated with little force. Such a setup, however, will increase the forces required for pushing/pulling the strings. A simple *thought experiment* may elucidate this: let us assume that the guitar is bolted down, and the string tension is provided by a weight – e.g. 6 kg for the G-string. **(a)** Pulling that string will, despite the exertion of force, not change the pitch since the force of the weight is not changed (given that the bearings have no effect), after all. **(b)** If now all 6 strings receive their tension combined from a single weight (e.g. 60 kg for a 009-string-set), the stiffness of the bearing of the G-string is given by the sum of the stiffnesses of the remaining 5 strings. **(c)** If the tension force is not generated by a weight but by a tension spring, we get, for the bearing stiffness of the G-string, the sum from 5 times the stiffnesses of the strings plus that of the tension spring. **(d)** If the tailpiece is fixed to the guitar body in a non-moveable fashion, the stiffness of the string bearing is infinite. Only this latter case provides for easiest string bending.

Case (a) is unusable for string bending: despite applying a bending force we do not get any pitch change. The last case (d) is ideal: due to the immobile tailpiece, the whole of the string bending force is used to change the tension force (via the force parallelogram). The practical situation with a vibrato-tailpiece remains in between: the stiffer the vibrato-strings, the easier the string bending gets. That's why some guitar players opt for the stiffest of all variants, and block the vibrato mechanism with a piece of wood.

Most vibrato-systems operate based on a simple principle: as the vibrato arm is pressed down, the pitch is lowered, if it is pulled up, the pitch increases. As a rule, the different strings are not equally de-tuned, and therefore the notes in a sounded chord will not remain relatively in tune when using the vibrato arm.

In the **Bigby vibrato**, the tailpiece (string retainer) is constituted by a metal cylinder around which the strings are run. As the vibrato arm is pressed down, the cylinder rotates: all strings are shortened by the corresponding (equal) distance. To obtain equal de-tuning for all strings, not the absolute strain ξ , but the relative strain $\Delta\xi/\xi$, would have to change by the same amount. Since the elongations for correct tuning amounts to, for example, 1.2 mm (E_2) and 4.8 mm (E_4), respectively, an absolute constant change of the elongation will give 4 times the detuning for the E_2 -string relative to the E_4 -string!

For the **Stratocaster**, the situation is mildly better: the bridge/tailpiece-contraption as a whole tilts around a straight line, with this line – defined by 6 (or 2) set screws – located just ahead of the string-anchoring points. Since the bearing point of the E_2 -string is set back (Chapter 7.2), the effect of the operation of the arm is a bit weaker for this string, and the relative detuning is not as strong compared to the Bigsby vibrato. Not that much is gained, though: instead of detuning the E_2 -string by a factor of 4 relative to the E_4 -string, the detuning is by a factor of 3 in the Strat. If indeed depressing the vibrato arm was to detune all strings by the same interval, an individual lever-transmission would be required for each string, and it would need to depend on the string data (κ). Luthiers did apparently not see much of a need for this, and neither did most guitarists.

As a typical example, we measured the **force** required on the vibrato arm of a Stratocaster to change the pitch by a $\frac{1}{4}$ -note (2.93%): E_2 : 2,9 N, A: 4,9 N, D: 5,2 N, G: 3,3 N, H: 3,9 N, E_4 : 8,5 N; (009 string set, 3 vibrato springs installed). The guitar was fitted with all 6 strings and the vibrato arm was depressed until a detuning of $\frac{1}{4}$ -note was reached for the respective string. A rough estimate shows that the main part of the spring stiffness results from the strings; the vibrato only provides about $\frac{1}{3}^{\text{rd}}$; it is relatively soft. However, the guitar player wishing for higher stiffness may increase the number of the vibrato springs up to 5 ... with corresponding provisions and suitable space already provided by the manufacturer.

A vibrato system is **detrimental** to the tuning process: as one string is tuned to pitch and the next string is tensioned, the spring-loaded tailpiece slackens, and the string already tuned up will be out of tune. Tuning up all 6 strings thus becomes an iterative process. If a string breaks during playing, the pitch of the remaining strings rises because the pull previously provided by the (now) broken string is lost. Last not least, worse **tuning stability** should be noted: due to unavoidable bearing friction, hysteresis appears. Depending on whether the vibrato arm is let go from the push- or the pull-position, different tuning will result. Resulting frequency deviations are inaudible only with high-grade vibrato systems. The **sound** may be influenced, as well: the spring-loading of the mass of the tailpiece may lead to low-frequency resonances, and spring vibrations may be transformed into electrical signals by magnetic pickups.

7.4.2 Bearing forces at the nut

Unless fretted, the middle, freely vibrating section of the string is delimited by two "supporting bars". The upper support bar (at the headstock) is called nut, while the lower one is termed bridge (sometimes specified by constructional details such as roller bridge, bar bridge, etc.). For the following considerations of the bearing forces, nut and bridge are not actually distinguished, and for this reason both shall be called saddle here.

In classic guitar design, the string experiences a (sharp) bend at each saddle, changing direction by the bend angle α (Fig. 7.10). Most electric guitars have adopted this bearing-principle. The clamping nut would represent an alternative – but this will not be considered here. The change in direction of the string leads to a spatial (vectorial) force-decomposition: besides the tension-force Ψ of the string, there is also the saddle-force F_S that rises as the bend angle increases. For our first considerations, let's assume that the string can slip across the saddle **without any friction**: the tension forces on both sides of the saddle are therefore equal in strength. The saddle force can be calculated from:

$$F_S = 2 \cdot \Psi \cdot \sin(\alpha/2) \approx \Psi \cdot \alpha / 57^\circ \quad \text{Saddle bearing-force}$$

Given a tension force of 850 N (013 set of strings, all 6 strings tuned to pitch), and a bend angle of 10° , a saddle force of 148 N results – corresponding to the weight of a mass of 15 kg. Back in the day when guitars did not have electric pickups, the guitar top needed to be made of vibration-happy wood as thin as possible – this in order to achieve a decent sound volume. The bridge (-saddle) therefore could not absorb strong forces, and the bend angle thus had to be small. For the above example, small bend angles involve a proportionality between bend angle and saddle-force: halving α to 5° will halve the saddle-force, as well – to 74 N. A high saddle-force is nevertheless desirable for the string to solidly lie on top of the saddle and not start any secondary motions that would kill off vibration-energy. Guitars with thin tops require a compromise between stability and sound: a large bend angle guarantees safe and solid bearing, but the resulting strenuous loading of the top may lead to fracture. Solid-body guitars do not have that problem; any bend angle is possible. However, for large bend angles, friction effects increasingly need to be considered. If the tension-force changes on one side of the saddle, the string slides length-wise across the saddle; in the case of considerable frictional forces, the string may be out-of-tune. Specifying the bend angle of the strings therefore is an important step in the design of a guitar.

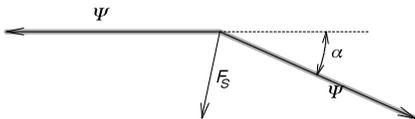


Fig. 7.10: Force-decomposition at the saddle.
 Ψ = tension-force of the string, F_S = bearing-force.

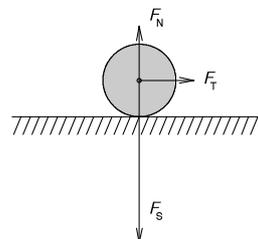


Fig. 7.11: Static and dynamic saddle-forces.
 F_T = tangential force, F_N = normal force.

At rest, two static forces act onto the string: the tension force and the saddle force. However, as soon as the string is plucked, a **dynamic*** load is added. In the general case, the string vibrates with a combination of a fretboard-parallel and a fretboard-normal vibration: the **normal force** F_N , with its direction perpendicular to fretboard and saddle, changes the bearing force (**Fig. 7.11**). If the normal forces get larger than the saddle-force, the string lifts off the saddle – it is imperative to avoid this. Undesirable string movements may however occur already when the normal force is not quite as big as the saddle force: the fretboard-parallel **tangential force** F_T acts against the static friction force F_R . As soon as F_T becomes larger than F_R , lateral movement of the string is possible if the notches in the saddle are too wide. If this movement happens, vibration energy is irreversibly transformed into caloric energy (heat), and the duration of the string vibration is shortened. Additionally, interfering noise may become audible.

As an example we can check out the B-string of a 010-string-set (13-mil). Its tension force Ψ amounts to 70 N; for a bend angle of 10° , the bearing force at the saddle will be 12.3 N. Let us further assume that, at a distance of 9 cm from the saddle, the string is **moved away from the fretboard** using a perpendicular force that rises from 0 to 5 N. As this plucking force increases, the saddle mounting force decreases at the same time by 4.3 N from 12.8 to 8 N (**Fig. 7.12**). At the moment when the plucking force jumps back to zero, wave movements start: given a dispersion-free model, the saddle force jumps back and forth between 8 and 13 N; if there is dispersion, additional oscillations occur (Chapter 1.3.1). In this example, we do not see a negative bearing force, though: the flow of force is not interrupted. However, if we recalculate using a bend angle of merely 4° (as it is the case for the *Gretsch Tennessean*), the static bearing force is merely 4.9 N – now a 5-N-plucking-force will already lead to short occurrences of lift-off of the string, resulting in a buzzing, less-than-clean sound.

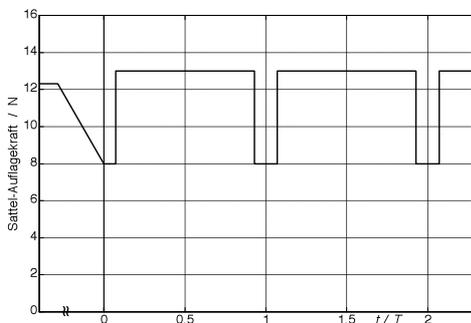


Fig. 7.12: Time function (modeled) of the saddle bearing force (“Sattel-Auflagekraft”). At $t < 0$, the string is pulled away from the guitar; the saddle force thus decreases to 8 N. At $t = 0$, the tension force jumps to 0; it takes 1/14 period (9cm / 126 cm) until this change arrives at the saddle.

If the direction of the plucking force is not perpendicular but parallel to the fretboard, shifts of the string to the side can occur. Given bearing at the rim of the saddle (**Fig. 7.11**), a tangential movement starts as soon as the static friction force is surmounted. In the individual case, the static friction coefficient depends on the specific pairing of materials – for a rough estimate we may assume $\mu = 0,1 \dots 0,5$. Ideal would be a small friction along the direction of the string, and a high friction in the perpendicular direction – but this only works if a groove is employed. Choosing $\mu = 0,1$ and a bearing force of 12.3 N (as we calculated it in the above example), it shows that already a tangential force of merely $0,1 \times 12,3 \text{ N} = 1,2 \text{ N}$ acting along the rim of the saddle may push the string back and forth on the saddle. A precisely machined V-shaped notch may prevent this back-and-forth sliding – however not every string bearing acts as a V-shaped notch!

* “Dynamic” is used here in contrast to “static”, and not in the sense of “dynamis = force”.

In the **V-shaped bearing** (**Fig 7.13**), the string rests in a defined fashion at two points. The sideways sliding is prohibited that way, however the static friction in the direction of the string is about double of what it would be using a rim bearing. The bearing in a **threaded groove** – as it is found in old Fender guitars – may result in two different states depending on whether the radius of the string is larger or smaller than the rounding-off of the groove of the thread. A bearing within a slot either has a clamping effect or a clearance – there are no special fits. A rounded-off bearing (fabricated using a round file) is similar to the bearing in a wide threaded groove – for heavy strings, there may however be clamping effects, as well.

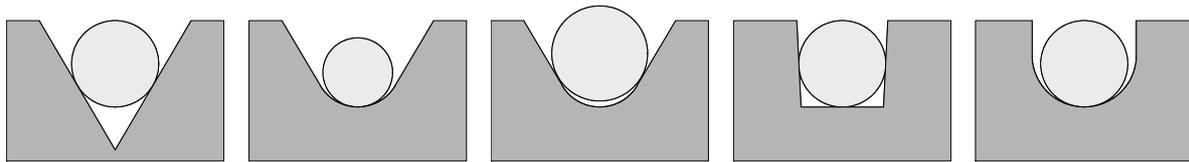


Fig. 7.13: String bearing: V-shape, threaded groove (w/light & heavy string), notch bearing, rounded bearing.

The ideal bearing should prohibit the string to move sideways. At the same time, it should have the least amount of friction in the direction of the string in order to allow for an unambiguous tension force. The tension force changes in particular during tuning of the guitar, and when bending strings. In case a vibrato-system is installed, operating it will also provide a change of the tension force. To illustrate the effects of bearing friction (in the direction of the string), we investigate a string with two fixed end-points and a saddle (**Fig. 7.14**); the main section of the string has a scale length M .

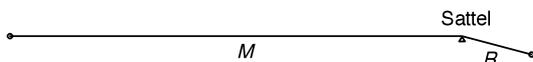


Fig. 7.14: String with scale length M and remaining section R . “Sattel” = saddle.

The maximum force F that can be applied to the saddle in horizontal direction* is the **static frictional force** F_R (= friction at the bearing). If this limit-value is surpassed, the string will slide across the saddle. Assuming that we are, force-wise, just below this occurrence, F_R acts onto two spring stiffnesses working in parallel: the longitudinal stiffness of the main section of the string (with the length M), and that of the residual R . Given smallish bend angles, we obtain (with S = area of the cross-section) the effective overall stiffness s :

$$s \approx S \cdot E \cdot (1/M + 1/R) \quad \text{Effective longitudinal stiffness}$$

The change in length of the main section of the string (due to the frictional force) amounts to $\Delta x = s \cdot F_R$. As we move the saddle to the right (in the figure) such that no string slippage across the saddle occurs, the main section of the string is elongated: its frequency rises. Moving the saddle to the left, the frequency drops. However, the relative frequency change does not depend on the relative change in length but on the relative change in *strain*. A string of the length M needs to be stretched by x to generate the frequency f_G . The strain for a B-string (247 Hz), for example, is 2.6 mm. Since the frequency is proportional to the square root of the strain, the relative frequency change corresponds to half the change in relative strain (differential for small changes).

* parallel to the longitudinal axis of the string.

Given strain x , Young's modulus E , scale length M , and density ρ , the fundamental frequency f_G is:

$$f_G = \sqrt{\frac{x \cdot E}{4\rho \cdot M^3}}; \quad \frac{df_G}{f_G} = \frac{1}{2} \cdot \frac{dx}{x}. \quad \text{Dependency of the frequency on the strain}$$

The change in strain Δx results from the frictional force* that in turn depends, via the frictional coefficient μ , on the tension force Ψ . The relative changes are:

$$\frac{\Delta x}{x} = \frac{\mu}{1 + M/R}; \quad \frac{\Delta f_G}{f_G} = \frac{1}{2} \cdot \frac{\mu}{1 + M/R} \quad \text{Relative changes}$$

To stick to our B-string example: given $M = 64$ cm, $R = 12$ cm and $\mu = 0,15$, we calculate a **relative detuning** of $\pm 1,2$ %, corresponding to ± 21 cent. Therefore, the actual fundamental frequency of the string is in fact indeterminate for a saddle having friction, with $\pm \Delta f_G$ as the frequency incertitude. There are three ways to decrease this incertitude: either the friction is made so strong that changes in the tension force will always remain smaller than the frictional force – this results in the **clamping saddle** (clamping nut, clamping bridge) in which the string cannot budge at all. Or the friction is reduced as far as possible by using low-friction saddle materials and/or small bend angles – this will increase the danger of relative movements between saddle and string, though. Or, the remaining string section between saddle and tailpiece is shortened to just a few centimeters (e.g. Les Paul with Stopbar tailpiece), and the longitudinal stiffness is thus increased. Besides these approaches, there have been experiments making the saddle (or the bridge as a whole) moveable in its bearings: examples are the Fender Jazzmaster and Jaguar ... not very successful, getting little love, and resurfacing more as a cult object. For further data see Table 7.1.

At this point, however, we need to remind ourselves of a fundamental law of sound generation: *individuality is imperfection*. Whether a bridge has little attenuation effect on the string vibration (i.e. long sustain), or much attenuation (i.e. percussive sound) – in the end that remains a matter of taste. The **sitar bridge** that can be retrofitted to Telecasters may serve as particularly succinct example: for it, a special string bearing intentionally generates “interfering noise”, kind of a “boiiiiinnnggg” ... not to be mistaken with a “bonk” ... you hear me, Mr. Clinton?

	Bend angle at the nut	Bend angle at the bridge
Stratocaster	7° – 9°	up to 90°
Telecaster	7° – 9°	9° (top loading), 34° (through-body)
Jazzmaster	7° – 9°	6° – 7°
Gretsch Tennessean	5° – 15°	4°
Rickenbacker 335	7° – 12°	4,5°
Gibson ES 335	15°	9° – 10°
Les Paul '59 reissue	17°	19° – 26°
Taylor PS-54-CE	13°	12° / 27°
Ovation SMT	11°	25° – 30°
Ovation EA68	11°	33°
Martin D45V	15°	25° – 30°

Table 7.1: Typical bend angles of strings

* The formula for wrap-around friction $\exp(\mu\alpha)$ lends itself for a more precise calculation.