The bandwidths of **1/3rd-octave filters** (23%) approximately correspond to the bandwidths of the filters found in the hearing system for frequencies above 500 Hz. Therefore, combining neighboring partials into one analysis channel rests on a similar basis. Also, a 1/3rd-octave filter will share with the critical-band filter the characteristic that a very low priority is given to phase responses.

The **Volagramm**♣ gives a clear (yet somewhat arbitrary) representation. It shows the decay of individual partials (in fact: DFT-lines) as a difference spectrum: $L(f, t + \Delta t) - L(f, t)$. **Fig. 7.61** conveys an idea of this approach: level differences were calculated for 4 DFT-spectra (determined for 0 / 170 / 340 / 510 ms) and outlined as polylines. On the left, we see a rather regular decay of the partials, as time passes, the polylines fan out downwards because higher-frequency partials decay more quickly that the lower-frequency ones. On the right, more pronounced irregularities can be seen – caused by fluctuating envelopes of the partials. This representation is not unambiguous because both the type of window and the time-spacing are chosen arbitrarily – but it does provide a quick orientation across frequency ranges of interest.

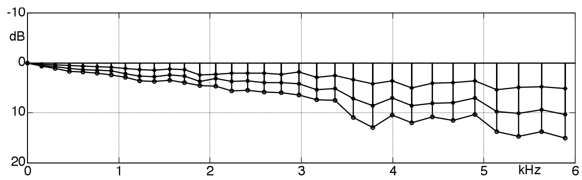

Fig. 7.61: Volagrams: string mounted on the stone table; ends of the string clamped (left) and supported (right). 0,7-mm-string, f_G = 150 Hz, Δt = 170 / 340 / 510 ms, N = 4096, Matlab-Kaiser-window (β = 12). As ordinate, the attenuation is shown; as time progresses, the polyline fans out downward.

7.6.3 The decay time T_{30}

There are several possibilities to quantitatively describe the attenuation in a resonance circuit: degree of damping, time constant, loss factor, logarithmic decrement, measure of decay, or Q-factor. The vibration of a spring-mass-system damped by Stokes-friction will decay exponentially after excitation by an impulse:

$$v(t) = e^{-t/\tau} \cdot \cos(\omega_0 t) \,; \qquad \tau = \text{time-constant}$$

The full designation for the time-constant used in this formula is amplitude-time-constant because it describes the decay of the amplitude. The power also decays exponentially for this vibration, but because power has a square-dependency on the amplitude, the time constant for the power decay will be different: this so-called power-time-constant is half the other time-constant. Standardized sound measurements use e.g. a power-time-constant of 125 ms in the "Fast"-mode of averaging; the corresponding amplitude-time-constant is 250 ms. A time-constant specifies the period of time during which the quantity characterized with it decays to $1/e$ = 0.368. Alternatively, a decay to other specific values may be given – such as is practice e.g. with the **reverberation-time** T_N used in room acoustics. During T_N, the signal level drops by 60 dB (i.e. the amplitude drops to 1/1000). Since such a drastic drop is lacking in practical relevance for musical tones, Fleischer [9] has proposed 30 dB as **decay-time** T_{30}.

♣ volare = to disappear, to be volatile, to decay (latin); graphein = to draw (greek).

The decay-time must, however, not be understood such that we pluck the string and then wait until the level has dropped off by 30 dB. Rather, we have to form a **smoothed straight line** in the *L*(*t*)-diagram, the gradient of which results in the decay-time. On the left in **Fig. 7.62**, we see a perfectly linear level decay. With an exponential decay of the amplitude over time, the level (i.e. the logarithm of the amplitude) will decrease linearly over time. The **decay-rate** – the negative gradient of the curve – is 8.7 dB/s in this example; the time-constant is 1 s and the decay-time is 3.45 s.

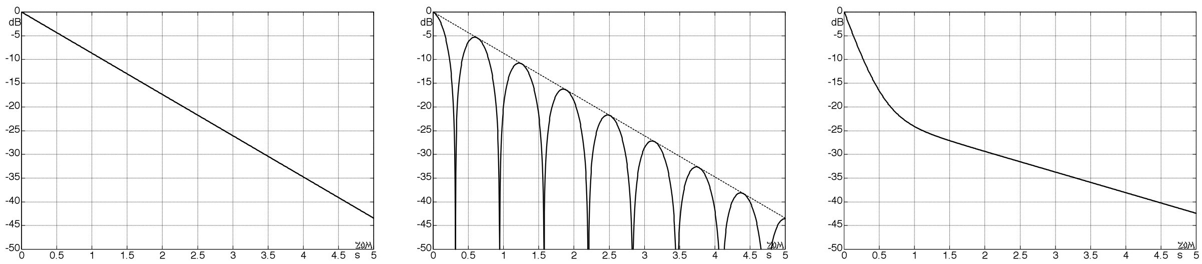

Fig. 7.62: Various decay processes.

The centre curve shows the level decay of a beating signal: after 0.31 s, the level has dropped (relative to the initial value) by 30 dB for the first time. This is, however, not the decay-time – that amounts to 3.45 s just as in the example on the left and is calculated via the (dashed) envelope. Such **beats** occur if two partials of the same initial amplitude and the same damping, but with slightly different frequencies, decay jointly. In this example it is not difficult to find an envelope for the maxima of the curve, and to determine its gradient. The process become more difficult if the periodicity of the beat is much longer, e.g. if the first minimum is only reached after 5 s. It may be impossible to determine the level values of subsequent maxima because the signal has already become too small and disappears in the ever-present noise.

The analysis of the decay becomes even more problematic if partials of very different time-constants decay (Fig. 7.62, right). We could determine T_{30} from the initial slope (as it would be done in room acoustics for the early-decay-time), or from the final slope, or we could – after all – take the point in time when *L* passes through -30 dB. In the case of a combination of beats and different time-constants this could easily lead to an unusable T_{30}-value, though. In most cases, the decay-time is a highly useful measure to describe decay processes or attenuation. Still, in some special scenarios it may not be purposeful. Therefore, caution is advised especially when using programs that automatically calculate d T_{30}.