1.2 Wound strings

The thinner strings of the electric guitar (E_4, B_3) consist of solid steel. If the thicker (bass-) strings (E_2, A_2, D_3, sometimes G_3 as well) were manufactured the same way, unavoidable flexural stiffness would result in considerable inharmonicities (Chapter 1.3). For this reason, a thin core made of steel is wound with a helically abutting winding (**Fig. 1.4**). For electric guitars, the winding consists of steel or nickel, while for acoustic guitars it is made of bronze. Using this construction, the flexural stiffness is determined mainly by the core. The winding merely contributes the required additional mass.

Several criteria are relevant for the relationship $\boldsymbol{\kappa = D_K/D_A}$ between core diameter D_K and the outer diameter D_A: in order to reduce the flexural stiffness, κ should be made as small as possible. However, the normal stress now very quickly approaches the limit of tensile strength even for high-strength steel. Simple machinery steel, for example, has a **minimum tensile strength** of around 430 N/mm^2 (St 44). For strings, this would be not adequate at all since – for regular tuning and in rest condition – up to 2000 N/mm^2 is required here. During playing, additional strain occurs that (in the interest of long durability) still needs to remain well below the breaking point. Moreover, high endurance towards changing strain is demanded as well. In addition, the string must not corrode too fast, it should not be too brittle (in order to agree with string bending), and it moreover needs to have certain magnetic properties. Overall, these are very challenging demands – not easily fulfilled by just any manufacturer of wires.

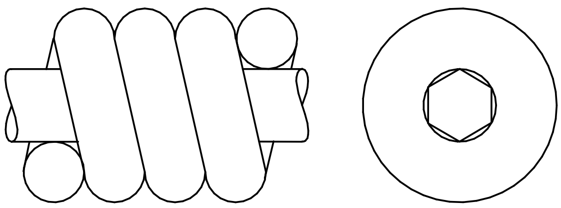

Fig. 1.4: Wound string.
The string-core is either round or polygonal (e.g. hexagonal).

For most wound strings, the **core-diameter** measures 1/3rd to 2/3rd of the outer diameter. In particular for the higher-frequency strings, a smaller κ-value leads to breakage, and moreover the winding-wire would have to be bent very strongly. Higher κ-values relieve the core but bring stronger inharmonicities, and also result in too small a diameter of the winding wire (this also calling for issues with durability). Besides the ratio of core diameter to overall diameter, the absolute values are significant, too. To generate a certain pitch (e.g. E_2), the heavier strings need to be (and may be) stretched more than the light strings. Doubling the diameter quadruples the mass; if the pitch is supposed to remain constant, the tension force also needs to be quadrupled – with the normal tension (pulling force / cross-sectional area) remaining unaffected by this.

The winding of a string often employs round wire; **flat wire** is used more rarely. Due to the oblique grooves, strings wound with round wire feel somewhat rough; strings wound with flat wire (flatwound strings) give a feel similar to the plain strings but they sound differently. Somewhere midway we find sanded-down strings: here the core is first wound with round wire, and subsequently the outer sections of the winding are slightly sanded in order to reduce the surface roughness.

 Translation by Tilmann Zwicker

On acoustic guitars, heavy strings facilitate a louder sound but require to be pressed down onto the fretboard with more force. The signals generated by electric guitars can be amplified to almost any degree, and therefore we frequently find, on these instruments, lighter strings than on acoustic guitars. In fact, it was only the reduction of the tension- and thus playing-forces by up to 50% that enabled the development of new techniques (bending strings, finger vibrato) on the electric guitar.

Every string manufacturer offers sets of strings with different diameters – designations are usually "heavy", "medium", "light", or "super light". For a more precise characterization, all string diameters are in addition given in mil (1 mil = 1/1000 inch = 25.4 mm /1000). On electric guitars, the so-called 009-set is found quite often, consisting e.g. of strings with the diameters 9-11-15-24-32-42. However, there are 009-sets also with different gradation, for example 9-11-16-26-36-46. In string sets with thinner strings ("light gauge strings"), the three treble strings are solid ("plain") while the heavier strings are wound; in heavier gauge string sets, the G-string is would, as well.

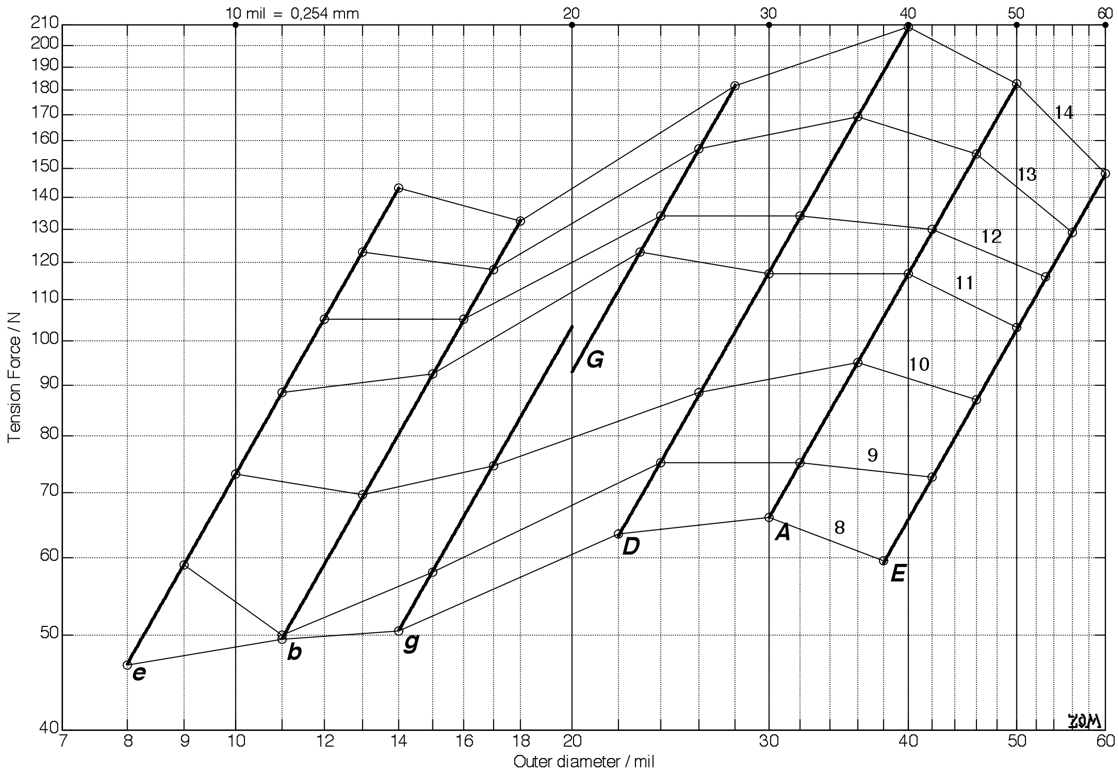

Fig. 1.5: Tension-force of a string dependent on the outer diameter. ρ_{core} = 7900 kg/m^3, $\rho_{winding.}$ = 8800 kg/m^3. For the string length, 25.5" = 64.8cm (e.g. Stratocaster) was taken; shorter lengths decrease the tension force Ψ. The effect of κ on Ψ is small. 013- and 014-string-sets are mainly found on steel-string acoustic guitars.

Fig. 1.5 shows how tension force Ψ and string diameter are related. The strings are depicted as steeply inclined lines, with the G-string shown both with and without winding. Frequently used diameter combinations are shown as a shallow curved line. The calculations are based on rigid (unyielding) string-bearings. Spring-loaded bearing (e.g. a vibrato system) necessitates higher tension forces. For frequency dependent spring effect see Chapter 2.5.2.

For solid strings, the **tension force of the string** Ψ is calculated from the density ρ, the fundamental frequency f_G, the (outer) diameter D, and the string length (scale) M:

$$\Psi = \pi \cdot \rho \cdot \left(f_G \cdot D \cdot M\right)^2$$ Tension force of the string

Due to the air enclosed in the winding, the **density** of **wound strings** is about 10% less compared to solid strings (given the same outer diameter):

$$\overline{\rho} = \rho_{wound} = \left[\kappa^2 + \left(1-\kappa^2\right)\cdot \frac{\pi \cdot \rho_W}{4 \cdot \rho_K}\right] \cdot \rho_{plain} \approx 0.9 \cdot \rho_{plain}; \qquad \kappa = D_K / D_A$$

In this formula, ρ_W is the density of the winding, ρ_K is the density of the core material. ρ_{plain} indicates the density of a solid string of the same outer diameter (used for comparison), $\overline{\rho}$ is the average density of the wound string. $\kappa = D_K/D_A$ = core- / outer-diameter. A more precise consideration requires minor corrections in case the core is not round but features a square or a hexagonal cross-section, and if the winding comprises sanded down round wire, or flat wire.

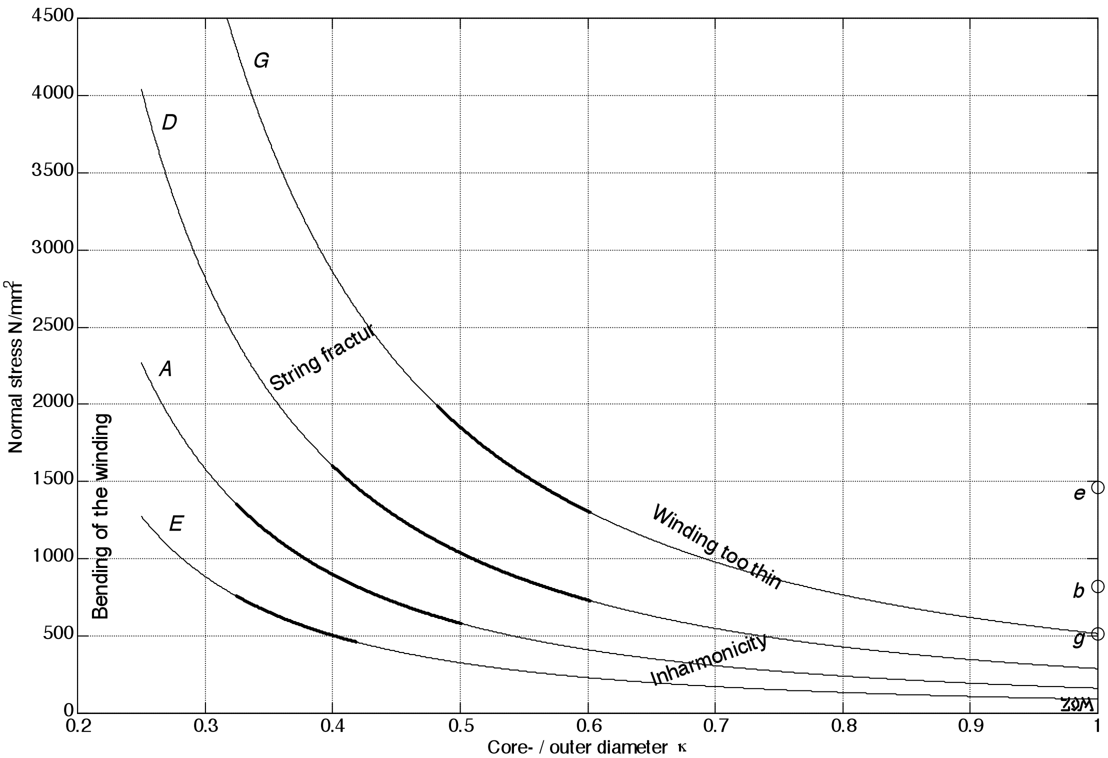

Fig. 1.6: Normal stress of the string dependent on κ. Customary values are shown in bold. The values for the (solid) e-, b- und g-strings are marked as a circle at the right border of the graph. M = 25.5" = 64.8 cm . The representations are valid for stiff (unyielding) string bearings; spring-loaded bearings (vibrato) yield increased normal tension.

 Translation by Tilmann Zwicker

For solid strings, the **normal tension** σ (tension force / cross-sectional) calculates as:

$$\sigma = \frac{4\Psi}{\pi D^2} = 4\rho \cdot (f_G \cdot M)^2$$ Normal tension (solid string)

Given equal fundamental frequency and length, the normal tension does not depend on the string diameter. If light strings seem to break more easily than heavy ones, this is due to the additionally acting plucking force – light strings offer little overhead here. For wound strings, σ calculates as:

$$\sigma = \frac{4\Psi}{\pi D_K^2} = \frac{4\overline{\rho} \cdot (f_G \cdot M)^2}{\kappa^2}$$ Normal tension (wound string)

An average density $\overline{\rho}$ reduced by 10% needs to be applied as density for wound strings. A particular influence is due to the ratio of the diameters κ. **Fig. 1.6** shows, for all 6 strings, the normal tensions; towards the top, the risk of breaking the string increases; towards the right, there is more inharmonicity (Fig. 1.7). Contrary to fracture of the string (which of course must be avoided), inharmonicity is not inherently a bad thing – it even may impart a special "liveliness" to the sound of the string (Chapter 8.2.5).

The **inharmonicity** that appears in particular for heavy strings in their higher partials is due to the flexural stiffness. According to [1], the frequency of the n-th partial calculates as:

$$\boxed{f_i[n] = n \cdot f_G \cdot \sqrt{1 + bn^2}} \qquad b = \left(\frac{\pi D}{8M^2 f_G}\right)^2 \cdot \frac{E}{\rho}$$ Spreading of partials

This formula (dating back to Lord Rayleigh) holds for solid strings with D as the string diameter. For wound strings we rearrange the math as follows:

$$b = \frac{\pi^2 B}{4M^4 f_G^2 m'} \qquad B = \frac{E\pi D_K^4}{64} \qquad m' = \frac{\pi D_A^2}{4}\overline{\rho}$$

Here, B is the flexural stiffness that depends on the core diameter D_K and on Young's modulus E, and m' is the **length-specific mass** depending on the outer diameter D_A. We obtain as the parameter of inharmonicity b:

$$b = \frac{\pi^2 E}{64M^4 f_G^2 \overline{\rho}} \cdot \kappa^4 D_A^2$$ Inharmonicity-parameter

In **Fig. 1.7**, several ranges are marked for b. These encompass, for wound strings, the range of customary outer diameters, and of customary values of κ (compare Fig. 1.6). For solid strings (lower-case letters), $\kappa = 1$ holds.

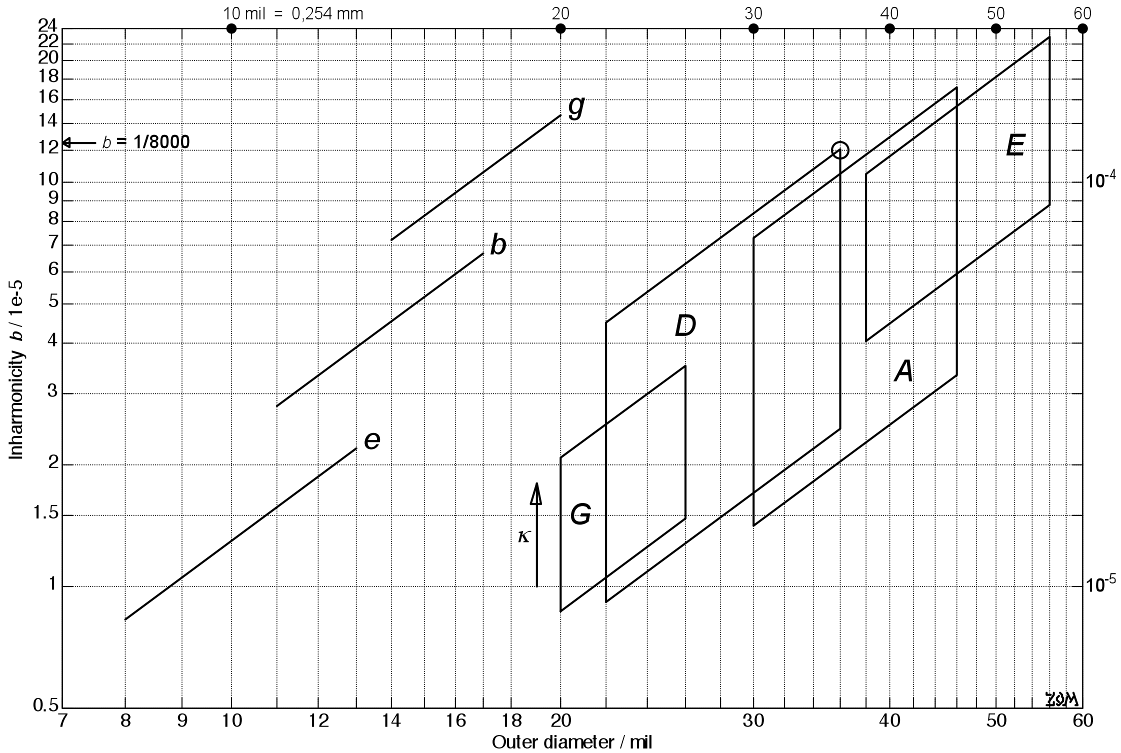

Fig. 1.7: Parameter *b* for the inharmonicity of partials of typical guitar strings; *E-A-D-G* = wound, *g-b-e* = plain. For a wound D-string with an outer diameter of 36 mil, we obtain: b = 12e-5 for $\kappa = 0.6$;
Core-/outer-diameter: κ_E = 0,33 – 0,42 κ_A = 0,33 – 0,50 κ_D = 0,40 – 0,60 κ_G = 0,48 – 0,60.
Scale = 65 cm. For a scale of 63 cm, all values for b need to be increased by 13%.

Material	Density ρ in 10^3 kg / m^3	Young's modulus E in 10^9 N / m^2
Steel	7,8 - 8,1	200 - 220
Nickel (Ni)	8,90	199
Copper (Cu)	8,92	120
Brass (Cu, Zn)	8,1 - 8,6	≈ 100
Bronze (Cu, Sn)	8,2 - 8,9	≈ 110
German silver (Cu, Zn, Ni)	≈ 8,6	≈ 130
Nylon (Polyamid)	≈ 1,2	≈ 3,5

Table: Material-data. Steel, Brass, Bronze and German silver are available in various compositions.