1.6.3 Partial and summation-levels

The real guitar string does not consist of a single concentrated mass and a single concentrated stiffness – rather, these quantities are continuously distributed along the length of the string. As a consequence of this spatial distribution, a multitude of Eigen-vibrations (natural vibrations) manifest themselves (Chapters 1.1. and 1.3), all of which decay with their individual frequency f_i, initial phase φ_i and damping ϑ_i. The actual overall vibration is a superposition (addition) of the individual vibrations that also appear in two planes each – again with different parameters. This already rather complex description is, however, still a simplification because we would have to consider non-linear behavior in addition, especially for strong plucking.

Typically, low-frequency partials show long sustain while high-frequency partials decay quickly – especially with old strings. The course of the levels of individual partials needs to be determined frequency-selectively, e.g. using a narrow band-pass filter with its center-frequency tuned to the frequency of the given partial. Choosing a filter bandwidth that is too wide will make the neighboring partial influence the measuring result; with too narrow a bandwidth, fast changes in level will not be captured correctly. From a systems-theory point-of-view, two filters are connected in series: the string and the band-pass. The output signal results from the filter input-signal (string vibration) convolved with the impulse response of the band-pass filter. The narrower the band of the filter, the slower its impulse response decays, and the less the course of the level of the partial is correctly captured.

This is an inherent problem existing irrespective of how the narrow-band filtering is achieved. A DFT (Direct Fourier Transform) can be interpreted as a filter-band: for this the DFT-window (e.g. Hanning) is moved along the time axis, and the now time-variant voltage of each discrete frequency point is interpreted as time-discrete output voltage of the filter (STFT = short-time Fourier Transform).

In the STFT, the time signal $\underline{u}(t)$ to be analyzed is first multiplied with a weighing window; this weighing function is different from zero only for a short time. The DFT is calculated across the signal weighted this way, resulting in a complex instantaneous value at the individual frequency *f*. Then, the window is shifted by one sample period, und again a DFT is calculated … and so on.

$$\underline{U}(t',\omega) = \int_{-\infty}^{\infty} \underline{u}(t) \cdot g(t'-t) \cdot e^{-j\omega(t'-t)} \cdot dt \qquad \text{STFT}$$

$$\underline{z}(t') = \int_{-\infty}^{\infty} \underline{x}(t) \cdot \underline{y}(t'-t) \cdot dt \qquad \text{Convolution}$$

Formally, the integration for the STFT happens across the infinitely lasting time *t*. De facto, however, this is done merely across the window-section that is shifted by *t*'; the *e*-function is due to the Fourier transform. The convolution integral has the same structure – its first factor is seen as time function to be filtered. Its second factor results – as impulse response – in a vibration of the circular frequency ω that is weighted with $g(t)$. This shows that the STFT works like a (digital) filter – including all associated system-typical selectivity-problems.

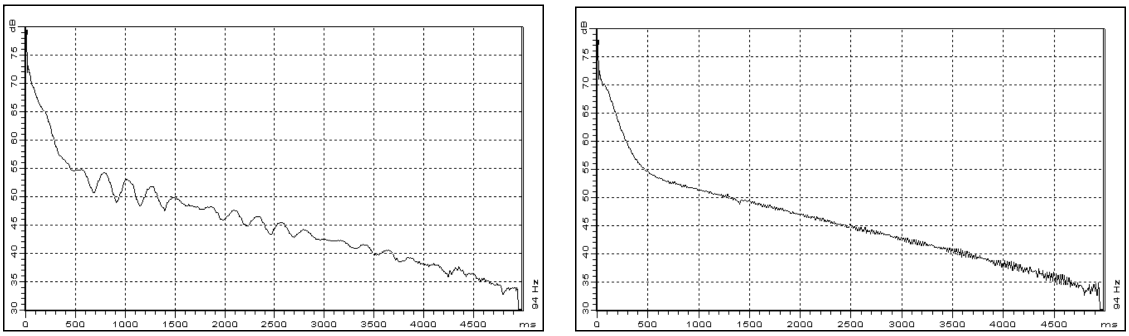

Fig. 1.50 Course of the level of the fundamental (*G#*): 40-dB-Kaiser-Bessel-window (left), 60-dB-Kaiser-Bessel-window (right).

While merely *one single* (theoretical) long-term spectrum exists, there are any numbers of short-term spectra that in some cases differ substantially. In **Fig. 1.50**, the same decay process is investigated using two different DFT-windows. The beats visible in the left-hand section of the figure are leakage effects of the DFT-window, as they would appear similarly also with the Hamming window and the 40-dB-Gauss-window♣. Although this analysis could not be actually termed 'wrong', it is more purposeful to use a window with stronger side-lobe attenuation (e.g. 60 dB; right hand section of the figure).

A 512-point DFT at 48 kHz sampling rate will have a frequency-line distance of 94 Hz. This frequency grid is too coarse to obtain a good resolution of an E2-spectrum (fundamental frequency 82,4 Hz). Using an 8k-DFT reduces the line distance to 5.9 Hz; however, at the same time the block length rises to 171 ms. Basis of the selective level measurement is now an averaging time of 171 ms (due to the filter, with a weighting corresponding to *g(t)*), and this smoothes out all quick changes in level. A compromise needs to be found between these two extremes.

The overall level can be calculated via summation of the temporal course of the partial-levels. However, this does not work by simply adding the dB-values; rather, it is necessary to add the individual *power* data (addition of incoherent sources). Since power is always positive, the overall level can never be smaller than the individual levels – if the latter are all measured using the same type of averaging, that is! Given different averaging, the value of the sum can indeed have a short-term value smaller than the individual values.

In summary, the following picture emerges: the *power* of the partials decays (in approximation) exponentially while the *level* of the partials decreases linearly. If the fretboard-normal and the fretboard-parallel components of the vibration show different damping, a kink can appear in the course of the level. If moreover the frequencies are also different, beats can result. Averaging techniques that are unavoidable when taking measurements will smoothen-out the course of the level. Directly after the plucking attack, the overall level is influenced strongly by the level of the high-frequency partials but these decay rather rapidly. After a short time, a few low-frequency partials dominate: they decay slowly. Therefore, the overall level often decays non-linearly – quickly at first, and then more and more slowly. Because many partials are involved, there is no sharp kink but a rounded off shape of the decay.

♣ More extensively elaborated in: M. Zollner, Signalverarbeitung, Hochschule Regensburg, 2010.

