1.6 The decay process

After being plucked, the sting vibrates in a free, damped oscillation process. "Free" implies that no further energy is injected; "damped" indicates that vibration energy is converted into sound and caloric energy (radiation, dissipation). Any further string damping (e.g. via the fingertips of the palm of the hand) shall not be considered here at this time.

1.6.1 One single degree of freedom (plane polarization)

The simplest oscillation system consists of a mass, a spring, and a damper. The mass force is proportional to the acceleration (inertia, NEWTON), the spring force is proportional to the displacement (stiffness, HOOKE), and the damper force is proportional to the (particle) velocity (friction, STOKES). The time derivative of the displacement yields the velocity; the time derivative of the velocity yields the acceleration [3].

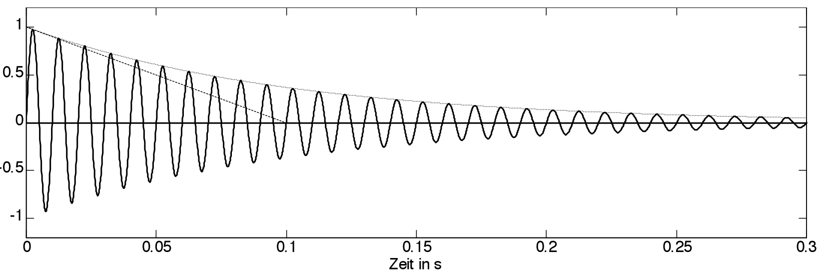
After the excitation a "periodic" oscillation of the frequency f_d results. Instants of equal phase (e.g. maxima, zeroes, and minima) occur at equal distances in time – which led to the us of the term **period** $T = 1/f_d$. However, signal theory does not actually see this decay process as a periodic signal: due to the exponential decay, the individual periods fail to be identical. Mechanics, on the other had, do use the term periodic vibration here because the duration of the periods in time-invariant (... non est disputandum).

The resulting envelope has three parameters: the frequency f_d, the initial phase φ, and the time constant of the envelope ϑ. In this general form, the equation for the oscillation is:

$$\xi(t) = \hat{\xi} \cdot e^{-t/\vartheta} \cdot \sin(2\pi f_d t + \varphi), \qquad t \geq 0 \qquad \text{Oscillation equation}$$

For $t = 0$, the e-function yields 1; with increasing time, it decreases towards 0. The phase shift φ may be taken to be zero for the first considerations. The time constant ϑ determines how fast the oscillation decays: the smaller ϑ is, the faster the decay. Instead of ϑ, literature offers a multitude of other parameters, as well – they can easily be converted into each other. The letter τ is frequently used for the time-constant; in the present context we will rely on this letter only when we get to the calculation of levels. What needs to be avoided in particular is confusion between the degree of damping and the decay-coefficient, since the latter is sometimes also designated with ϑ!

It may be the displacement, the (particle) velocity, or the acceleration that represents the physical oscillation. A sensor converts these quantities into a voltage $u(t)$ that subsequently is analyzed.



$f_d = 100\,\text{Hz}$

$\vartheta = 0{,}1\,\text{s}$

Fig. 1.41: Damped oscillation of 100 Hz; exponential decay; time-constant $\vartheta = 0{,}1\,\text{s}$.

Given mass m, spring-stiffness s, and friction W, we calculate frequency and time-constant:

$$f_d = \frac{1}{2\pi}\sqrt{\frac{s}{m} - \frac{1}{\vartheta^2}} \qquad \vartheta = \frac{2m}{W}$$

Parameters of the oscillation

If the friction W is set to zero, the un-damped system results. It has an infinite time-constant: the e-function now has the constant value 1, and the vibration does not decay anymore. A weakly damped vibration with a frequency f_d = 100 is shown in **Fig. 1.41**. The shape of the e-function is indicated as a dashed line with its tangent crossing zero at ϑ. At the point in time of $t = \vartheta$, the envelope has decreased from 1 to $1/e \approx 0{,}37$.

In instrumentation, the decay process is often depicted as **level-curve**. Level is a logarithmic measure that may be determined in various ways. It always constitutes a time-average over a weighted measurement interval; the averaging is done using the squared signal quantity. We often see an exponential averaging where the weighting is of exponential form, and is done such that the signal components lying further back in the past contribute less prominently to the measurement. The **averaging time constant $\boldsymbol{\tau}$** is specified as parameter of the exponential averaging; the value $\tau = 125\,\text{ms}$ is used frequently, with the corresponding standardized way of averaging being labeled **FAST**. The decay constant ϑ of the dampened oscillation must not be confused with the averaging time constant τ of the level measurement.

The level measurement comprises three consecutive operations: squaring, averaging, and logarithmizing. Squaring and logarithmizing are non-linear operations; the order of sequence must therefore not be interchanged. It is only the averaging that is a linear filter operation: a 1^{st}-order low-pass in the case of the level measurement. In the time domain, the averaging is described by a convolution [6]: the result of the averaging corresponds to the convolution of squared signal and impulse response $h(t)$ of the averager. For damped oscillations we get:

$$m(t) = h(t) * u^2(t) = \int_0^t \left(\frac{1}{\tau} e^{\frac{\psi - t}{\tau}}\right) \cdot \left(\hat{u} \cdot e^{-\psi/\vartheta} \cdot \sin(\omega_d \psi)\right)^2 \cdot d\psi$$

(for causal signals)

$$h(t) = \frac{1}{\tau} e^{-t/\tau} \qquad u(t) = \hat{u} \cdot e^{-t/\vartheta} \cdot \sin(\omega_d t) \qquad \omega_d = 2\pi f_d$$

Here, $h(t)$ is the impulse response of the averager, $u(t)$ is the damped oscillation, the star symbol stands for the convolution. The average $m(t)$ is calculated for the point in time t with the time-variable ψ integrated from 0 to t. Therefore, the **average value** $m(t)$ does in this case not indicate the average over the whole decaying oscillation but the average from the excitation to the (variable) point in time t. The averaging time constant τ is large compared to the oscillation period T; the contribution of the sine function can thus be disregarded in good approximation. Using this, the time-variant average is:

$$m(t) = \frac{\tilde{u}^2}{1 - 2\tau/\vartheta} \cdot \left(e^{\frac{-2t}{\vartheta}} - e^{\frac{-t}{\tau}}\right) \qquad \tilde{u} = \hat{u}/\sqrt{2} \qquad \text{for } 2\tau \neq \vartheta$$

 Translation by Tilmann Zwicker

When calculating levels we need to consider that we are working with a squared signal, which is why we need to opt for the formula for power levels. The reference value needs to be chosen such that the correct absolute value results for the steady case ($\vartheta \to \infty$). Using, on the other hand, $u_0 = \tilde{u}$, we get the relative level that decays starting from 0 dB.

$$L(t) = 10\lg\left(m(t)/u_0^2\right)\mathrm{dB} \qquad \mathrm{dB} = \text{decibel} \qquad u_0 = \text{reference value}$$

Fig. 1.42 shows the course of the level of a damped oscillation determined via exponential averaging. The time-constant of the damping is $\vartheta = 4\,\mathrm{s}$. Having an understanding of the equation of the oscillation, we could also give the exact course of the level. To do that, it is merely necessary to logarithmize the *e*-function (shown as a dashed line). The level determined via measurements deviates significantly from this calculation. In the figure, we see two graphs with the averaging time-constants 0,125 s and 0,5 s, as well as the theoretical behavior (dashed).

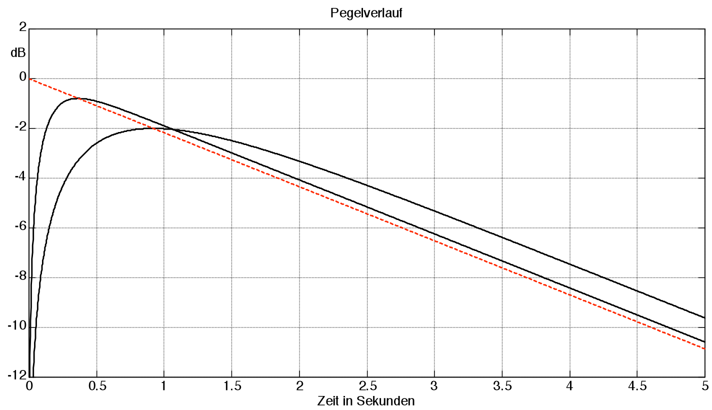


Fig. 1.42: Level of an exponentially damped oscillation. Damping time-constant ϑ = 4 s, averaging time constanr, τ = 125 ms and 500 ms. For 500 ms, the asymptote is too high by 1,2 dB, and for 125 ms, it is too high by 0,3 dB.
"Pegelverlauf" = course of the level; "Zeit in Sekunden" = time in seconds

After a short attack phase (mainly determined by τ), the level drops off with approximately the time constant ϑ. As is evident, the measurement curves run in parallel to the exact values after a short time, but remain too high. Therefore the slope – and thus the system damping – can be determined with good accuracy; for measurements of absolute values, however, considerable errors may arise. Using $L(t)$, the level difference is calculated as:

$$\Delta L = 10\lg\frac{1}{1 - 2\tau/\vartheta}\mathrm{dB} \qquad\qquad \vartheta = 10\tau \;\Big\}\; \Delta L \approx 1\mathrm{dB}$$

The shorter the averaging time-constant gets relative to the damping time-constant, the more exact the tracing of levels via measurements becomes. Still, the averaging time-constant must not be chosen too short, either, because then the (squared) oscillation may not be fully averaged anymore, and ripples in the level-graphs would result.

Moreover, Fig. 1.42 indicates that the measured **level maximum** is lower than expected. The position of the maximum is determined via differentiating and zeroing:

$$t_{\max} = \frac{\ln(2\tau/\vartheta)}{2/\tau - 1/\tau} \qquad\qquad m_{\max} = \tilde{u}^2 \cdot \left(\frac{\vartheta}{2\tau}\right)^{\frac{1}{1-\vartheta/2\tau}}$$

The larger the averaging time-constant is chosen, the lower the maximum.

From a signal-theory point-of-view, a damped oscillation belongs with **energy signals**. The signal energy is derived as integral over the squared signal value; it differs from the physical energy:

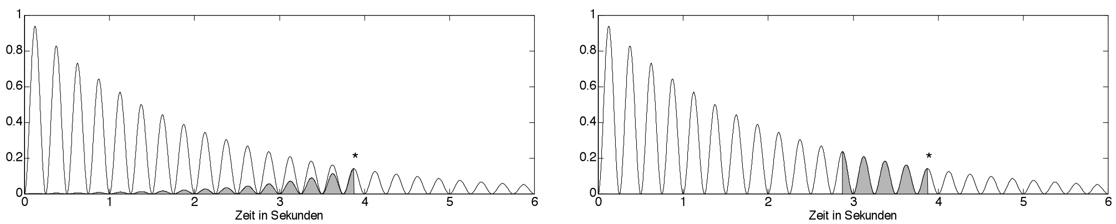
$$E_{Signal} = \int_{-\infty}^{\infty} u^2(t)\,dt \qquad E_{phys} = \int_{-\infty}^{\infty} F(t)v(t)dt = \int_{-\infty}^{\infty} v^2(t)\cdot Z\cdot dt \qquad Z = \text{impedance}$$

The signal energy of the damped oscillation may be calculated from the equation of the oscillation using integration:

$$E = \int_0^{\infty} \left(\hat{u}\cdot e^{-t/\vartheta}\cdot \sin(2?f_d t)\right)^2 dt \quad \xrightarrow{\vartheta\cdot f_d\, >>1} \quad E = \hat{u}^2\cdot\vartheta/4$$

The average value across $m(t)$ yields the same signal energy irrespective of τ. If the energy is derived via $m_{\max}$, however, a correction is required due to $m_{\max} < \tilde{u}^2$.

Besides the exponential averaging there are also other ways to average: block-averaging is done with constant weighting across a fixed time interval, Hanning-averaging uses a sine-shaped weighting. Block averaging is also called **linear averaging**, a rather confusing term that is common in the area of spectral analysis, though. While the exponential averaging is always run from the start of the signal to the point in time of the measurement (marked with a star on Fig. **1.43**), linear averaging is done from the start of the signal over an interval of fixed duration (1 s in the figure). In exponential averaging, only the end of the interval is shifted, in linear averaging, however, this is done to both start and end. **The Hanning-averaging** uses a fixed duration of the averaging (2 s in the figure), as well, but weighs the signal with a $\sin^2$. Hanning-averaging is often deployed in DFT-analyzers – as are many other DFT-windows (Blackman Kaiser, Bessel Gauß, Flat-Top, etc.).



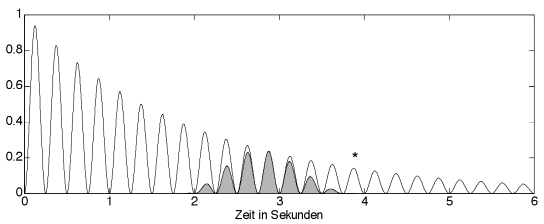


Fig. 1.43: Different ways of averaging:
exponential averaging (upper left),
linear averaging (upper right),
Hanning-averaging (lower left).
"Zeit in Sekunden" = time in seconds"

All ways of averaging are calibrated such that for steady signals (constant level), equal results are obtained. With levels varying over time, differences occur. In frequency-selective analyses (DFT, 1/3rd-octave, etc.), also further system-immanent errors contribute: a filter will react more sluggishly to the input signal as the filter band becomes narrower. In broadband level-measurements (e.g. 10 Hz – 20 kHz), no significant errors will occur, but in selective measurements of partials (e.g. 2500 Hz – 2519 Hz), they might creep in, depending on circumstances.

1.6.2 Spatial string vibrations

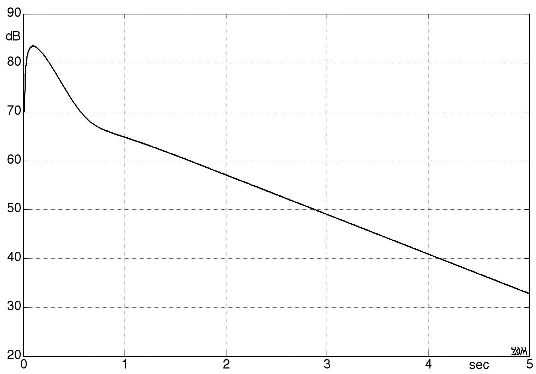
After a guitar string is plucked, spatial vibrations will propagate on it. The transversal waves introduced in Chapter 1.1 are of particular significance. Given that the axis along the string is taken as z-coordinate, transversal waves can propagate both in the xz-plane and the yz-plane; superpositions are possible, as well. For electric guitars, the vibration plane perpendicular to the guitar top is especially important, while for acoustic guitars the vibration parallel to the guitar top also has effects.

The wave equation includes a dependency both on place and time. However, investigations into the vibrations of guitar strings are mostly based on a fixed location (the place of e.g. pickup, or bridge) so that merely the time remains as variable. As a simplification, the string vibration occurring at a given location tends to be seen as superposition of many exponentially decaying partials (Chapter 1.6.3). In this scenario we need to consider, though, that for each partial, vibrations may appear in two planes. Sometimes one of the two vibrations has next to no effect and may be disregarded, but in some cases both need to be taken into consideration.

The following approaches first start from the assumption that plucking the string will result in two *same-frequency* vibrations orthogonal in space. The time constants of the damping ϑ are still different for the two vibrations, the effect on the output is different, and they may be phase-shifted relative to each other. At the output, both are superimposed:

$$u(t) = \hat{u} \cdot \left(e^{-t/\vartheta_1} \cdot \sin(\omega t) + d \cdot e^{-t/\vartheta_2} \cdot \sin(\omega t + \varphi) \right) \qquad d = \text{top-parallel part}$$

Particularly in acoustic guitars, the top-normal vibration is tightly coupled to the resulting sound field, and therefore vibration energy is relatively quickly withdrawn, and the damping time-constant is short. The top-parallel vibration does not lead to as efficient a radiation (d is smaller); it thus has a longer time-constant. In the level-analysis, the decay shows up with a characteristic kink (**Fig. 1.44**).



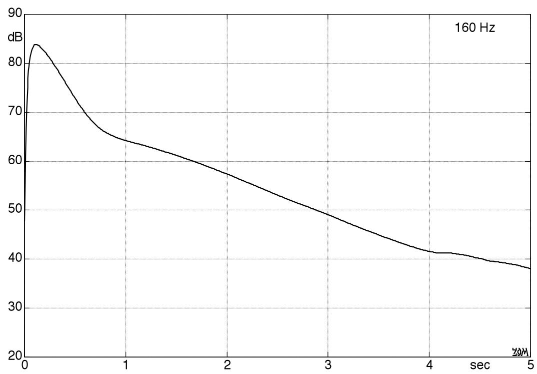


Fig. 1.44: Open E_2-string, FAST-level of the 2nd partial; *left*: calculation; *right*: measurement (Martin D45V).

To confirm our hypotheses about the vibrations, two experiments were carried out. In order to adjust the neck, the OVATION Adamas SMT allows for the removal of a cover plate (of ∅13cm) in the guitar body. This detunes the Helmholtz resonance and thus changes the low-frequency coupling to the sound field. With the cover taken off, the low frequencies receive weaker radiation; the time constant should therefore be longer.

This figure is reserved for the print version of this book

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Fig. 1.45: Left: Ovation Adamas SMT, level of fundamental (F#$_2$), with closed ("Deckel geschlossen") and removed ("Deckel geöffnet") cover plate.
Right: Ovation Viper EA-68, level of fundamental (F#$_2$), with ("mit Magnetfeld") and without influence of the magnetic field.

Fig. 1.45 (left) depicts the decay curves for the fundamental of the tone F# fretted at the 2nd fret on the low E-string. The measurements confirm the assumption. In a second experiment, a **permanent magnet** was brought close to the low E-string on an OVATION Viper. Due to the attraction force the stiffness of the string is reduced in *one* plane of vibration – the vibration frequency is thus reduced in that plane. This leads to a **beating** of the orthogonal fundamentals now slightly detuned relative to each other (**Fig. 1.45**, right).

However, even without any magnetic field, the top-normal vibration of a particular partial does not necessarily occur at the exact same frequency as that of the top-parallel vibration of the same partial. This is due to the reflection factors of the string clamping (nut, bridge) – the former are dependent on the vibration direction. The spring-stiffnesses at the edges may be different for the two directions of the vibration, resulting in slight differences in the vibration frequencies. The decay process will then include beatings that render the sound more "lively". **Fig. 1.46** shows results of calculations and, for comparison, sound pressure levels measured with an acoustic guitar (MARTIN D45V, anechoic room, microphone at 1 m distance ahead of the guitar). Various patterns emerge:

The level differences between the two sub-vibrations determine the *strength* of the interference. At a difference of 20 dB, the amplitude fluctuates merely by 10%, while at 6 dB difference the fluctuations grow to 50%. Differences in the damping determine for which *period* the beating persists. If both sub-vibrations decay with the same damping, the level-difference does not change, and neither does the beat-intensity. Conversely, if the decay is different, the beats are strongest at the instant when both levels are equal. The frequency difference determines the *periodicity in the envelope*: the larger this difference, the faster the fluctuations. Moreover, the *phase* of the sub-vibrations is of significance – in particular if different damping occurs i.e. if the beats are limited to a short time-interval. The interference-caused cancellation will only present itself if both sub-vibrations are in opposite phase during said time-interval.

Another degree of freedom comes into play if we allow for **non-linearities**. For example, the friction may depend on a higher order of particle velocity, or the spring-stiffness may depend on the displacement. This may cause, for example, that the level of a mono-frequent vibration does not decay linearly with time but shows a curvature. Addressing such aspects requires considerable effort – no corresponding investigations were carried out in the present framework.

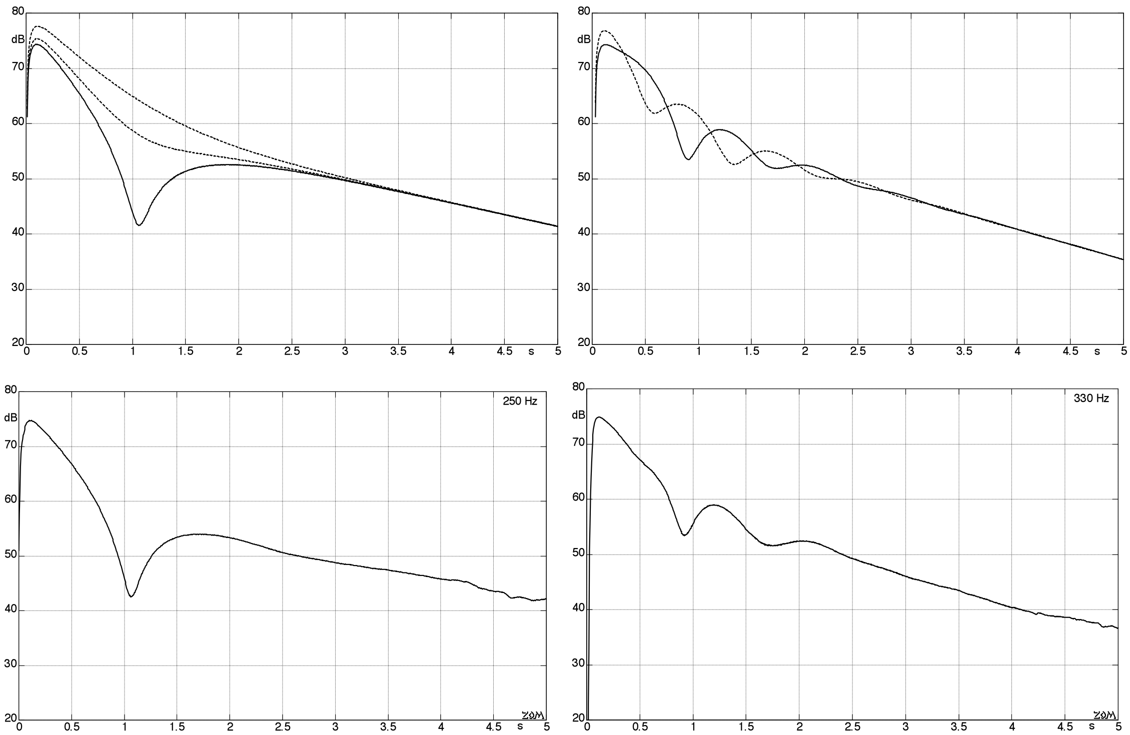


Fig. 1.46 Top: Decay processes given phase-differences. Left: both vibrations with the same frequency; right: beats due to a frequency differencde of 1.2 Hz. The damping cannot be determined precisely anymore from the initial slope of the curve. **Bottom**: measurements with a MARTIN D45V

An interesting set of curves emerges if the excitation energy remains constant while the string damping varies. First, however, we need to define more precisely the term "damping": any real string executes a damped vibration. In this case, **damping** means that vibration energy is continuously withdrawn from the string, with displacement amplitude (potential energy) and velocity amplitude (kinetic energy) decreasing over the course of time. Springs and masses store energy while resistances "remove" energy. Sure, energy cannot actually be removed – rather its mechanic incarnations are converted into caloric energy (heat); but in any case the "removed" energy is not available anymore to the vibration of the string.

In the **acoustic guitar**, we need to distinguish between the 'good' and the 'bad' losses. If all of the energy in the string is converted to sound-energy with an efficiency of 100%, we do have damping (a loss), but the objective of generating sound has been achieved with the utmost efficiency. If, conversely, 90% of the energy in the strings is converted directly into heat due to inner friction, and only 10% are radiated, we have an undesirable loss. To illustrate this with an EXAMPLE: a watering can supplies water to a flowerpot. If the water flows through a small cross-section, it will take a long time until the can is empty. With a larger cross-section, the process will be quicker – but it's always the whole of the water that arrived in the flowerpot. This situation changes if there is a hole in the bottom of the can – an additional degree of freedom is now present that influences the efficiency ◊. Applying this to the string: via tight coupling between string and sound field, the energy flows from the string quickly – the string is damped strongly but all energy reaches the sound field (100% efficiency). The efficiency drops only as friction-resistance is included in the guitar.

In **electric guitars**, the objective is entirely different. They do not need to radiate sound energy – that's taken care of by the loudspeaker. Due to the lack of radiation loss, the string damping is lower, the decay is longer – the guitar has longer/better **sustain**.

Several quantities are disposable in order to describe damping: one is the time constant of the damping (or **time constant of the envelope**) ϑ of the individual partials. During the length of a time constant, the level of the respective partial drops by 8,686 dB. A vibration with a level dropping off by 60 db within 10 s has a time constant of 1,45 s. The duration of time that it takes a level to drop by 60 dB is – in room acoustics – also called the **reverberation time T_N**. The latter is suitable to describe a damping, as well: the formula $T_N = 6{,}91 \cdot \vartheta$ holds. **Fig. 1.47** shows the course of the levels of the fundamentals (*G#*) measured via the piezo pickup. During the initial second, the time constants differ by a factor of 18.

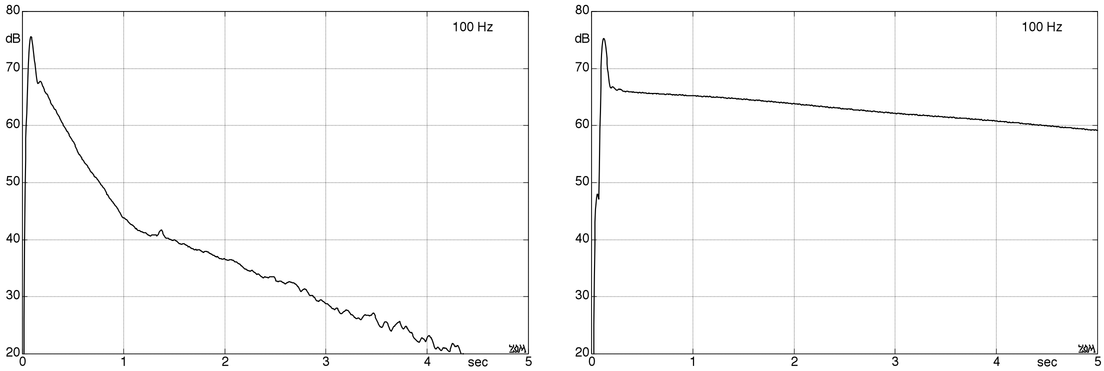


Fig. 1.47: Measurements with Ovation guitars: SMT (acoustic guitar, left); Viper (electric guitar, right).

The following considerations are based on the law of conservation of energy. In the plucking process, the string is given a certain potential energy that is in part dissipated and in part radiated. As an EXAMPLE, a string is to be plucked with 5 mWs; it then decays in different ways. Which sound pressure level is generated at a distance of 1 m if we assume – to begin with – that 100% of the vibration energy is radiated as **sound wave**?

For any exact calculation we would have to know about the beaming – as a simplification let us assume an omni-directional characteristic here. In fact, this assumption is a good approximation for the (quite level-strong) 2nd partial of the E-string [1]. The energy E of the spherical wave [3] is calculated as:

$$E = \frac{4\pi R^2}{Z_0} \int_0^\infty p^2(t)dt = \frac{4\pi R^2}{Z_0} \cdot \frac{\hat{p}^2}{4} \cdot \vartheta \qquad \text{with } Z_0 = 414 \text{ Ns/m}^3$$

Herein, $p(t)$ is the sound pressure at the distance R = 1m; the integral over the damped vibration was already calculated at the end of Chapter 1.6.1. The equation can be solved for the sound pressure amplitude:

$$\hat{p} = \sqrt{\frac{Z_0}{\pi R^2} \cdot \frac{E}{\vartheta}} \quad \text{in the example} \quad \hat{p} = 0{,}57\,\text{Pa} \quad \text{for } \eta = 100\% \text{ und } \vartheta = 2\,\text{s}.$$

From the (now known) sound pressure, the level can be calculated e.g. for exponential FAST-averaging (**Fig. 1.48, left section**, different ϑ). ◊

$$L(t) = 10\lg\left(\frac{Z_0 E/p_0}{2\pi R^2(\vartheta - 2\tau)} \cdot \left(e^{-2t/\vartheta} - e^{-t/\tau}\right)\right) \qquad \vartheta \neq 2\tau$$

The time constant ϑ of the damping influences both the maximum value and the speed of decay. The luthier can increase the peak sound pressure level via high mechano-acoustical coupling – the loudness will then decrease more quickly, though. Lower coupling will enable him (or her) to achieve longer sustain, but then the guitars is not as loud. The plucking energy is present only once, after all. Now, if we allow the string to vibrate in two planes, the seemingly impossible is in reach: a loud guitar with long sustain. The top-normal vibration generates a loud attack. The quick decay of this loud attack is "drowned out" after a short time by the more slowly decaying top-parallel vibration.

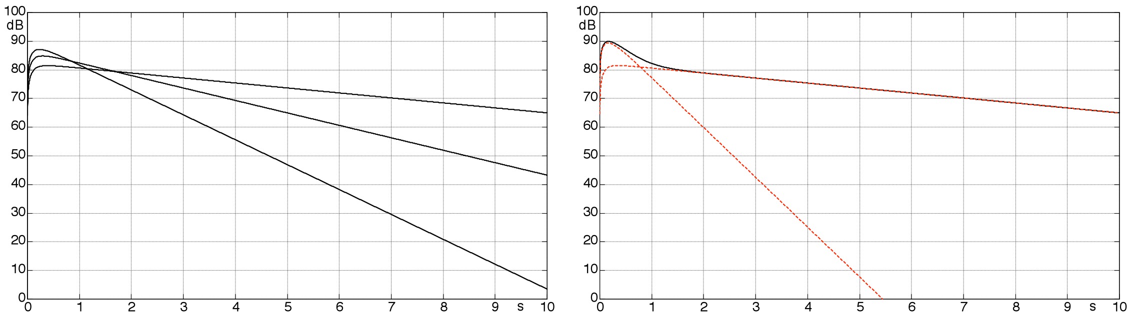


Fig. 1.48: Left: FAST SPL for different degrees of coupling between string and sound field (η = 100%). Right: FAST-SPL for two superimposed orthogonal vibrations (η = 100%). Equal energy.

Fig. 1.48 (right hand section) shows an example with both vibrations being excited with 5 mWs. The quicker decay happens at a time constant of the damping of 0,5 s, the longer decay has a time constant of 5 s. The dashed lines indicated the levels of the individual vibrations. An efficiency of 100% is assumed again for both vibrations.

Of course, in practice an **efficiency** of 100% is not achievable; part of the vibration is converted into caloric energy already within the string, and in the guitar body, as well. Reducing the efficiency to 50% will also reduce the time constant of the decay by half (this may be deduced via the transmission-line equation). The course of the level will then be determined by two parameters: the mechano-acoustical matching, and the dissipation in the guitar (**Abb. 1.49**).

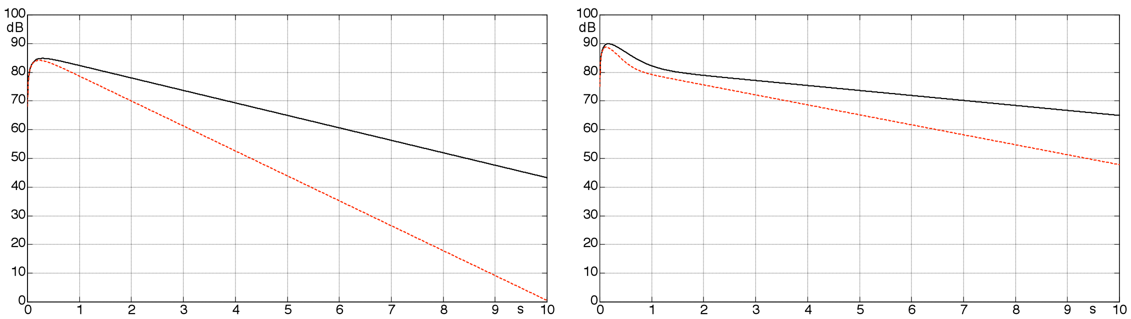


Fig. 1.49: Calculated SPL for an excitation energy of 5 mWs (left) and 2.5 mWs (right). The solid line indicates an efficiency of 100 %, the dashed one an efficiency of 50%.

1.6.3 Partial and summation-levels

The real guitar string does not consist of a single concentrated mass and a single concentrated stiffness – rather, these quantities are continuously distributed along the length of the string. As a consequence of this spatial distribution, a multitude of Eigen-vibrations (natural vibrations) manifest themselves (Chapters 1.1. and 1.3), all of which decay with their individual frequency f_i, initial phase φ_i and damping ϑ_i. The actual overall vibration is a superposition (addition) of the individual vibrations that also appear in two planes each – again with different parameters. This already rather complex description is, however, still a simplification because we would have to consider non-linear behavior in addition, especially for strong plucking.

Typically, low-frequency partials show long sustain while high-frequency partials decay quickly – especially with old strings. The course of the levels of individual partials needs to be determined frequency-selectively, e.g. using a narrow band-pass filter with its center-frequency tuned to the frequency of the given partial. Choosing a filter bandwidth that is too wide will make the neighboring partial influence the measuring result; with too narrow a bandwidth, fast changes in level will not be captured correctly. From a systems-theory point-of-view, two filters are connected in series: the string and the band-pass. The output signal results from the filter input-signal (string vibration) convolved with the impulse response of the band-pass filter. The narrower the band of the filter, the slower its impulse response decays, and the less the course of the level of the partial is correctly captured.

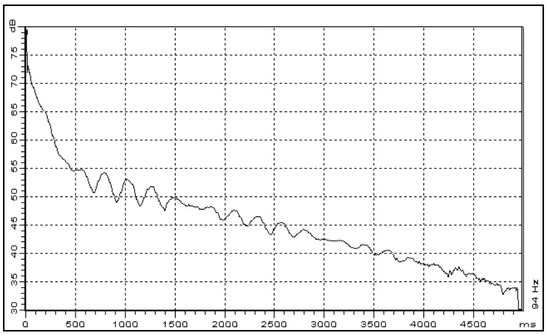
This is an inherent problem existing irrespective of how the narrow-band filtering is achieved. A DFT (Direct Fourier Transform) can be interpreted as a filter-band: for this the DFT-window (e.g. Hanning) is moved along the time axis, and the now time-variant voltage of each discrete frequency point is interpreted as time-discrete output voltage of the filter (STFT = short-time Fourier Transform).

In the STFT, the time signal $\underline{u}(t)$ to be analyzed is first multiplied with a weighing window; this weighing function is different from zero only for a short time. The DFT is calculated across the signal weighted this way, resulting in a complex instantaneous value at the individual frequency f. Then, the window is shifted by one sample period, und again a DFT is calculated … and so on.

$$\underline{U}(t',\omega) = \int_{-\infty}^{\infty} \underline{u}(t) \cdot g(t'-t) \cdot e^{-j\omega(t'-t)} \cdot dt \qquad \text{STFT}$$

$$\underline{z}(t') = \int_{-\infty}^{\infty} \underline{x}(t) \cdot \underline{y}(t'-t) \cdot dt \qquad \text{Convolution}$$

Formally, the integration for the STFT happens across the infinitely lasting time t. De facto, however, this is done merely across the window-section that is shifted by t'; the e-function is due to the Fourier transform. The convolution integral has the same structure – its first factor is seen as time function to be filtered. Its second factor results – as impulse response – in a vibration of the circular frequency ω that is weighted with $g(t)$. This shows that the STFT works like a (digital) filter – including all associated system-typical selectivity-problems.



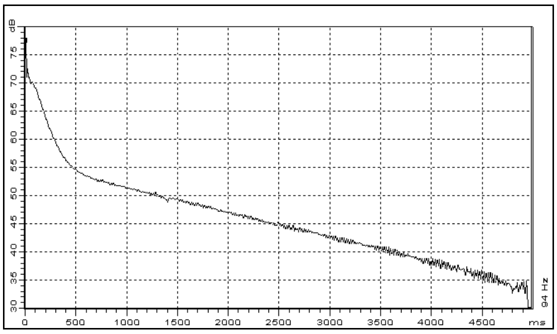


Fig. 1.50 Course of the level of the fundamental (*G#*): 40-dB-Kaiser-Bessel-window (left), 60-dB-Kaiser-Bessel-window (right).

While merely *one single* (theoretical) long-term spectrum exists, there are any numbers of short-term spectra that in some cases differ substantially. In **Fig. 1.50**, the same decay process is investigated using two different DFT-windows. The beats visible in the left-hand section of the figure are leakage effects of the DFT-window, as they would appear similarly also with the Hamming window and the 40-dB-Gauss-window♣. Although this analysis could not be actually termed 'wrong', it is more purposeful to use a window with stronger side-lobe attenuation (e.g. 60 dB; right hand section of the figure).

A 512-point DFT at 48 kHz sampling rate will have a frequency-line distance of 94 Hz. This frequency grid is too coarse to obtain a good resolution of an E2-spectrum (fundamental frequency 82,4 Hz). Using an 8k-DFT reduces the line distance to 5.9 Hz; however, at the same time the block length rises to 171 ms. Basis of the selective level measurement is now an averaging time of 171 ms (due to the filter, with a weighting corresponding to *g(t)*), and this smoothes out all quick changes in level. A compromise needs to be found between these two extremes.

The overall level can be calculated via summation of the temporal course of the partial-levels. However, this does not work by simply adding the dB-values; rather, it is necessary to add the individual *power* data (addition of incoherent sources). Since power is always positive, the overall level can never be smaller than the individual levels – if the latter are all measured using the same type of averaging, that is! Given different averaging, the value of the sum can indeed have a short-term value smaller than the individual values.

In summary, the following picture emerges: the *power* of the partials decays (in approximation) exponentially while the *level* of the partials decreases linearly. If the fretboard-normal and the fretboard-parallel components of the vibration show different damping, a kink can appear in the course of the level. If moreover the frequencies are also different, beats can result. Averaging techniques that are unavoidable when taking measurements will smoothen-out the course of the level. Directly after the plucking attack, the overall level is influenced strongly by the level of the high-frequency partials but these decay rather rapidly. After a short time, a few low-frequency partials dominate: they decay slowly. Therefore, the overall level often decays non-linearly – quickly at first, and then more and more slowly. Because many partials are involved, there is no sharp kink but a rounded off shape of the decay.

♣ More extensively elaborated in: M. Zollner, Signalverarbeitung, Hochschule Regensburg, 2010.

1.6.4 Old strings

For wound strings, the energy share converted into heat depends strongly on the age of the strings. Dirt and remains of skin are deposited in the grooves of the winding; this causes additional damping. Corrosion may also contribute. The mass introduced into the winding has the effect of a detuning; however, the strongest impact is perceivable in the damping of high frequency partials: an old string sounds dull. With electrically amplified guitars it does not help to turn up the treble control, because the decay constant cannot be extended that way.

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Fig. 1.51: The decay of an open E_2-string: left for a low-frequency partial, right for a high-frequency partial. "Alte Saite" = old string; "neue Saite" = new string.

In **Fig. 1.51** we see the course of the partial-levels of a decaying E_2-string. For the 2^{nd} partial (164,8 Hz), the differences between old and new string are within the limit of reproducibility: the vibrations decay with practically the same speed. This is very different at high frequencies: the decay duration for the old string is reduced to $1/7^{th}$. The time constant for the decay of the old string is merely 0,1 s; under no circumstance must any measurement of the decay therefore be taken with the FAST setting.

For the E_4-string, no ageing could be found: neither with the fundamental, nor for the higher harmonics. The string had been wiped with a cloth before the measurement, and apparently any residue lets itself readily enough be removed from the solid strings. In contrast, simple wiping does bring only very mild relief for the wound strings. Better results are said to be obtained by ultrasonic baths, or boiling the strings in suitable solvents; we did not carry out any analysis to that end.

Besides corrosion and residue, a further ageing process is to be considered: over time, the frets grind small **transverse grooves** into the strings – action and homogeneity consequently change. Mass and stiffness are not distributed uniformly along the string anymore but depend on the location. For the model of the string, an inhomogeneous transmission line with location-dependent wave-impedance results. Each groove makes for a small mismatch and thus triggers minor reflections. This effect was not analyzed in the scope of this present work.

In conclusion, Chapters 1.5.3 , 7.7.6, and 7.12.2 should be mentioned: for old strings, it is not only the decay process that is different but also the excitation. New strings sound more brilliant because every **bounce** off a fret generates a broadband impulse. In old strings, the deposits act as treble-attenuating buffer.

 Translation by Tilmann Zwicker