2. The string as a transmission line

Within the terminology of systems theory, a special transmission channel that transmits signals from the source to the receiver constitutes a **transmission line**. In the framework of the electric guitar, our thinking in terms of a transmission line will in the first place probably be target the guitar cable. However, while the latter does transmit electrical signals from the guitar to the amplifier in the sense as given above, we do not need the general line theory in order to describe its function. This is because for short lines, a simplification to concentrated line elements is adequate. The guitar cable indeed is a short line – short relative to the *electrical* wavelength that is in excess of 30 km. Transmission line theory is supposed to describe predominantly long lines with dimensions in the order of the wavelength or a length longer than that. In this sense, the **guitar string** does represent a long *mechanical* transmission line. The source of the propagating mechanical wave is the place where the string is plucked. Receiver of the signal transmitted via the string is the bridge that decouples part of the incoming signal energy and feeds it to the guitar body. The remaining part of the signal energy is fed back to the string as reflection. The nut (or the "active" fret) reflects, as well, leading to the manifestation of a **standing wave** on the string.

String vibrations are the basis for all musical signals generated in the pickup; the following section is dedicated to these vibrations. A pickup may also generate interference, but this will be investigated elsewhere (Chapter 5.7). The guitar string is a mechanical system that, strictly speaking, reacts non-linearly in a complicated manner; we will assume it to be linear and time-invariant in order to simplify things. Given such boundary conditions, we can define – as **system quantities –** masses, stiffnesses and resistances, and acting on these we have the **signal quantities** of force, and of vibration velocity = particle velocity. The local distributions of the signal quantities run along the string as a **wave** – the propagation speed being *c*. On electrical lines, we find very similar relationships: here the system quantities are capacitance, inductance and resistance, and the signal quantities are current and voltage. Using the analogous mathematical description, mechanical and electrical lines will be juxtaposed in the following. The mechanical line is the guitar string; the analogous electrical line is supposed to serve as **model** for illustration it does not actually exist, and it certainly is not the guitar cable!

Translator's remark: *in this chapter, again often the bridge and the nut of the guitar are taken as the points between which the guitar string vibrate i.e. as the string bearings. Of course, all basic considerations apply to the fretted string in the same way – the bearings are then bridge and fret. This is not always explicitly indicated, and therefore the term "nut" should be considered to appropriately include the term "or fret", as well.*

2.1 Transversal waves

On a mechanical transmission line, mechanical waves propagate. These waves may be longitudinal or transversal waves, or a combination hereof. In a pure transversal wave, the differentially small line particles oscillate laterally relative to the direction of propagation, either in a planar movement, or in rotating fashion. In a pure longitudinal wave, the particles oscillate in the direction of propagation; for a guitar string this would be along the string axis, having rather minor significance compared to the transversal wave. In a simple electrical line, an electrical field is generated between two parallel conductors. *Within* the conductors, currents are flowing, and differences in electrical potential (i.e. voltages) result *between* the conductors.

Electrical line theory distinguishes between various conductor geometries – this will not be required for the fundamental considerations to be discussed here.

It is the local distribution of the signal quantities that propagates along the transmission line with the speed c. Defined as a function of place and time, the **force** F is a signal quantity on the mechanical line: $F(z,t)$. Herein, z is the place coordinate in the direction along the string, and t is the time. A first reason for misunderstandings pops up: it is not the tension force Ψ of the string that is meant here but the wave force F. Tuning the string, a tension force Ψ is exerted onto the string; after conclusion of the tuning process this will (ideally) remain constant. In addition, plucking the string will introduce a lateral **transverse force**; this force is meant with F. On top of the force distribution across place and time we require also a movement quantity to describe the changing geometry. For this we basically look at the distribution of the **lateral velocity** that may be converted into acceleration via differentiation and into displacement via integration. To avoid confusion with the propagation speed c (which is signal-independent constant), this signal speed is termed (**particle) velocity** $v(z,t)$. The signal-carrying **wave quantities** are thus the force $F(z,t)$ and the velocity $v(z,t)$. In the important transversal wave, the direction of the latter is transverse to the string axis, for the longitudinal wave, it is in parallel.

Either wave quantity may not be directly observed. Even as we see *that* a string indeed vibrates, it is impossible to say whether the particle velocity is 1 m/s or 5 m/s. Conversely, the displacement can be estimated – at least if it is sufficiently strong. Easiest to interpret are therefore graphical representations of the **displacement** which is often designated with x or ξ. However, ξ, is dependent on place *and* time: $\xi(z,t)$. This function could be represented in space via a z,t,ξ-coordiate-system, with ξ being the elevation above the z,t-plane. Sections along $t = t_0 = const$ result in a place-function $\xi(z,t_0)$; sections along $z = z_0 = const$ yield a time-function $\xi(z_0,t)$. The **place-function** is a snapshot showing the location-distribution of the displacement at *one* point in time. The **time-function** is a snapshot indicating the course of the displacement of *one* special point on the string. Spacial representations above a z,t-plane do, however, have the big disadvantage that the time t is in fact *not* a space-coordinate. This is not a problem for the general definition of the term "space" but it is not very descriptive for fundamental considerations. A real problem, though, is simplifying $\xi(z,t_0)$ to $\xi(z)$ = position function, and simplifying $\xi(z_0,t)$ to $\xi(t)$ = time-function. Indeed, t_0 and z_0 are both constant quantities, but $\xi(z)$ and $\xi(t)$ remain two distinct, different functions that should not be designated with one and the same letter ξ. We will write $\xi_{ZF}(t) = \xi(z_0,t)$ for the displacement-time-function in order to facilitate that distinction, and $\xi_{OF}(t) = \xi(z,t_0)$ for the displacement-place-function.

For three different wave-shapes, **Fig. 2.1** shows the place-function of the displacement at seven different points in time. In each of the three graphs, a transversal wave runs from right to left. As a contrast to the real string, the wave propagation depicted in Fig. 2.1 is not dispersive i.e. the wave maintains its shape. In the real string, the propagation happens with a frequency-dependent speed (**dispersion**), and the wave changes its shape during the propagation, because higher frequencies propagate with higher speed. For introductory considerations, we may neglect dispersion, but for more exact analyses it will have to be taken into account, with c being not a constant but dependent on frequency.

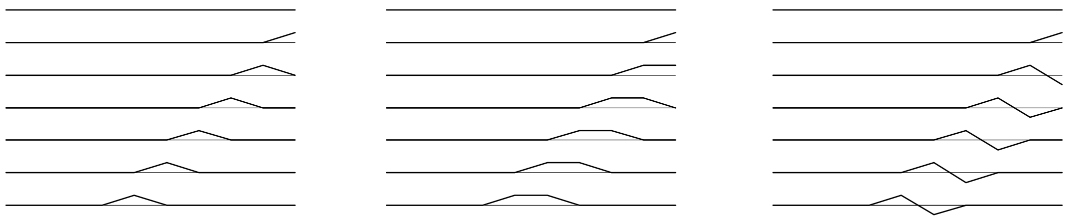


Fig. 2.1: Transversal waves. Each line shows the lateral displacement of the string at <u>one</u> point in time. The wave propagates (in each of the three columns) from right to left; the lower lines show later points in time. At the far right, a short-duration lateral excitation happens, causing a wave running to the left with constant propagation speed. The three graphs depict three different excitation functions.

In the following, $\xi = \xi_{OF}(z)$ is to be interpreted as the analytical representation of a **function**, with **Fig. 2.2** showing the corresponding graphical representation (for a specific example). A function is a rule that unambiguously allocates to each **argument** z a **function value** ξ. Rather than the term ”allocate”, we often use “map”, and thus a function is also a **mapping**: the set of z-points is mapped onto the set of ξ-points.

A **transformation** also is a mapping, because again sets are mapped onto each other. In the following, the term “transformation” is – as a specialization – defined as describing the shifting of the $z\xi$-plane. Each point on this plane is described as a pair of values; the origin e.g. by z = 0, ξ = 0. Shifting every point on the $z\xi$-plane by the same distance in the same direction results in a special transformation that in this case is termed **shift** or **translation**. Analytical geometry of the plane calls this a *parallel shift of the plane in itself* – the shift belongs to the class of concordant **congruent mappings**.

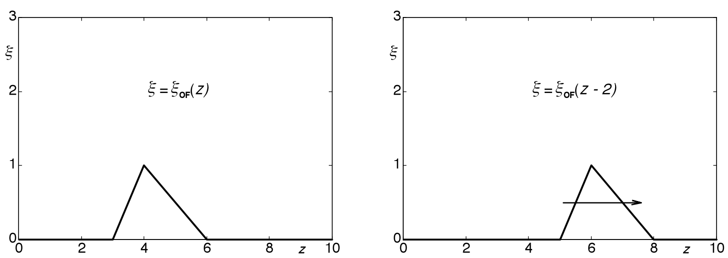


Fig. 2.2: Graphical representation of the function $\xi = \xi_{OF}(z)$. Applying the transformation shifts the function graph in the positive z-direction.
Right: ct = 2.

Functions, mappings and transformations are allocation rules. For the following considerations we will use these specializations of the terms: the z-ξ-allocation is termed *function*, while the shift of all z,ξ-points that leads to a shifting of the function graph (the function curve) is designated a *transformation*. The shift of the function graph in the direction of the z-coordinate is of particular importance since this is the axis of the string (i.e. the direction along the string), with elastic waves running along the string in that direction. The place-function of the displacement describes the connection between the place z and the displacement ξ. For the string, each z is tied to a distinct ξ for any special point in time. Analytically described by $\xi = \xi_{OF}(z)$, the function graph is a depiction of the string displacement.

Depending on the changing time t, the function graph changes its position; it shifts along z. Mathematically seen this shift is a time-dependent transformation (specifically: a translation). It is either termed a coordinate transformation or an **argument transformation,** because the transformation rule changes merely the function-argument z.

All values (ξ) of the function are retained; they are, however, mapped to new z-values via the transformation.

$$\xi = \xi_{OF}(z - ct_0) \qquad \text{Time-dependent translation}$$

The transformation changes the argument of the function: z becomes $z - ct_0$. Herein, c is the **propagation speed**[♣] of the wave, and ct_0 is the distance covered during the time t_0. We may interpret $\xi_{OF}(z)$ as place-function at the time $t = 0$, and $\xi_{OF}(z - ct_0)$ as place-function at (a different) time t_0. The function graph defined by $\xi_{OF}(z)$ is shifted in the z-direction by the transformation: if c is positive, the shift is towards the right, and for negative c it is towards the left. Besides the place-function that describes the displacement for a fixed point in time t_0 as a function of place, we may also consider the time-function giving the displacement for a fixed location z_0 as a function of time. If one of the two functions is known, the other can be calculated from it.

Fig. 2.3 exemplarily depicts a triangular place function $\xi_{OF}(z)$. The location (z) is newly defined relative to the specific location $z_0 = 8$ on the string: $z = z_0 - ct$. Basis for this substitution is the consideration that it does not make any difference for the calculation whether the wave runs towards the location z_0 or whether the observer moves towards the wave starting from location z_0. $\xi_{OF}(z_0 - ct)$ becomes the new function $\xi_{ZF}(t)$ that originates from $\xi_{OF}(z)$ via argument-transformation: $\xi_{OF}(z) \Leftrightarrow \xi_{ZF}(t)$. More generally: **the place function becomes the time function via argument transformation, and vice versa**. ξ_{OF} and ξ_{ZF} show a similar behavior but they are not identical.

For a positive c (with the wave running towards the right), one function originates from the other via horizontal stretching, via mirroring relative to the ordinate, and via horizontal shifting. Although other mapping steps would also be definable, these three partial mappings are to be considered. The horizontal stretching (performed in the direction of the abscissa) allocates a new scaling to the abscissa: the place becomes the time, and vice versa ($z = ct$). The mirroring results in a reversal of the direction of the abscissa. Both partial mappings could also be called "stretching with negative coefficient". As a last step, the curve – mirrored and stretched in the direction of the abscissa – is subsequently also shifted in the direction of the abscissa; the place function becomes a time function (or vice versa). For the wave running towards the left (negative c), the mirroring is omitted, i.e. the direction of the abscissa is not inverted. Both graphs in Fig. 2.3 are displacement functions; the functional connection between abscissa and ordinate is, however, different.

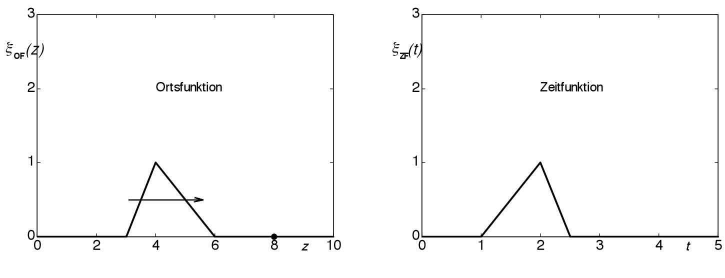


Fig. 2.3: Place und time function. The wave runs towards the right to the point $z_0 = 8$; the displacement of that point is shown in the time function. For the physical units see the text.
"Ortsfunktion" = place function; "Zeitfunktion" = time function

[♣] In literature equations are also found that fundamentally start with a positive c and use a plus- or a minus-sign depending on the propagation direction.

In Figs. 2.2 and 2.3, the variables do not possess any physical units – this is not unusual for mathematical representations. We could add **units**, or interpret the location coordinate z in a normalized manner … e.g. normalized to 1 m. That would make $z_0 = 8$ in fact mean $z_0 = 8\,\text{m}$. If, in addition, we assume that the time t is normalized to 1 s, the propagation speed for this example would be $c = z/t = 2\,\text{m/s}$.

In Fig 2.23 it does not make any difference whether the wave (on the left) runs towards the observer located at the fixed point $z_0 = 8$ at a speed of 2 m/s, or whether the observer runs (starting at $z_0 = 8$) towards the motionless (!) wave at a speed of 2 m/s. In both cases the observer sees the same time function. Also (please do remain calm now, dear physicists): waves on guitar strings do not run at light speed. Not even approximately.

The graphs shown so far have represented place- and time-functions of the **displacement** because the latter is easily observed on vibrating strings. From the point of view of systems theory, however, the **(particle) velocity** v is of greater importance because power and impedance result from it (along with the force). The velocity v (at the place z_0) is the partial temporal derivative of the displacement ξ (at the same place):

$$v(z,t) = \frac{\partial}{\partial t}\xi(z,t)$$ Time function: displacement → velocity

With both v und ξ depending on two variables in the general representation, a partial derivative for t is required. In it, the differentiation is done merely for t with the condition that $z = z_0$ remains constant:

$$v_{\text{ZF}}(t)\Big|_{z=z_0} = \frac{d}{dt}\xi_{\text{ZF}}(t)\Bigg|_{z=z_0}$$ Both Functions for the same place z_0

However, place and time are interdependent via the propagation speed: $z = z_0 - ct.$ It therefore is possible to reshape the time-differentiation d/dt into a place-differentiation d/dz, and with this to move from the place-function of the displacement $\xi_{\text{OF}}(z)$ directly to the place-function of the velocity $v_{\text{OF}}(z)$ (chain rule of differential calculus):

$$v_{\text{OF}}(z)\Big|_{t=t_0} = -c \cdot \frac{d\xi_{\text{OF}}(z)}{dz}\Bigg|_{t=t_0}$$ Place-function: displacement → velocity

In all these equations, the **sign** of the velocity v is oriented relative to the direction of ξ: movement in the direction of ξ yields a positive v. The conversion of the velocity-place-function is done, just as for the displacement, via substitution: $z = z_0 - ct$.

$$v_{\text{OF}}(z) \Rightarrow v_{\text{OF}}(z_0 - ct) \Rightarrow v_{\text{ZF}}(t)$$ Place-function → time-function

For known place-function and known propagation speed, the time-function is unambiguously defined – and vice versa. For known displacement and known propagation speed, the velocity is unambiguously defined, and vice versa.

Fig. 2.4 shows the place-function of triangular-shaped displacement waves; the corresponding velocity waves have a square shape. In the figures, z is the abscissa; seven subsequent points in time are shown in the figures. Despite the shape of the displacement-place-function being the same, the velocity-place-function differs in the sign. In the formula used so far, this change of the sign has been covered by c: for waves running towards the right, c has been defined as being positive; for waves running to the left it was negative.

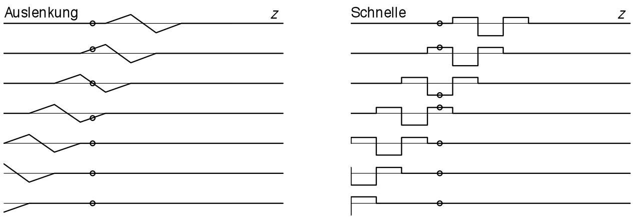


Fig. 2.4a: Place-functions of the wave running towards the left. The marked point is first moved *upwards*: the velocity of this point starts in the positive.
"Auslenkung" = displacement
"Schnelle" = (particle) velocity

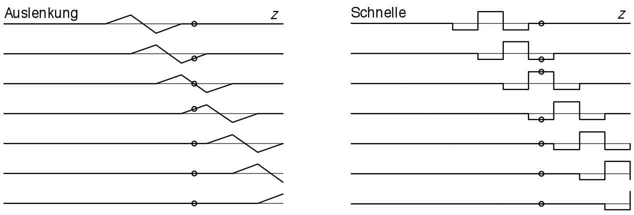


Fig. 2.4b: Place-functions of the wave running towards the right. The marked point is first moved *downwards*: the velocity of this point starts in the negative.
"Auslenkung" = displacement
"Schnelle" = (particle) velocity

N.B.: At the left and right border, the wave disappears from the picture frame; there is no reflection.

Displacement ξ and velocity v describe the deformation of the string; the force F may be interpreted as their cause. As was already mentioned, it is not the tensioning force that is meant here, but the transverse force. It is purposeful at this point to look at the electrical transmission line rather than at the mechanical one. At the root of both lines we have the same type of differential equation (it is merely the system parameters that are designated differently). **Considerations of analogy** enable us to extrapolate from the behavior of one line to the behavior of the other [3]. It is particularly obvious to transfer the insights gained from the electrical line theory [5] to the mechanical line using the force-current-analogy. Doing this, the following correspondences result: capacitance ↔ mass, inductance ↔ spring, electrical admittance ↔ mechanical impedance, electrical voltage ↔ (particle) velocity, current ↔ force. For reasons of simplification, we are exclusively looking at loss-free lines with negligible short-term signal damping. Dispersion is not included in the considerations.

As a wave propagates along an electrical line, voltage and current are linked at every position on this line by the **wave impedance** Z_{Wel}: $\underline{U} = Z_{Wel} \cdot \underline{I}$. For loss-free lines, the wave impedance is of purely resistive character (i.e. it is real). There is no contradiction here: the line indeed accepts energy – however, this energy will not be dissipated as heat but will be transmitted. In order to avoid reflections, we usually assume an *infinitely long line*. This is not mandatory, though: as long as the wave is not facing any 'obstacles', we can do calculations using the wave impedance. Applying the *F-I*-analogy to the electrical line yields:

$$\underline{F} = Z_W \cdot \underline{v} \qquad \text{Mechanical line quantities}$$

Distinguishing it from the electrical wave impedance Z_{Wel}, we term the mechanical wave impedance Z_W.

With the length-specific mass *m'*, and the length-specific compliance *n'*, the mechanical wave impedance Z_W is calculated as:

$$Z_W = \sqrt{m'/n'} = \sqrt{\tfrac{1}{4}\rho D^2 \pi \cdot \Psi}$$ Mechanical wave impedance

In this formula, Ψ represents the tensioning force of the string, ρ the density, and D the diameter. For a 009-gage string set, 0,68 Ns/m (E2♣) and 0,14 Ns/m (E4) result – see also Chapter A.5.

In the wave propagating without perturbation, this real quantity connects the force F and the velocity v at every location. As example: the E4-string vibrates with an amplitude of 1 mm; its velocity amounts to $2\pi \cdot 330\text{s}^{-1} \cdot 0{,}001\text{m} = 2{,}07$ m/s (330Hz, sine-shape, peak value). Given $Z_W = 0{,}14$ Ns/m we obtain for the peak value of the force-wave: $F = 0{,}29$ N. Because Z_W is real, force and velocity are in phase at every location. However, this holds only for the wave propagating without perturbation. As soon as reflected waves are superimposed, there are other dependencies. The table below indicates the connections between the wave quantities:

Place-function → time-function: $z = z_0 - ct$. Time-function → place-function: $t = t_0 - z/c$

	Place-function	Time-function
Displacement	$\xi_{OF}(z)$	$\xi_{ZF}(t)$
Velocity	$v_{OF}(z) = -c \cdot \dfrac{d\xi_{OF}(z)}{dz}$	$v_{ZF}(t) = \dfrac{d\xi_{ZF}(t)}{dt}$
Force	$F_{OF}(z) = Z_W \cdot v_{OF}(z)$	$F_{ZF}(t) = Z_W \cdot v_{ZF}(t)$

Applying the formulas introduced so far, place- and time-functions can be converted into each other, and relationships between displacement, velocity and force can be set up. We have, however, not paid sufficient attention to the **sign** – its definition is not as trivial as it first may seem. For the displacement, we obtain still relatively simple relationships: the displacements in the ξ-direction are defined positively. For a wave progressing in the +*z*-direction, a positive displacement therefore implies: seen in the direction of the propagation, the displacement is 'to the left", while for the wave running in the –z-direction, positive displacement means 'to the right", seen in the direction of the propagation.

Evidently, there are two different possibilities for the **definition of the sign**: either referring to the absolute coordinates, or referring to the direction of propagation. If waves propagating in different directions are to be superimposed, **absolute coordinates** are more purposeful; with them, the superposition can be done – independently of the propagation direction – as a simple addition. For **displacement**, this definition is obvious: displacements in the ξ-direction are positive. For the **velocity** and the acceleration this approach is recommended, as well. Positive **acceleration** therefore implies that the string moves in the ξ-direction with increasing velocity. For the **force**, the following holds: a positive force generates a state of pressure in the spring. In an upright-standing coil spring, a *pressure* state can be generated as the upper end is pressed downward, or the lower end upward – both cases have the effect of a positive force.

♣ For wound strings, the calculation needs to consider a density reduced by 10% due to the encased air.

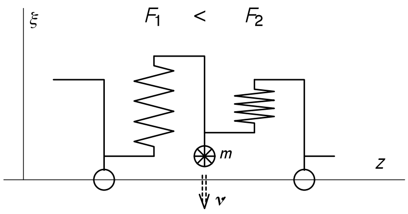


Fig. 2.5: Line-element. The circles are transversely movable masses; the springs model the transverse stiffness. $F_2 > F_1$ indicates that there is a greater pressure force within the spring on the right. Consequently, a downward-directed acceleration (indicated by the arrow) acts onto the mass. Given the sign convention explained above, this acceleration is negative (negative ξ-direction).

In order to illustrate the transversal forces, the spring-mass-model according to **Fig. 2.5** may serve. The displacement ξ of the points of mass is to be seen directly as distance to the zero-line, and the transversal force F acting within the springs can be taken from the deformation of the springs. The acceleration forces relevant for the masses result as the difference of the two adjacent spring-forces. The force-difference $F = F_2 - F_1$ has the effect of an acceleration directed downwards; the inertia-formula therefore requires a minus sign. Dividing the equation by the differential length dz of the line element, the force becomes the length-specific force, and the mass m becomes the length-specific mass m'.

$$F_2 - F_1 = -\dot{v} \cdot m \quad \Big\} \quad \frac{dF}{dz} = -\dot{v} \cdot \frac{dm}{dz} = -\dot{v} \cdot m' \qquad \text{Law of inertia}$$

The transversal force F acting in a spring depends, via the compliance n, on the change of the length $\Delta\xi$ $F = \Delta\xi/n$. The change of the length is the difference between two adjacent displacements; by relating it to dz, the compliance n becomes the specific compliance n'.

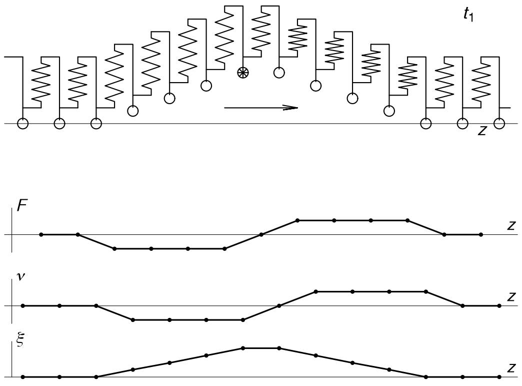
$$F = -\frac{\xi_2 - \xi_1}{n} \quad \Big\} \quad F = -\frac{d\xi}{dz} \cdot \frac{dz}{dn} = -\frac{d\xi/dz}{n'} \qquad \text{Hooke's law}$$

The specific compliance (compliance per length) is the inverse of the tension force Ψ of the string (to be discussed later). A further differentiation of the spring force yields two terms that can be put into an equation:

$$\frac{\partial F}{\partial z} = -m' \cdot \frac{\partial^2 \xi}{\partial t^2}; \qquad \frac{\partial F}{\partial z} = -\Psi \cdot \frac{\partial^2 \xi}{\partial z^2}; \qquad \text{this yields:} \quad \boxed{\Psi \cdot \frac{\partial^2 \xi}{\partial z^2} = m' \cdot \frac{\partial^2 \xi}{\partial t^2}}$$

The differential equation derived this way is called the **wave equation**. It interconnects the second place-derivative (curvature) with the second time-derivative (acceleration). The general solution consists of the superposition of an arbitrary number of waves that each may run towards the left or towards the right. However, the magnitude of the propagation needs to be equal for all waves because it depends – as a constant – on the transmission line parameters (string parameters). For waves running towards the right, we defined c (arbitrarily) as positive, and for waves running towards the left as negative. The wave impedance $Z_W = F/v$ is also carrying a sign; given the sign-convention used previously here, a positive wave impedance is for the wave running towards the right, and a negative wave impedance is for the wave running towards the left.

$$c^2 = \Psi / m'; \quad c = \pm\sqrt{\Psi / m'} \qquad\qquad Z_W^2 = \Psi \cdot m'; \quad Z_W = \pm\sqrt{\Psi \cdot m'}$$



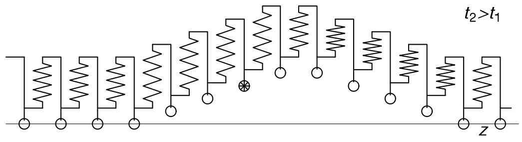
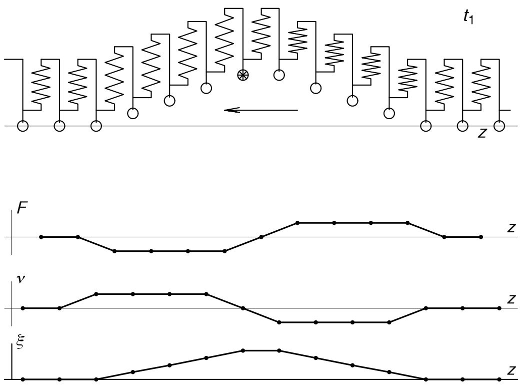


Fig. 2.6a: Transversal wave running towards the right. The propagation speed *c* is positive, as is the wave impedance.



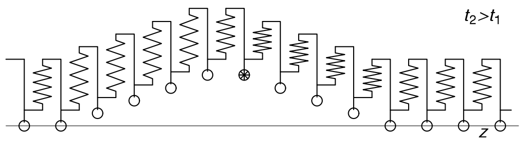


Fig. 2.6b: Transversal wave running towards the left. The propagation speed *c* is negative, as is the wave impedance.

Fig. 2.6 depicts a progressing wave at the two points in time of t_1 and t_2; the difference in displacement allows for deduction of the momentary velocity. For example, for the wave running towards the right, the mass tagged with * moves downwards, and its velocity therefore is negative. However, the force *F* shown here is not the inertia force but the force transmitted in the springs. Via the place-function, the displacement *F* is unambiguously determined; in order to determine *v*, though, we need to additionally know *c*.

It is not obligatory to connect the springs as shown in Fig. 2.6. Alternatively, the upper end of the spring could be connected to the mass positioned adjacent, and the lower end could be connected to the mass on the right. However, this connection would require reversal of the sign of the force! As a consequence, the wave running towards the left would have a positive wave impedance, and the wave running to the right a negative one. Both changes do not represent a contradiction: the spring-mass-model is a direct visualization of a mechanical tension state. To start with, the sign in this model may be arbitrarily defined – subsequently, however, all following calculations are committed to this definition. Instead of the spring-mass-model, it would also be possible to define place-discrete shear stresses, but again this would entail freedom in setting the sign.

The following graph (**Fig. 2.7**) gives an overview for different triangular displacement waves. Seven function graphs – positioned one above the other – indicate seven consecutive points in time; the start is in the uppermost line each. All depictions show the place-functions along the *z*-coordinate. For all examples it is assumed that a transversal wave only moves along the string. As soon as we allow for a superposition of waves running in different directions, a new degree of freedom is introduced for the velocity (Fig. 2.8). The force, however, is always connected unambiguously with the displacement.

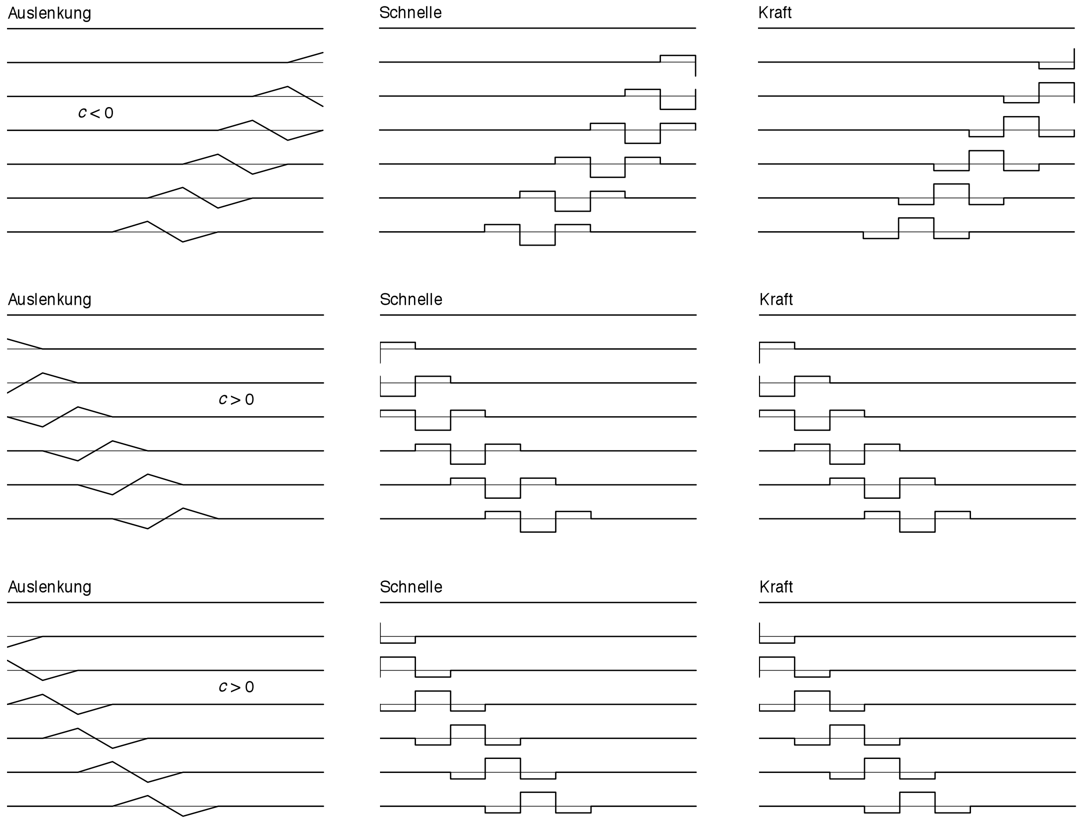


Fig. 2.7: Place-function of the displacement (= "Auslenkung"), the (particle) velocity (= "Schnelle"), and the transverse force (= "Kraft") for three different waves.

In **Fig. 2.8** we see the superposition of two waves running in different directions. At the fifth point in time, the velocity is zero for all points in the string. This special condition cannot be realized with *one single* wave; for $c \neq 0$ the displacement would otherwise have to be always zero for the whole of the string.

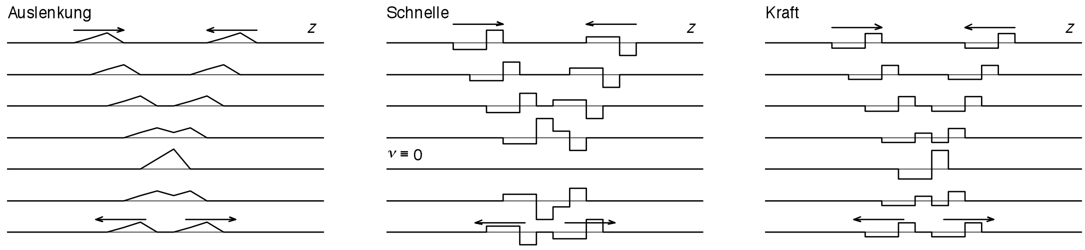


Fig. 2.8: Place-function of the displacement (= "Auslenkung"), the (particle) velocity (= "Schnelle"), and the transverse force (= "Kraft"). The sum of the force cannot be calculated from the sum of the velocity anymore.

