The magnitude of an all-pass function is 1, and the phase shifts by $n \cdot \pi$ for $0 : f : \infty$, n being the order of the all-pass function. For $f = 0$, $r_v = -1$ holds: the velocity wave is reflected with opposite sign. For $f = f_r$ we obtain $r_v = +1$; for $f \rightarrow \infty$ we again get $r_v = -1$.

Therefore, having a resonator terminating the transmission line has the effect of an additional phase shift. Natural vibrations (partials) occur at those frequencies where the phase shift for a full travel-path on the string (2L; back and forth) is an integer multiple of 2π. Assuming dispersion-free wave propagation on a fully clamped-down string, partials at integer multiples of the fundamental frequency result. However, if a bearing acts as a resonator, an additional phase shift is introduced that generates (in our example) an **additional partial**. For resonators of higher order, several additional partials occur.

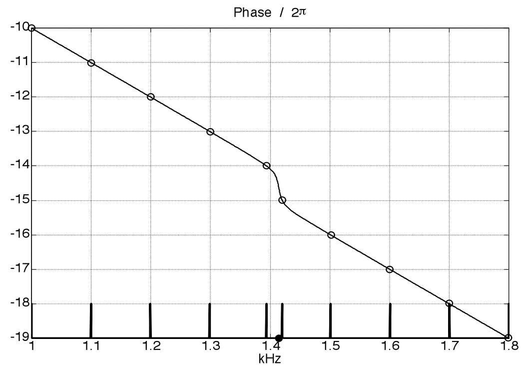

Fig. 2.21: Phase shift along a full travel path along the string. One string bearing is configured as a resonator resonating at 1,415 kHz. An additional natural frequency is the result of the narrow-band additional phase shift.

Fig. 2.21 shows the phase shift occurring for a string vibrating at a fundamental frequency of 100 Hz and a full travel path (double the string length). The phase is negative as it is customary for delays in recent literature. One string bearing is configured as a resonator with a resonance frequency of 1,415 kHz (dot on the abscissa). At the bottom of the graph, the frequencies of the partials are indicated with bars. The partial at 1,4 kHz is substantially detuned downwards by the bearing resonance, and an additional partial is generated at 1,42 kHz. All other de-tunings are too small to be recognizable in the figure.

The spectral derivative $-d\varphi/d\omega$ yields the **group delay** (Chapter 1.3.1). The slope of the phase function is virtually constant with the exception of the range around the bearing resonance. Thus, the group delay is also generally constant – only in the range of the bearing resonance it becomes longer. This leads to a warping in the spectrogram (Fig. 1.8).

2.6 Line losses

Ideal masses and springs store energy but do not dissipate them as heat. These elements are therefore termed “loss-less”. In contrast, any real string also features friction-resistances that irreversibly convert the vibration energy into caloric energy. Line theory considers these energy losses via distributed, differentially small resistances. It is insignificant for the model whether the losses are due to mechanical friction in the string (inner damping), or result from the string *directly* radiating sound energy (i.e. without detour through the guitar body).

With a level decreasing by 10 dB within 10s in the decay process, a low E-string looses about 2,8% of its vibration energy per period of the fundamental. The string therefore is included in the groups of weakly damped systems (Q-factor $Q = 2249$) – it may be seen as a transmission line with low losses in good approximation. Considering moreover that the main share of the measured losses is not from the string itself but from the bearings shows that this approximation is justified to a high degree.

For **transmission lines with low losses**, the assumption is that phase- and group-velocity are practically not affected by the damping. It is only the vibration amplitude that decreases slightly as the signal passes through the line. For line lengths in the range of the length of a string, the amplitude damping is so small that it may be disregarded altogether in many cases. However, if the signals are subjected to numerous reflections, and if the objective of the investigations is a decay process lasting several seconds, then the amplitude damping must not be ignored anymore. It is not always necessary, though, to formulate a differentially distributed damping: insofar as merely discrete points on the string are of interest, ladder networks consisting of loss-free delay elements and delay-free damping elements provide a useful model (Chapter 2.8).

Trying to calculate the **internal losses in the string** brings curious issues to light: the loss factors for steel given by different books differ by a factor of 14. Even in one and the same book we may find differences of 600%. That may be because microphysical loss effects depend on manufacturing processes, or because there is not *the one* steel. It is more likely though, that 'internal' losses also include radiation losses. A loss factor of $d = 0{,}0001$ (*Gahlau et al., Geräuschminderung durch Werkstoffe und Systeme, Expert Sindelfingen 1986*) appears plausible; it yields a level decay of 0,22 dB/s for 82,4 Hz – significantly less than that of typical measurement results (0,6 dB/s), and leaving room for further damping mechanisms. The $d = 0{,}0006$ specified only 14 pages on in the same book, however, is too high (1,3 dB/s).

We probably better abandon hope for any consistent terminology – all too entrenched are the habits. Terms like damping factor, damping coefficient, degree of damping, loss factor, etc. may certainly (?) be applied in a consistent manner within one and the same publication, but interindividual differences are the rule. It is therefore not surprising that an author specifies the aperiodic boundary case (called critically damped oscillation elsewhere) with $d = 1$, while another (equally renowned) colleague specifies $d = 2$ for the same case. You can live with such a scenario 💣 – but you gotta be aware (sapienti sat).

The situation is more conducive for the calculation of direct **radiation losses**. Under the heading "air damping", we find in [9] formulas for the radiation of active energy, and evaluations for bass strings. The losses mount with increasing frequency, and decrease as the string-diameter grows. The calculations in [9] relate to the damping of the fundamentals – higher harmonics tend to be radiated less well implying lower string damping♣. For guitar strings, calculations yield radiation-induced time-constants of the amplitude in a range of 20 s (open E_2-string) to 2 s (open E_4-string). We can therefore disregard radiation losses for the low guitar strings, while for the high strings these losses are at the borderline (measurement values are e.g. 1,7 s).

♣ In addition, we can consider that fretboard and guitar body are located in close vicinity of the string and act as reflectors. This compounds the calculation of the radiation impedance.

As a **bottom line**, we may state: inner damping and radiation losses may be disregarded as long as merely the wave propagation along short sections of the string is discussed. When analyzing vibrations of longer duration, we find – in electric guitars – damping mechanisms having a greater effect towards the higher frequencies (Chapter 7.7), and additional frequency-selective absorptions (e.g. resonances of the bridge). For acoustic guitars, we need to expect substantial absorptions in the low-frequency range, as well, since a non-negligible share of the vibration energy is fed to the bearings (bridge, frets).

2.7 Dispersive bending waves

The simple transmission line theory assumes place-independent wave impedance and frequency-independent propagation speed. However, the transversal waves of the guitar string propagate in a dispersive fashion, i.e. with frequency-dependent speed. The high frequencies run faster than the low ones (Chapter 1.3.1). The reason is the bending stiffness that increases the transverse stiffness, the latter in turn depending on the tensioning force.

Modeling the string as a dispersive transmission line takes much effort and is not always necessary. In most cases, only two or three points on the string are of interest (nut/fret, bridge, and point of plucking). Possibly, the position of the pickup also needs to be added in. It is easy to model the parts of the line between the discrete points via all-passes (Chapter 2.8). However, if precise description of the reflection conditions is required, we need a more detailed model. The simplest solution is found for steady-state (mono-frequent) partials: propagation speed and wave impedance are only weakly dependent on the frequency. For narrow-band considerations they may in fact be assumed to be constant. Transient processes extend across a frequency *range*, though; in such cases we need to apply frequency-dependent quantities.

We had introduced a simple element for modeling the dispersion-free string in Abb. 2.5. As characterizing quantities, force and velocity were sufficient (both quantities being signal-, place- and time-dependent). However, the rigidity of the real string requires that in addition to the (transverse) **force *F***, a place- and time-dependent **bending moment *M*** is specified, and also that we introduce an **angular speed *w***. This gives us a frequency-dependent phase delay (Fig. 1.6). The dispersive line element cannot be described as a quadripole (two-port network); rather, we need to specify a **four-port network** (octapole) [11]. The input quantities of the latter are F_1, M_1, v_1, w_1; its output quantities are F_2, M_2, v_2, w_2. Because the transverse dimensions of the string are small relative to the wavelength, we may disregard shear deformations and rotational inertia moments (Euler-Bernoulli theory for beams). Thus, the length-specific **mass *m'***, the length-specific **compliance *n'***, and the **bending stiffness *B*** remain as the system quantities (inside the four-port network).

The rigid string features *two* **wave impedances** $Z_F = F/v$ and $Z_M = M/w$, and *two* wave powers $P_F = Fv$ and $P_M = Mw$. *Two* bearing impedances each are active at both string bearings (nut/fret, bridge), and in addition the four signal quantities may be intercoupled in each bearing. For example, the edge-force may generate an edge-moment, or a displacement will necessarily lead to torsion. Since all these relationships appear depending both on frequency and direction, simplifications and approximations are indispensable.

