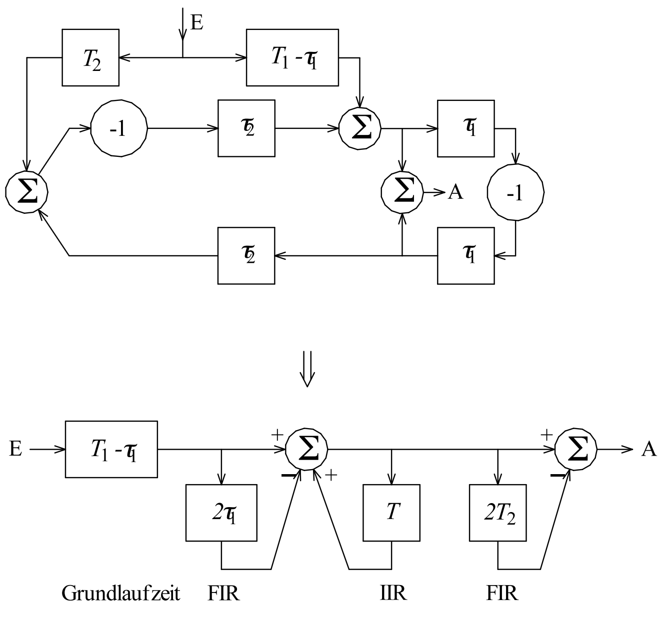
2.8.2 String with single-coil pickup

The SFD presented in Fig. 2.26 is now extended by the output of a magnetic pickup, assuming that the pickup will not influence the vibration of the string. This assumption is not fundamentally justified, because the attraction force of the permanent magnet does change the string vibration, and moreover the law of energy conservation demands that the string delivers the electrical energy generated. While the latter effect may be neglected when high-impedance pickups are deployed, strong magnets are indeed known for their interference when adjusted too close to the strings (Chapter 4.11). However, on order to explain the transfer characteristic in principle, the attraction does not need to be modeled.

Fig. 2.30 depicts the simplified model for the ideal string and a single–coil pickup. T_1 and T_2 designate the delay from the plucking point to the bridge and the nut, respectively. τ_1 and τ_2, respectively, is the delay from the location of the pickup to the bridge and the nut. Multiple rearranging of the drawing yields a ladder network consisting of four different filters:

- A basic delay from plucking point to pickup
- An FIR-filter with the long delay $2T_2$ (or $2\tau_2$, respectively)
- A recursive IIR-filter to model the string vibration
- An FIR-filter with the short delay $2\tau_1$ (or $2T_1$, respectively)

The sequence of these four subsystems may be changed arbitrarily. The pitch depends on the IIR-filter, and the sound color depends on the FIR-filters with their interference effect retraceable to the delay times T_1 and τ_1. There are three cases for the position of the pickup and the plucking point: $T_1 < \tau_1$, $T_1 > \tau_1$, and $T_1 = \tau_1$. It is immaterial whether pickup or plucking point is located closer to the bridge. For example, the pickup may be mounted 10 cm off the bridge, and the string is plucked 4 cm from the bridge, or the pickup may be mounted 4 cm from the bridge and the plucking may happen 10 cm from the bridge – in a *linear model*, the result will be the same (Fig. 2.35). What is not modeled: the string hitting and bouncing off the frets.



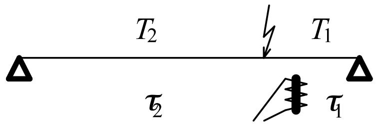
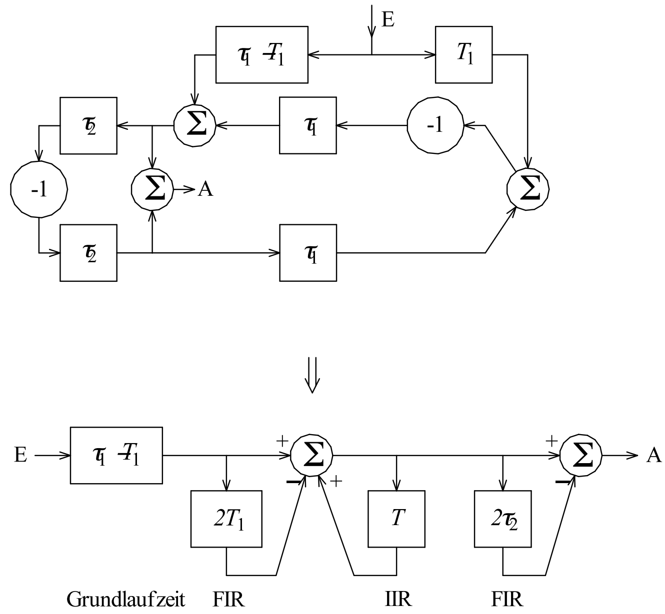


Fig. 2.30a: Ideal string with single-coil magnetic pickup, $T_1 \geq \tau_1$.
Reflections at bridge and nut are taken to be loss-free ($R = r = -1$).
"Grundlaufzeit" = basic delay time



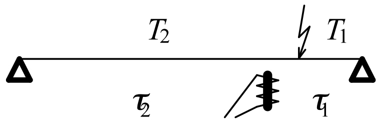


Fig. 2.30b: Ideal string with single-coil magnetic pickup, $T_1 \le \tau_1$.
Reflections at bridge and nut are taken to be loss-free ($R = r = -1$).
"Grundlaufzeit" = basic delay time

The **step response** associated with the step excitation is indicated in **Fig. 2.31**. Like Fig. 2.30, Fig. 2.31 shows that when changing from $T_1 < \tau_1$ to $T_1 > \tau_1$, merely the delay times T_1 and τ_1 need to be interchanged. The periodicity of this dispersion-free filter is $T = 2(T_1 + T_2) = 2(\tau_1 + \tau_2)$. Two square impulses are located within that period, centered around the point in time t_0, and $T - t_0$, respectively. For $T_1 < \tau_1$ we get $t_0 = \tau_1$, while $t_0 = T_1$ results for $T_1 > \tau_1$. The impulse width amounts to $\Delta t = | T_1 - \tau_1 |$.

The impulse width corresponds to the delay time of the transversal wave running from plucking point to pickup. If this distance is e.g. 4 cm, the impulse width calculates as 4·T/2·64 = T/32. Herein, the scale is assumed to be 64 cm. If the string is plucked exactly over the pickup, the two square impulses are perfectly contiguous.

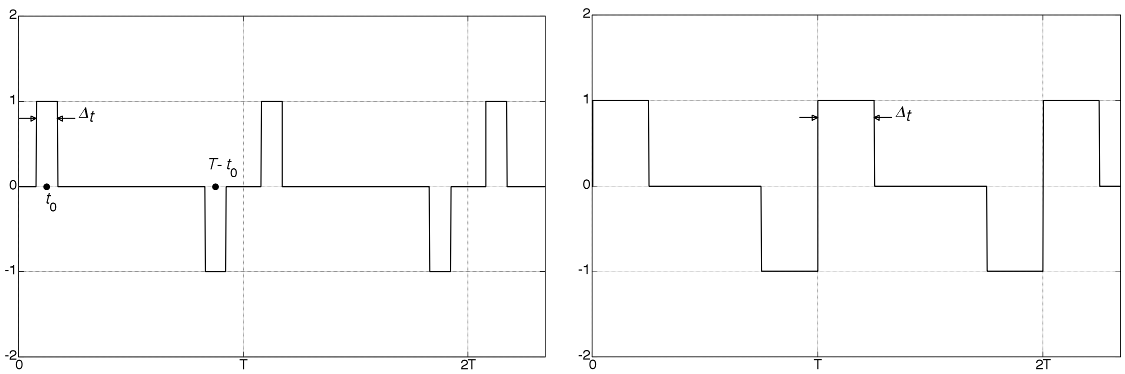


Fig. 2.31: Step response of the filter from Fig. 2.30. Left: $T_1 \ne \tau_1$; right $T_1 = \tau_1$. Input quantity for the filter is a force step at the plucking point. Output quantity is the string velocity over the magnet of the pickup – the source voltage of the pickup is proportional to this velocity. The terminal voltage results from low-pass filtering of the source voltage (Chapter 5.9). In particular for the low strings, the frequency-dependent propagation velocity (dispersion, Chapter 2.8.4) takes care of reshaping the rectangular waveform. In order to model this effect, the delays in Fig. 2.30 need to be realized as all-passes (Fig. 2.39).

The calculation of the **overall transfer function** of the 4 serially connected individual filters requires a multiplication of the individual transfer functions, resulting in somewhat more complicated frequency responses (**Fig. 2.32**).

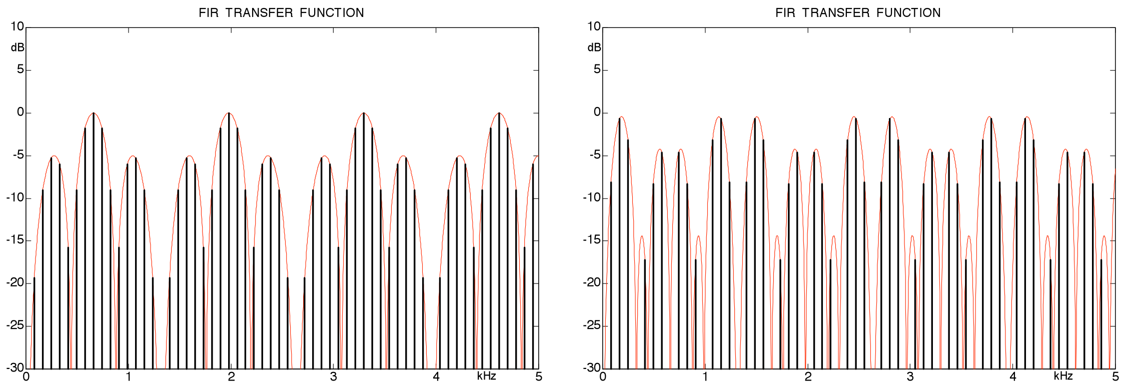


Fig. 2.32a: Transfer frequency response, E_2-string plucked 12 cm away from the bridge. Scale = 64 cm. Left: bridge-pickup (4 cm distance from the bridge); right: neck-pickup (16 cm distance from the bridge).

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Fig. 2.32b: Transfer frequency response, string plucked 12 cm away from the bridge; bridge pickup (5cm distance from the bridge, scale = 64 cm). Left: E_2-string, right: A-string.

It should be noted as particularly important that the two FIR-filters act **string-specifically** and do not have a global filter effect (as the magnetic pickup discussed in Chapter 5 would show it). The winding of the pickup coil is permeated by field-alterations of all 6 strings, and thus the resonance peak of the pickup will affect all 6 strings in the same way. The cancellations of the FIR-filter, however, are based on the propagation speeds of the waves, and these are string specific. As already elaborated, these propagation speeds do not depend on the (fretted) pitch, but on the pitch of the open string. The latter determines the propagation speed c_P, after all. It is therefore not possible to generate the FIR-characteristic electronically with an effects device … not with your regular pickups, anyway.

Fig. 2.33 shops the FIR frequency responses of a **Stratocaster** dependent on the pickup position. The effect of the second FIR-filter (plucking location) was not included in the calculations. To ensure a clear representation, the minima are only shown to a depth of 18 dB; according to the theory, the graph should extend to $-\infty$ dB in the minima.

 Translation by Tilmann Zwicker

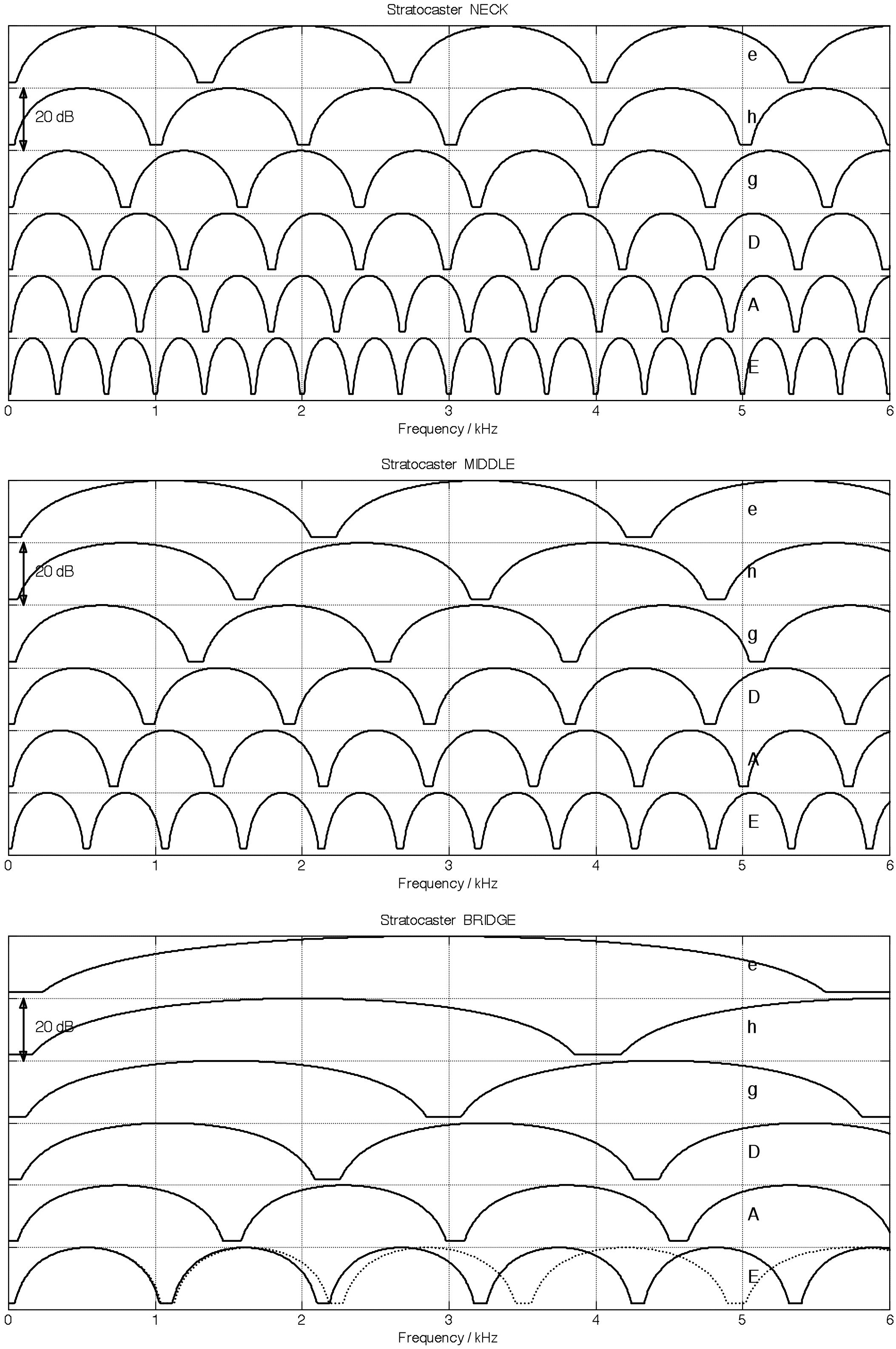


Fig. 2.33: Calculated FIR frequency responses for the Stratocaster; without dispersion. The dynamic is limited to 18 dB. In the lowermost graph, the effect of dispersive propagation is shown as a dotted line (compare to Chapter 1.8.4).

In **Fig. 2.34**, we see a comparison between measurement and calculation. A Stratocaster is connected to an instrumentation amplifier (input impedance: 100 kΩ) via a cable of a capacitance of 200 pF. The E_2-string is plucked directly at the bridge with a plectrum, and the signal of the bridge-pickup was evaluated.

The comparative calculation of the line spectrum includes both FIR-filters, the IIR-filter, and the equivalent circuit diagram of the pickup (Chapter 5.9.3). In addition, a small treble-attenuation was included to emulate the window of the magnetic field (Chapter 5.4.4). There is a clear correspondence. The measured spectrum nicely depicts the spreading of the frequency of the partials; it was emulated for the calculation using a simple model. The simulation easily reproduces the comb-filter structure, as well – at high frequencies, however, differences between measurement and calculation become evident. For a further improvement, e.g. the reflection coefficients would have to be adapted.

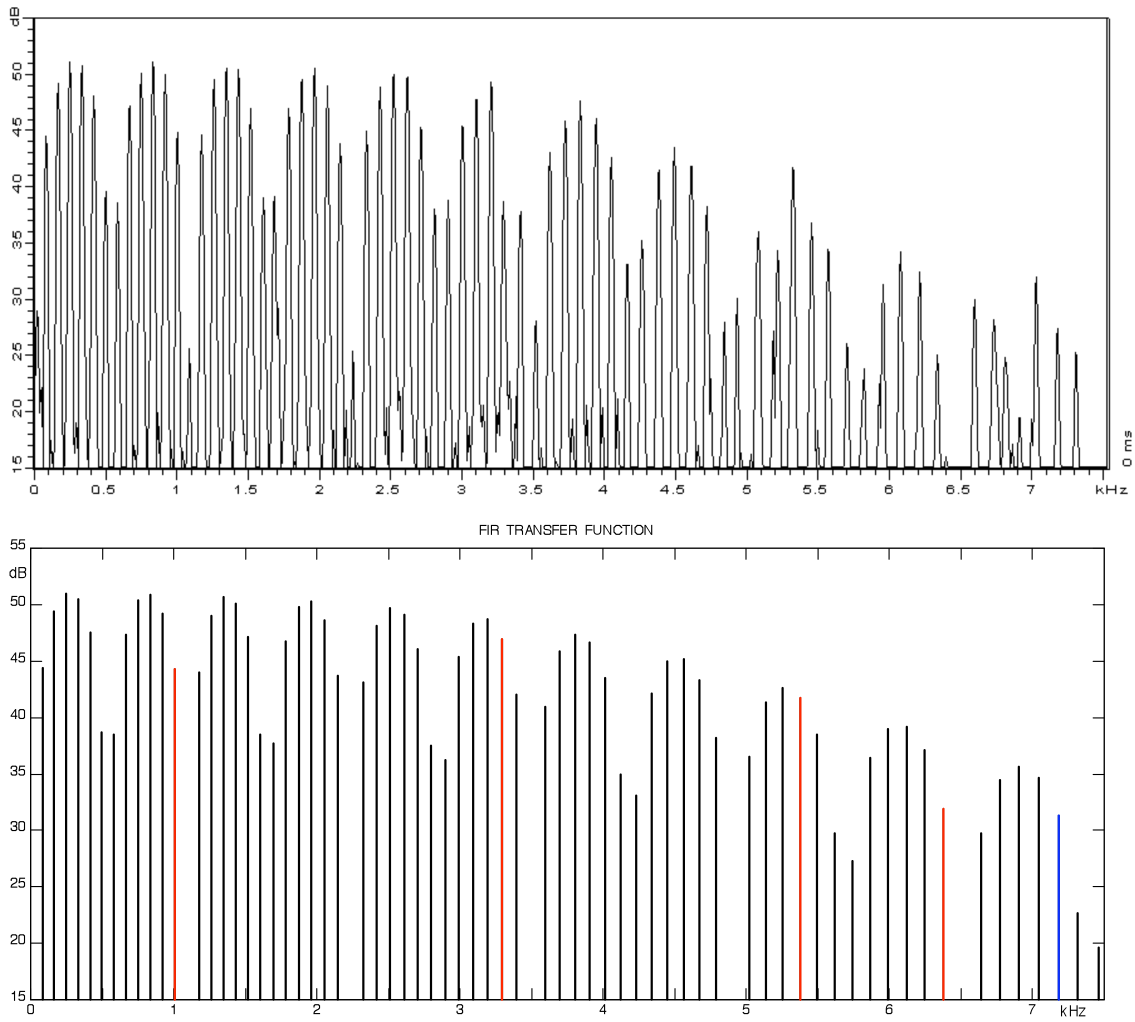
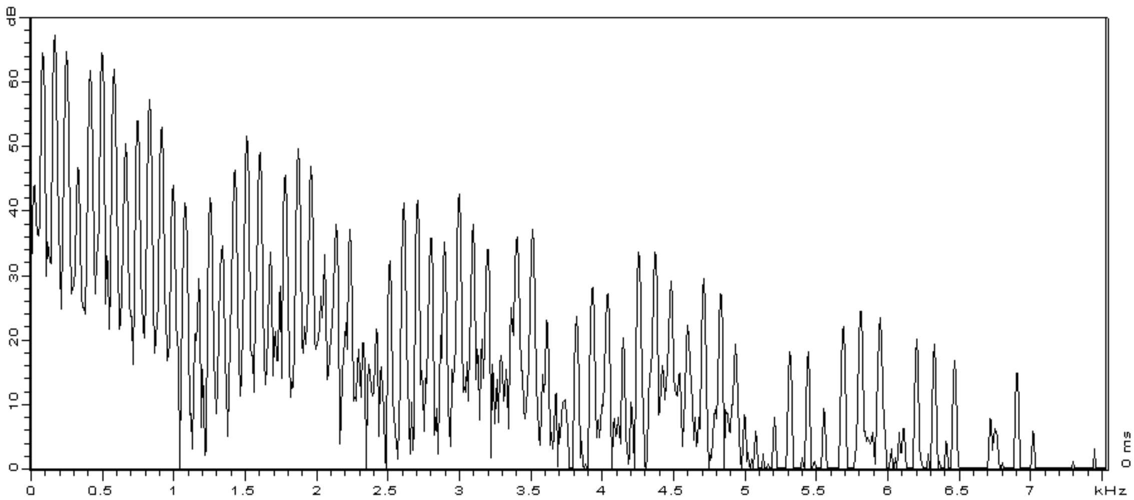


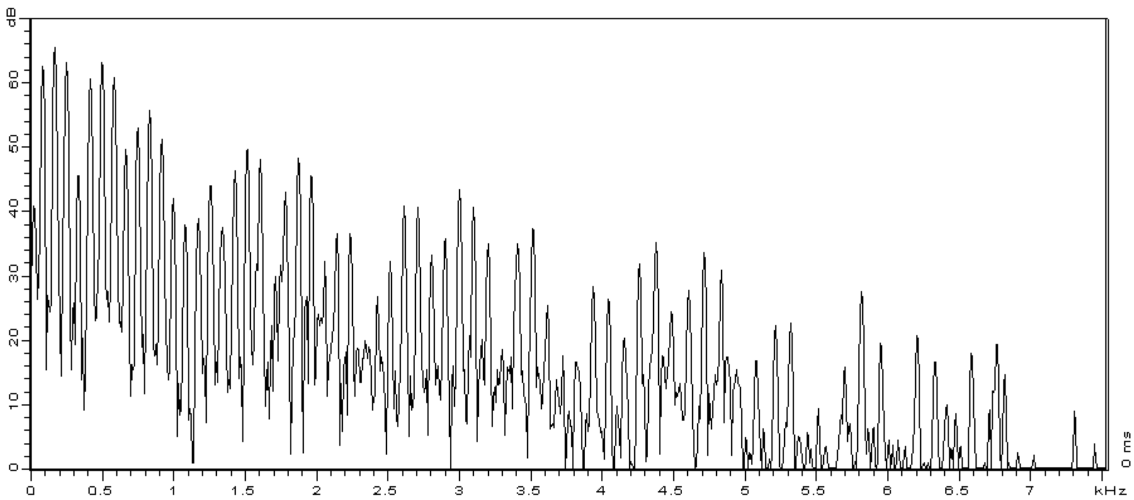
Fig. 2.34: Spectrum of an E_2-string plucked directly at the bridge (Stratocaster, middle pickup).
Top: measurement (with DFT-leakage). Bottom: calculation (with dispersion, compare to Chapter 2.8.4).
The inharmonic spreading is considerable; the 70^{th} “harmonic” is at 7,37 kHz rather than at 5,84 kHz.

It was already noted with respect to Fig. 2.30 that the plucking point on the string and the location of the pickup result in an FIR-filter each (with different delay times). Two serially connected filters represent two *commutatively* connected mappings the sequence of which may be interchanged. It therefore should not make any difference whether the string is plucked at point A with the pickup being located at point B, or the string is plucked at point B with the pickup located at point A.

To check this hypothesis, the E_2-string of a Stratocaster was plucked over the neck-pickup while the signal from the bridge-pickup was recorded. Subsequently, the E_2-string was plucked over the bridge-pickup and the signal of the neck-pickup was recorded. **Fig. 2.35** shows the DFT-spectra of both signals. The agreement is uncanny – especially considering that the reproducibility of the plucking process is not particularly good.



Measured signal of the bridge-pickup; string plucked over the neck-pickup.



Measured signal of the neck-pickup; string plucked over the bridge-pickup.

Fig. 2.35: Spectrum of the E_2-string of a Stratocaster; pickup and plucking position interchanged.

