2.8.4 Dispersive line elements

In Chapter 1.2, we had discussed that the propagation speed of the transversal waves is frequency dependent (**dispersion**), leading to a “spreading out” of the frequencies of the partials. This effect may be modeled in the SFD using frequency-dependent delay times. If we first assume the string to be loss-free, the magnitude spectrum will not change during the wave propagation. The phase spectrum does change – but not with a linear-phase characteristic like it would in a delay line. Rather, it assumes the characteristic of an **all-pass function** due to the frequency-dependent delay time. From the spreading of the partials, we can deduce the all-pass transfer-function (Chapter 1.3.1), and from this the all-pass impulse response (Chapter 1.3.2) via inverse Fourier transform. The simulations shown in Chapter 1 were calculated using such an SFD.

All-pass: linear system with a frequency-independent magnitude transmission coefficient and frequency-dependent phase shift.
Minimal-phase system: linear all-pass-free system.
Linear-phase system: linear system with frequency-proportional phase shift.
System order: number of the independent storage elements in the system.

For a wound E_2-string (b = 1/8000), **Fig. 2.42** shows the phase shift φ as it appears in a transversal wave running the distance of 8,65 mm (Chapter 1.3.1). Cascading 74 of the digital filters indicated in the figure yields a good approximation of the overall phase-shift of an E_2-string of 64 cm length (single travel path). The relatively high number of filters is due to the chosen sample frequency: a 2^{nd}-order all-pass can turn shift the phase by no more than 2π.

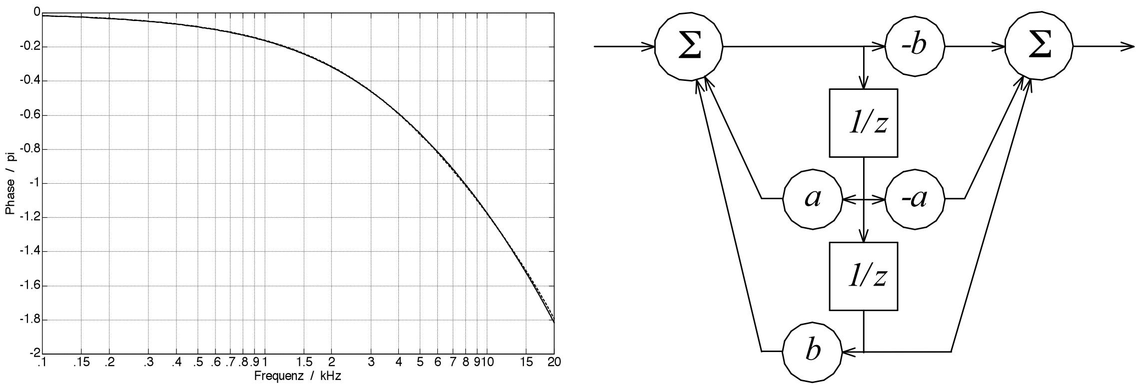

Fig. 2.42: Block-diagram and frequency response of the phase of a 2^{nd}-oder canonic digital all-pass filter. Sample frequency: f_a = 48 kHz, a = 0,5378, b = -0,03668. The frequency response of the filter phase is indicated as a dashed line; the differences to the phase of the string (——) are insignificant. “Frequenz” = frequency.

Given f_a = sample frequency, the transfer function of the digital all-pass is:

$$\underline{H}(z) = \frac{1 - az - bz^2}{z^2 - az - b}; \qquad \left|\underline{H}(z)\right| = 1; \qquad z = \exp(j\omega / f_a)$$

If the sample frequency is changed, the parameters a and b need to be adapted, as well.

The phase delay of the all-pass filter shown in Fig. 2.42 features the same tendency as it is found in dispersive waves on strings: high frequencies get to the output of the filter faster than low ones. Given a step excitation, we will therefore see a reaction in the high frequency range first; the low frequency components follow with a delay (**Fig. 2.43**).

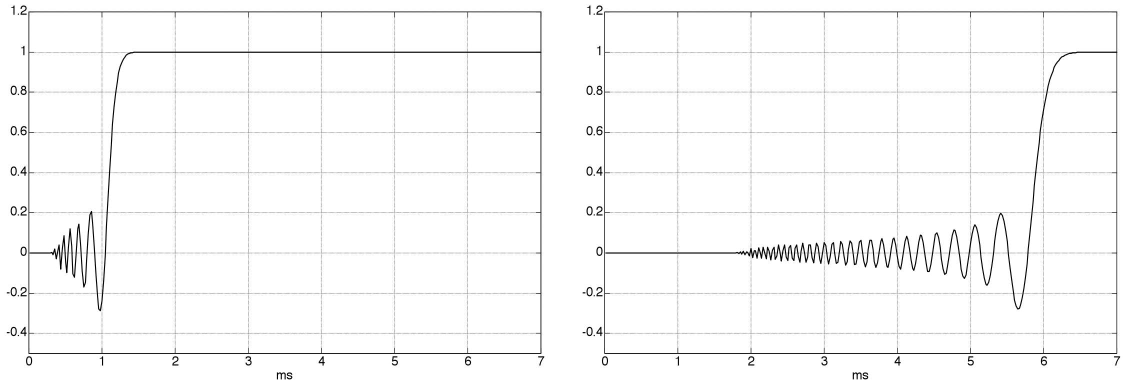

Fig. 2.43: Step response of a cascade of 14 (left) and 74 (right) all-pass filters. Data as in Fig. 2.42. In addition to the all-passes, a slight treble attenuation was included (*one* 1st-order low-pass at 10 kHz).

On the one hand, **dispersion** has the effect of a progressive spreading of the frequency of the partials. For the perceived sound, it is more important, though, that the FIR-filters depicted in Figs. 2.30 and 2.36 are subject to the same mechanism, as well: their interference effect happens progressively spread out towards higher frequencies. Given a dispersion-free E_2-string, the bridge pickup of a Stratocaster would feature an interference cancellation at $3 \cdot f_G \cdot$ 65cm / 5cm = 3214 Hz. However, your commercially available string is not free of dispersion, and therefore the interference cancellation mentioned above will happen somewhere in the range of 3330 – 3520 Hz, depending on the specific design of the string. In case the loudspeaker contributes narrow-band resonances in that same frequency range, a change of the make of strings may indeed bring audible differences. In this context, it should not be left unmentioned, though, that moving the guitar loudspeaker may well lead to changes in the sound: the room represents an FIR-filter, as well – due to the various occurring sound paths.

