2.9 Magnetic pickup with excitation by dilatational waves

Does an axial shift in the string induce an electrical voltage in the magnetic pickup? The distance between string and pole-piece of the pickup does remain constant, after all – which is why we would not expect any voltage. Measurements do not support this hypothesis, though. Apparently, the distance between string and pole-piece is not the only criterion for the generation of a voltage: due to hysteresis and associated memory processes, a dilatational wave running along the string may indeed induce a voltage in the pickup, as well. The following model considerations discuss the basic context:

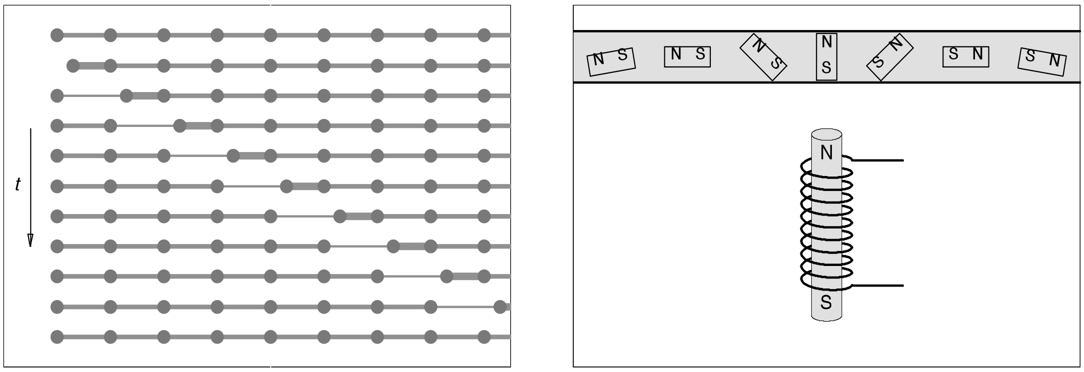


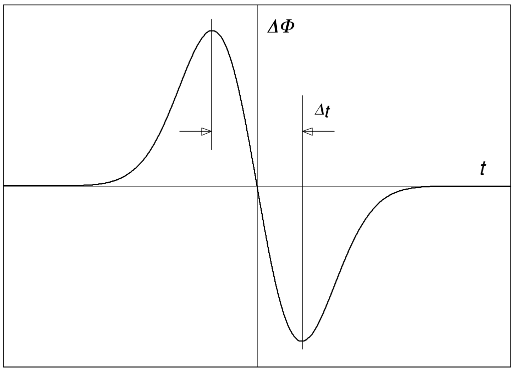
Fig. 2.44: Dilatational wave (left), string with elementary magnets and pickup coil (right). Both figures show considerably simplified, discretisizing models.

In the left-hand section of **Fig. 2.44**, we see a model of a string depicted at 11 different times; the bold points are masses, and in between them there are springs♣. On the top left, a compression impulse is generated that propagates along the string with progressing time (dilatational waves are generally free of dispersion). A pickup mounted beneath the string generates a permanent magnetization within the string – this is shown in the right-hand graph with a few elementary magnets. The dilatational-wave impulse sequentially shifts each of the elementary magnets: first a little to the right, then back to the original position. This shift varies the magnetic flux axially penetrating the coil. Looking at the right-hand graph seen in Fig. 2.44: for the elementary magnets shown towards the left, the variation of the location (resulting from the impulse) causes an increase in the magnetic flux penetrating the coil, and a decrease for the elementary magnets shown on the right.

The efficiency of the voltage induction caused by this effect depends on many factors: the magnet, the turns-number of the coil, or the material of the string. Of particular importance for the above model consideration are two parameters: the distance between the elementary magnets and the coil, and the angle between the axis of the elementary magnets and that of the coil. The compression impulse running along the string from left to right generates – in the coil – first an increase in the flux, and then a decrease. These variations in the flux induce an electrical voltage in the coil (law of induction: the voltage induced per turn of coil-winding corresponds to the temporal derivative of the magnetic flux penetrating this turn).

♣ The shown change in diameter is strongly exaggerated.

For a half-wave-shaped displacement impulse, **Fig. 2.45** presents the time functions of the flux change $\Delta\Phi$, and the corresponding temporal derivative. The graphs – put together from simple functions – are meant to merely familiarize us with the shape in principle; an exact calculation would require considerable effort. Given a geometric distance of the ranges of maximum sensitivity of about 1 – 2 cm (the typical dimensions of pickups), we obtain the distance in time of the extrema of about Δt = 2 – 4 µs (with a propagation speed of dilatational waves of about 5 km/s).



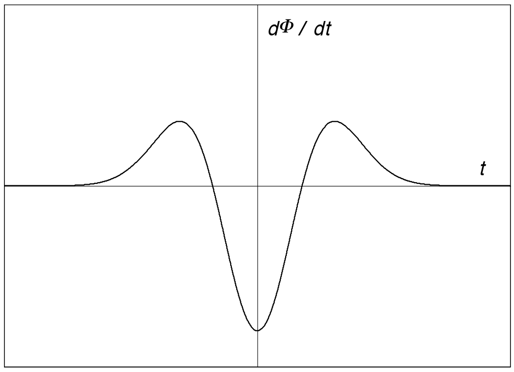


Fig. 2.45: Variation of flux (left) and its temporal derivative, caused by a compression impulse.

The signal shown in the right-hand part of Fig. 2.45 may be interpreted as impulse response $h_{U\xi}$. The first index (U) points to the pickup voltage U being seen as the output value that results from the differentiation of the magnetic flux. The second index ξ relates to the source signal: a *displacement* impulse. From the impulse response $h(t)$ of an LTI-system [6, 7], and using the help of a Fourier-transformation, we arrive at the transfer function $\underline{H}(j\omega)$ of this system♣. Herein, input and output signals remain the same; they are merely represented in different "domains": the impulse response connects (via the convolution) the input- and the output-**time-function**, and the transfer function connects (as a multiplication) the input- and the output-**spectrum**. The Fourier transform of the impulse response $h_{U\xi}$ is therefore the transfer function $\underline{H}_{U\xi}$. Model-considerations for equivalent circuits have, however, shown that $\underline{H}_{Uv}$ represents the more easily interpretable transfer function (Chapter 5.9.3), rather than $\underline{H}_{U\xi}$. Instead of the displacement impulse, a (*particle-*) *velocity* impulse is applied as trigger of the dilatational wave (the corresponding displacement function is the step). Instead of exciting a dilatational wave within the string with an excitation impulse, the temporal integral (the displacement step = velocity impulse) of the dilatational wave is impressed optionally. This additional integration is taken into consideration in Fig. 2.45 by requiring that the induced voltage shown in the right part of the figure is subject to an integration (commutativity of LTI-systems). Since the right-hand part of the figure was derived from the left-hand part via differentiation, we can use the left-hand graph to establish the time-course of h_{Uv} – merely the units are different. The following summary results:

A dilatational wave resulting from a displacement impulse induces the pickup voltage shown in the right-hand graph of Fig. 2.45. A dilatational wave resulting from velocity impulse induces the voltage shown in the left-hand graph of Fig. 2.45.

♣ The additional low-pass filtering occurring in pickup and cable is ignored here to begin with.

The Fourier-transform of h_{Uv} (i.e. the transfer function $|\underline{H}_{Uv}|$) is depicted in **Fig. 2.46**. We can see a frequency-proportional rise in the frequency range particularly relevant for the magnet pickup (< 10 kHz) – corresponding to the magnitude frequency-response of a differentiator. It becomes clear that the exact shape of the h_{Uv}-curve is of minor importance: any (odd numbered) origin-symmetric impulse response will exhibit the characteristic of a differentiator in the low-frequency range. Due to the high propagation speed of the dilatational waves, the maximum of the transfer characteristic is located at such high frequencies that its specific range does not need to be determined.

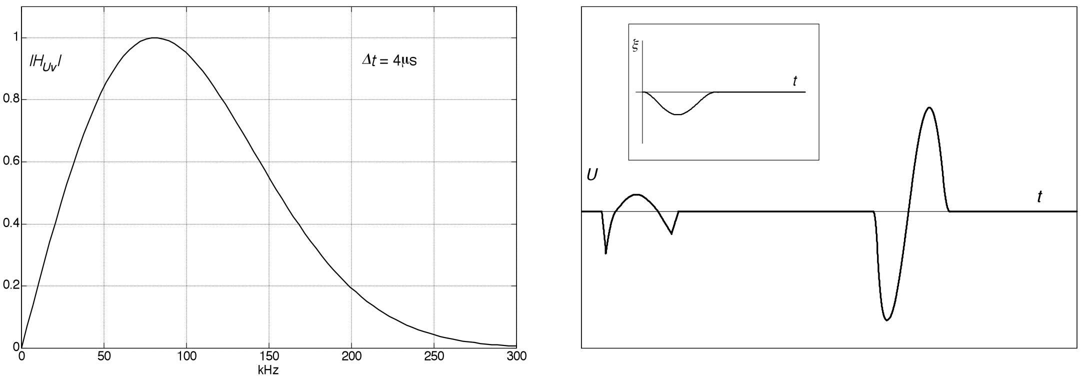


Fig. 2.46: Magnitude of the transfer function of (particle) velocity → voltage, without LC low-pass (left). Voltage induced in the pickup: string-excitation by a drop hammer, w/out dispersion, w/out LC low-pass (right).

Excitation of the string via a **drop hammer** generates two subsequent impulses in the pickup winding: first, we get the impulse induced by the dilatational wave, and then the impulse induced by the (slower) flexural wave. If at first the dispersion (that occurs only for the flexural wave) is disregarded, a voltage shaped similarly to the one shown in Fig. 2.46 would be expected. A displacement impulse shaped similar to a sinusoidal half-wave (small figure) runs along the string both as a (simplified) dispersion-free, slow transversal wave, and as fast dilatational wave. The first temporal derivative of this impulse corresponds to the voltage induced by the transversal wave; the second derivative corresponds to the voltage generated by the dilatational wave. However, the **dispersive propagation** of the flexural wave leads to a considerable reshaping of the impulse. Therefore, the shape of the voltage shown in Fig. 2.46 on the right will not occur during measurements in reality. Rather, all-pass-induced impulse deformations appear (Chapter 1.3.2, Chapter 2.8.4). In order to be able to compare the above theoretical model-calculations with measurements, the transversal-wave impulse (that looks similar to a full sine oscillation) needs to be first sent through an **all-pass filter**.

The measurements used for comparisons in the following were done using a 30 m long string of 0,7 mm diameter mounted below a Jazzmaster pickup. Due to its very low winding capacitance, and given suitable electrical loading, this pickup allows for broadband measurements up to about 20 kHz. While not exactly typical for use in electric guitars, the corresponding circuitry is highly qualified for measurements. At 3 mm from its mounting point (clamp), the string was excited by a short displacement impulse, leading to the propagation of a dilatational and a flexural wave along the string. At a distance of 68 cm from the clamp, the transversal velocity was sampled both with a laser vibrometer and with the Jazzmaster pickup, and the resulting signal was digitally stored.

In **Fig. 2.47**, we see on the left the transversal velocity measured by the laser vibrometer. The dilatational-wave impulse reaches the measuring point 0,13 ms after the drop hammer has struck the string – this instant represents the origin of the time-scale. The laser vibrometer practically ignores the dilatational-wave impulse; the pickup, however, shows an impulse that resembles a twice-differentiated sinusoidal half-wave impulse (Fig. 2.46). After about 1 ms, the high-frequency components of the flexural wave reach the measuring point, and the low-frequency components follow after about 6 ms (dispersive propagation); these waves are received by both sensors in a similar way.

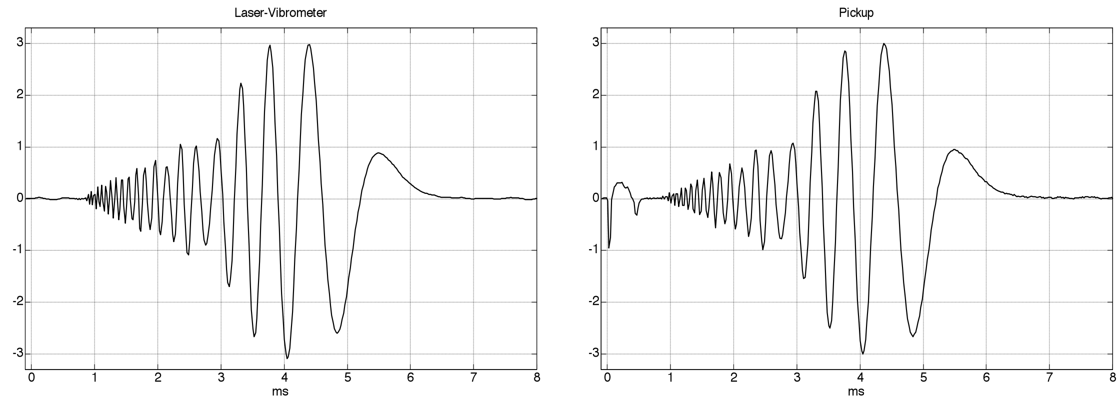


Fig. 2.47: Time function measured after impulse excitation of the string; laser (left), pickup (right).

From the point of view of **systems theory**, the tensioned string represents – with good approximation – an LTI-system that maps input quantities onto output quantities. A separation according to the two wave types yields two sub-systems: a dispersion-free delay line (dilatational wave), and a dispersive delay line (flexural wave). **De-convoluting** the output quantity of the system measured at the pickup gives the input quantity of the system. The effect of this de-convolution is shown in **Fig. 2.48**: of the pickup voltage indicated on the right in Fig. 2.47, the time-snippet between 1 ms and 7 ms was de-convoluted with the impulse response of the all-pass (Chapter 1.3.2). The result was drawn into the right-hand half of the left-hand section of Fig. 2.48; for comparison, the original dilatational-wave impulse is presented on the left. The part of the figure on the right shows the twice-integrated functions corresponding to the displacement. While the curves juxtaposed in the figure are not identical, they still are very similar – this could not be expected given the original functions that are, after all, of an entirely different character.

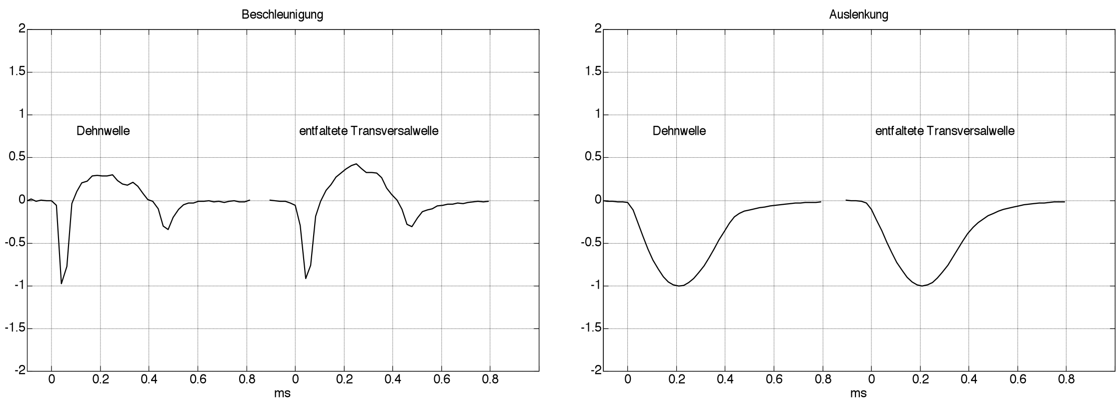


Fig. 2.48: Comparison between measured dilatational-wave impulse and de-convoluted flexural-wave impulse. “Beschleunigung” = acceleration; “Auslenkung” = displacement; “Dehnwelle” = dilatational wave; “entfaltete Transversalwelle” = de-convoluted transversal wave.

The pronounced similarity of the shape of the curves presented in Fig. 2.48 leads to the following conclusion: **dilatational wave and flexural wave have approximately the same time-function at the moment of their formation**. This hypothesis may be further corroborated via mapping the dilatational-wave impulse onto the flexural-wave impulse. For this, the section from 0 ms to 1 ms of the impulse shown in Fig. 2.47 on the right is integrated and convoluted with the impulse response of the all-pass: the signal depicted on **Fig. 2.49** on the right results. This latter signal corresponds with good approximation to the signal of the flexural wave (on the right in Fig. 2.47; repeated in Fig. 2.49 on the left). An example for which measurement and calculation correspond even better still is given in **Fig. 2.50**.

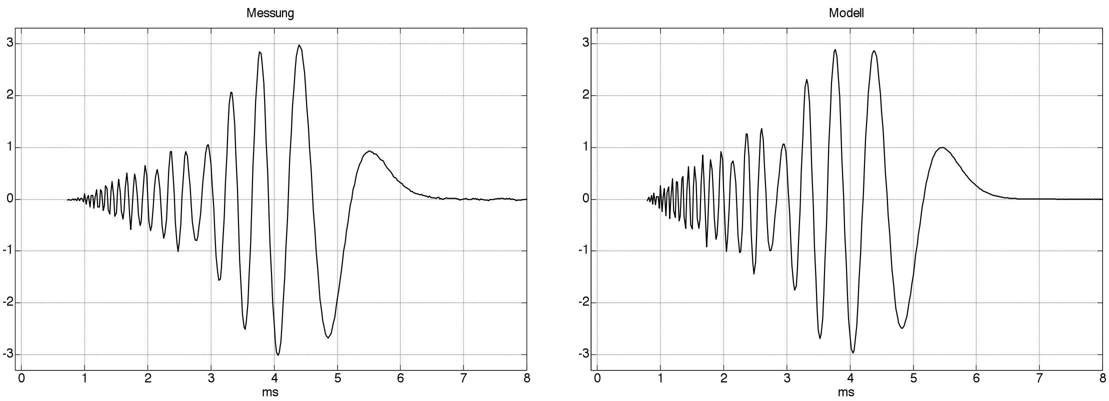


Fig. 2.49: Pickup voltages: flexural wave (left); impulse derived from the dilatational wave (right).

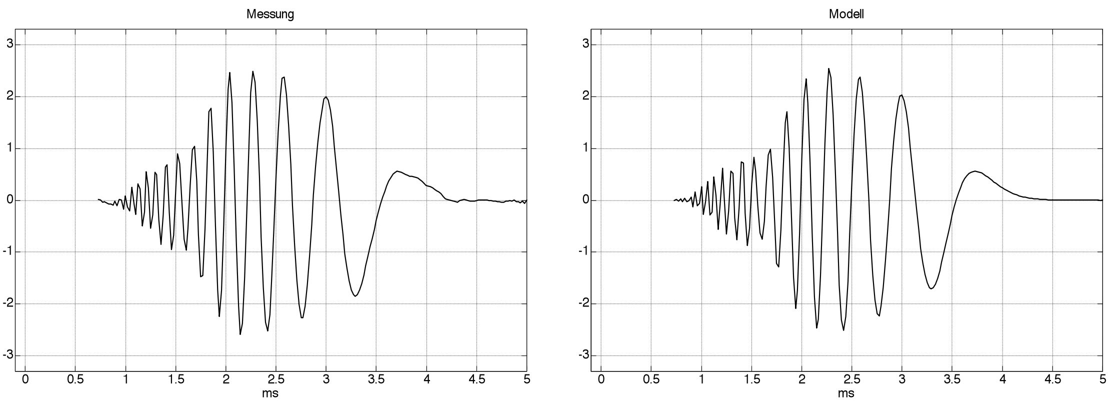


Fig. 2.50: As in Fig. 2.49, but established at a different pickup-position (55 cm instead of 68 cm).
"Messung" = measurement, "Modell" = model.

There is no absolute scaling of the ordinate in the above figures – for that a transfer coefficient for the individual pickup would be required. To get an impression of the wave-parameters, the following table lists typical (rounded-off!) values. The relationship of the two wave energies depends on the respective string bearing.

	Flexural wave	Dilatational wave
Maximum displacement	30 µm	5,7 µm
Maximum (particle) velocity	0,4 m/s	0,07 m/s
Maximum force	0,2 N	1,2 N
Wave impedance	0,5 Ns/m	17 Ns/m
Maximum power	88 mW	88 mW
Impulse-energy	8,0 µWs	8,5 µWs

