3. Magnetics of the string

In order to be able to change the magnetic resistance in the magnetic circuit, the vibrating string needs to consist from ferromagnetic material. Ferromagnetics come in great variety – if they are to be suitable as basic material for guitar strings, one feature is a predominant requirement: they have to withstand the extremely high tensile stress. Just about every guitarist will have broken a string during play at least once; that clearly shows how close to the limit we are operating! Typical tensioning forces of strings fall into the range of between 50 N and 140 N. Given the rather small cross-sectional areas this implies **tensile stresses** of up to 2000 N/mm^2. Given to such high stress, only high-strength ferromagnetic **special steel** qualifies as material for strings. As a protection against corrosion, the surface of the string is usually coated with a thin layer of nickel or gold; this layer has no magnetic effect due to its small thickness. Wound strings behave differently: their core diameter is about 30 – 60% of the overall diameter, with the **winding** consequently giving a substantial contribution to the cross-sectional area (the latter growing with the square of the diameter). Testimony to this issue is the effect we get when trying to use – on an electric guitar – strings with steel core and bronze winding as manufactured for acoustic guitars. Compared to the solid treble strings, such wound strings are picked up with too little volume – because bronze is not magnetic. The three bass strings of the electric guitar (E-A-D) are therefore wound with a magnetically conductive material: usually with nickel, nickel-plated steel, or special non-corrosive steel. In the following paragraphs, the magnetic properties of typical steel strings are discussed. Subsequently, **Chapter 4** will contribute a detailed description of magnetic fields.

3.1 Steel, nickel, bronze

High tensile strength requires a smooth surface because cracks and pores would increase the risk of breakage. As a protection against corrosion, the string surface may be coated (TINNED MANDOLIN WIRE); there are also uncoated strings, though. “Tinned” does not compulsorily imply that the surface is coated with *tin*: in fact the coating of typical guitar strings is formed of nickel (NICKEL PLATED STEEL). The two highest treble strings (E_4, H_3) are always solid (PLAIN), and the three bass strings (E_2, A_2, D_3) always sport a winding (WOUND); the G-string is solid (plain) in light string sets, and wound in heavy ones. The winding does not absorb tensile forces but merely serves to increase the mass. Other than steel, less stress-resilient materials may be used for the winding, as well.

Without doubt, the material of the strings does influence the **sound** of the guitar. The reason for this is, however, not that self-evident. Obviously, we will think of the inner damping of the material. When bending steel, nickel, copper, or other metals, different amounts of energy are converted into heat (dissipated). The decay of vibration therefore is material-dependent. The differences between the customary metals are, however, not pronounced to the extent that an audible difference in sound will result in tones of short duration.

The main effect results from the string bouncing off the frets. Even with regular strength of plucking/picking, the string will hit and bounce off the frets many times (Chapter 7). In this process, the winding (or coating) acts as elastic and therefore sound-determining buffer between fret and core of the string. An exact description of the **string-bounce process** is only possibly with a very high effort: each individual string/fret contact is a non-linear occurrence that will rule out the otherwise so helpful principle of superposition. The great number of these non-linear contacts can only be described in a non-linear, stochastic model – which would include a frightful variety of parameters.

Every string/fret-contact implies a mechanical impact. Mechanics know two kinds of impacts: the elastic one, and the inelastic one. For the **elastic** impact, there is no generation of thermal energy during the contact phase – it is termed the **loss-free** condition. However, this does not mean that the string is not loosing any energy, but only indicates that the sum of the energy in both partners involved in the impact is constant! The vibration energy transferred to the fret is lost to the string at first: the string experiences a damping from the elastic impact. Also, we may not expect that the vibration energy stored in the fret is re-transferred to the string later – in fact, a substantial portion of the energy is lost in the fretboard and the neck of the guitar. Given an **inelastic** impact, energy is dissipated irreversibly already in the deformation of the material during the impact phase, i.e. it is irretrievably converted into caloric energy.

Each contact between string and fret is also a source of two fresh **secondary waves** running in opposite directions. The energy contained in these secondary waves is not introduced to the system from the outside but withdrawn from the original wave-energy. *After* each contact, the system is again a linear one, and all waves may be superimposed. The contact phase itself, however, is a non-linear, drive-level-dependent process that cannot be described via superposition. The multitude of contacts renders the system non-linear during the first 10$^{\text{ths}}$ of a second; only the subsequent decay process is linear.

A string/fret contact (other than where the string is actually fretted) may only be avoided with very slight plucking of a (normally adjusted) string; in this case every analysis shows that the levels of the higher-frequency partials decay substantially faster than the low-frequency ones. The short impact of the string on the fret during the string-bounce represents a broad-band excitation that "refreshes" the treble, in a manner of speaking. Instead of being plucked one single time, countless "pickings" rain down on the string and make for a treble-rich, brilliant sound.

Auditory experiments with a E_2-string confirm this hypothesis: between a string wound with *nickel-plated steel* (Fender 250) and *pure-nickel-wound* string (Fender 150), there is a just-about significant, noticeable difference. However, raising the height of the bridge to the extent that any post-plucking string/fret-contact is avoided makes the two string-types sound the same. It needs to be emphasized here that the string/fret contacts are not generally perceived as string-buzz or clatter. Rather, these contacts merge, as auditory events, to a single homogenous plucking sound (ATTACK), as long as the contact noises do not dominate too strongly, or are audibly modulated by low-frequency components. Each string/fret contact transforms part of the low-frequency vibration energy into high-frequency vibration energy; therefore the attack of "bouncing" string sounds more trebly. Nickel – as a material that is the softer compared to steel – at the same time absorbs more of this add-on treble, and therefore nickel-wound strings have a sound not quite as brilliant as steel-wound strings.

On guitars having a piezo pickup mounted rather than a magnetic pickup, the magnetic conductance of the string winding does not play any role. Strings for these guitars therefore typically sport a winding made from brass or bronze. What again holds: harder, low-loss winding materials result in a more brilliant sound; softer winding materials also sound “softer”, i.e. not as brilliant.

The “**Zebra**”-strings made by DR with their double-start winding represent a peculiarity: they are manufactured with two different winding threads positioned next to each other. The bronze-wire is supposed to generate the sound typical for flattop steel-string guitars, the steel-wire is supposed to score with the magnetic pickup (see Chapter 3.2).

"Every other coil is nickel-plated steel, every other coil rare phosphor bronze, wound on hex cores", it says in the Internet ad. Only on the packaging we then read: "...by winding phosphor-bronze plated steel wire side-by-side with 8% nickel plated steel wire. Phosphor-bronze brings out the acoustic tones of your guitar. 8% nickel plated steel is designed to increase the response of a Piezo pickup in the bridge, or a magnetic pickup mounted in the soundhole, as well as the pickups in the archtop guitars." Nickel for the piezo? Be that as it may … However: a bronze wire, as it is customary for an acoustic guitar, turns into a bronze-coated steel wire. To meet the cosmetic expectations, the flimsiest of coating is sufficient … there’s that reddish gleam. It musn’t be much more, either, because bronze is a magnetic insulator! Just imagine that across the winding, an electric current would have to flow (along the string) … and then the guys wind around the core once bare copper wire, and alternately a combination of copper wire and enameled copper wire. This example speaks for itself. While bronze is not a perfect magnetic insulator, it still is less efficient than steel or nickel by several orders on magnitude. **Fig. 3.1** shows the approximate shape of the magnetic flux – strongly simplified in order to keep the calculation effort at bay. Finding: the magnetic resistance of the winding is determined predominantly by the surface touching the winding (Hertzian stress). In this range, the flux density is high, the material is magnetically saturated, and the exact calculation proves time-consuming.

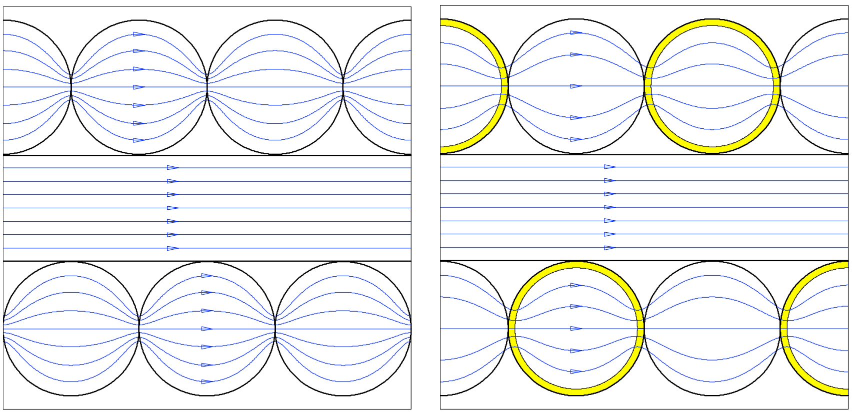

Fig. 3.1: Magnetic flux in a wound string. Single-layer winding (left), double layer winding with a bronze-coated winding wire (right). The lines of flux are not calculated precisely; in a real string, core and winding influence each other mutually.

Measurement on a 0,042"-**Zebra-string** showed that it is less sensitive by 2 dB compared to a steel-wire-wound 0,042"-Fender-string (Type 350). The core wires of both string have the same diameter and the same magnetic properties – the difference results from the winding exclusively. If one to the two winding wires were indeed made from solid bronze, the magnetic efficiency of the remaining other winding wire would practically disappear. Whether bronze-*coated* steel wire actually has a significant influence on the acoustic sound … that would be a topic for more extensive experiments. The issue was not looked into, though.

Unfortunately, not all manufacturers of strings give information regarding the actual build of their strings. Tom Wheeler uses the heading "Welcome to Fantasyland" for the chapter on strings in his reference oeuvre "Guitar Book". And he continues: “Advertisements for string often bristle with misleading information; one almost forgets that the only serious path to a good sound is paved with auditory experiments”. Indeed – it ain’t easy. Gerken at. al opine: “phosphor-bronze strings sound a little more mellow that 80/20 bronze or brass strings”; in Day et al., it conversely reads: “Phosphor-bronze sounds more brilliant than bronze”. Both books were issued (in Germany) by the same GC-Carstensens publishers within only 2 years.

Often, the declarations about materials used flounder on the marketing primacy: brass (which is a copper-zink-alloy), for example, turns into “bronze”. The reason might simply be that brass is also the term for horn instruments … as played in that other kind of “band” … the one in the football stadium. Do guitarists seek association with that scenario? Probably not, the contrary may actually be true. *(The translator recalls Pat Metheny’s “Forward March” here ...)* So: “bronze” rather than “brass”. This ab-use has even migrated in German guitar-“literature”. Now, how do you call the winding made of “real” bronze (a copper-tin-alloy), then? Right: name it “bronze”, as well! Or maybe “phosphor bronze”, to distinguish it from the (boring) other “bronze”. Come to think of: the mentioning of phosphor is not necessarily off, because bronze tends to become porous … indeed, phosphor is added: has a cleaning effect and reduces the porosity, and the high hardness of Cu_3P brings more brilliance to the sound. How much P the manufacturers add – that remains shrouded in the mystery that is string marketing.

Similar vagueness is found in “pure **nickel** strings”. Strings made from pure nickel could never, ever withstand the high tensile load – you <u>have</u> to use steel. Only the surface (nickel plated) or the winding (nickel wound) may consist of nickel. The winding may be made from pure nickel or form nickel-coated steel. The manufacturers are reluctant to hand out the specifics, though. Only the advertisement for most recent development is clear about which side one’s bread is buttered on: “special strings for lefties” …

3.2 The loudness of the strings

If you exchange on your guitar the 009-string-set for an 011 one, will it sound louder? Practical experience says: yes – theoretical considerations advise caution, though. First, we should look at a meaningful intermediate quantity rather than the loudness that is difficult to establish. Using the AC-component of the force at the bridge (acoustic guitar, pickup built into the bridge) come to mind, or the induced AC-voltage (magnetic pickup). Keeping the boundary conditions constant (!), there is no way around realizing that neither the bridge-force, nor the pickup voltage includes any dependency on the string diameter.

The **force at the bridge** first: the excitation force transferred to the string as it is plucked may be modeled as sum of two sub-components of equal value causing transversal waves running in opposite directions (Chapter 2). These two waves superimpose at the bridge with equal phase: the force at the bridge (only the AC component is of interest here) thus corresponds to the plucking force – that’s independent of the string diameter. Still, the diameter of the string has an effect on the sound because it affects the transverse stiffness (see appendix), and thus the displacement of the string The heavier the string, the larger the plucking force for a given displacement can be, and the louder the guitar will sound – if the guitarist takes advantage of this. With *equal* plucking force, heavier strings bounce less (Chapter 1.5.3) and sound fuller. We could have analyzed the dependency of internal damping mechanisms and radiation losses on string diameter – but that had less priority and was put on the backburner.

In the **magnetic pickup**, the vibrating string induces an electrical voltage that is proportional to the velocity of the string. Redoubling the amplitude of the string displacement leads to double the velocity and thus to double the induced voltage – at least as long as we take the linear model as a basis. However, a number of other factors enter into the transfer coefficient of a pickup, as well: winding- and magnet-parameters, the distance between string and magnet, the direction of the string vibration, and the string diameter … to name but the most important ones. Our first considerations are directed to the induced voltage and its level.

The dependency of the pickup voltage on the diameter of the string was experimentally determined using a test bench fitted with a shaker. For all measurements, a Stratocaster pickup was deployed, with a string being sinusoidally moved up and down over its D-magnet at a frequency of 85 Hz. The direction of the vibration was along the axis of the magnet, with a displacement amplitude of 0,22 mm. Varying the amplitude between 0,15 and 0,50 mm gave no indications of any substantial non-linearities: the voltage remained proportional to the displacement in this range. The clear width between magnet and string was 2 – 5 mm; no abnormities could be detected for these distances. The pickup-voltage level changed with about 2,1 dB/mm for light strings, and with about 2,7 dB/mm for heavy strings. Solid strings with diameters between 0,23 and 0,66 mm yielded proportionality between pickup voltage and **cross-sectional area of the string**. Redoubling the string diameter quadruples the output voltage (all other parameters remaining constant).

The proportionality between voltage and cross-sectional area only holds for solid strings, though. In **wound strings**, the winding is magnetically not fully effective. In the experiment, the core wires of Fender strings type 150 (pure nickel wrap), type 250 (nickel plated steel wrap), and type 350 (stainless steel wrap) were compared. The core wires are hexagonal with a diameter of about 0,4 mm. In terms of figures, the winding increases the cross-sectional area by a factor of seven – the measurement shows merely double the voltage, though, for the core with winding compared to the core without winding.

Fig. 3.2 explains why the winding is so inefficient magnetically: the individual layers only touch at narrow fringe areas, and this is what predominantly determines the magnetic resistance (Hertzian stress). While a part of the magnetic flux will find its way without air gap via the helix-shaped path along the winding, this path is much longer and shows, relative to the core, a magnetic resistance larger by a factor of 10. The magnetic effectiveness of the winding depends, other than on the permeability, also on the mechanical tension in the winding. If all windings are densely and tautly placed next to each other, larger areas of contact result, with the string representing a smaller magnetic resistance. The annular area marked grey in Fig. 3.2 is to be seen as an equivalent: a corresponding hollow cylinder would have the same magnetic properties as the winding (measurement results from the Fender strings).

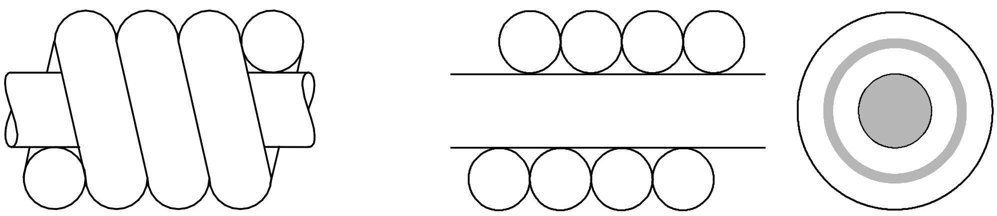

Fig. 3.2: Wound string: the areas indicated in grey on the right are magnetically effective (compare to Fig. 3.1) In contrast to the figure, the core of the Fender strings is hexagonal.

The **winding** of a string contributes to the sound in more ways than one: the *mass* of the winding increases the mass of the string, but it does so without substantially increasing the string stiffness. The *hardness* of the winding determines the harmonic content generated as the string bounces off the frets. The *magnetic* characteristics of the winding determine the (electric) loudness of the string. Now, **loudness** is a quantity that is not easily described and that depends on many parameters, e.g. on the levels of the partials that in turn may be traced back to the electrical partial-voltages generated by the pickup. Assuming a fretboard-normal string vibration, the voltage of the fundamental depends on the cross-section of the string, on the string velocity, and on the string-to-magnet distance. In the frequency range of the fundamentals, the transfer coefficient of the pickup is still substantially independent of the frequency and may be seen as constant (although it could well be modeled as frequency-dependent, see Chapter 5). The clear width between string and magnetic pole of the pickup is – for the time being – also seen as constant, so that merely string velocity and string cross-section remain as parameters to be considered.

The voltage of the fundamental is proportional to the particle velocity of the string (law of induction) and to the string cross-section (measurement results): $U \sim v{\cdot}S$. The string velocity depends on the fundamental frequency and the string displacement, the latter being traceable back to plucking force and transverse stiffness s_Q. For a constant distance to the bridge, the transverse stiffness is directly proportional to the tensioning force of the string. This force has similar values for all 6 strings.

Assuming a constant plucking force, we obtain for the string displacement ξ:

$$\left.\xi = F/s_Q\,; \qquad s_Q \sim \Psi; \qquad \Psi \sim S \cdot f_G^2\,; \qquad\right\} \qquad \xi \sim \frac{1}{S \cdot f_G^2}$$

The string velocity is proportional to the product of displacement and frequency. What therefore remains for the tension is a simple frequency dependency that is independent of the cross-section:

$$\left.v \sim \frac{1}{S \cdot f_G}; \qquad v \cdot S \sim 1/f_G; \qquad U \sim v{\cdot}S \qquad\right\} \qquad U \sim 1/f_G$$

If all 6 strings on the guitar were solid, and given the above conditions, the E_2-string would generate the quadruple voltage relative to the E_4-string. Because in each string the second harmonic is of double the frequency of the fundamental, the same relationship would be found here, as well. This simple consideration may not readily be transferred to *all* partials, but we can already say without diving into the depths of loudness-calculation that the bass-strings would be too loud in comparison to the treble strings. However, the wound strings are magnetically less efficient than the solid treble strings, and therefore all strings generate (via pickup, amplifier, and loudspeaker) a similar loudness as a first approximation.

Fig. 3.3 presents the dependency of the level of the fundamental on the frequency. This graph may serve as rough orientation regarding the loudness of the strings (although of course loudness and level are two different quantities). If all strings were solid, the dashed $1/f$-line would result. The measurement values (gathered with a Fender 150 string set: 042-032-024-016-011-009) are indicated as the bold line. All measurements were performed over one and the same magnet of a 1972-Stratocaster-pickup. The figure on the right shows the results taken from a typical bronze-wound string set (again measured with the Stratocaster pickup).

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Fig. 3.3: Level of the fundamental of the 6 guitar strings (magn. pickup). Winding: nickel (left), bronze (right).

The ratio of core diameter to overall diameter presents a significant parameter of the wound string. For E_2-strings this ratio is in the order of 0,33, for G-strings rises to about 0,6 – with manufacturer-typical variations (Chapter 1.2). The winding-inefficiency is predominantly due to the geometry and can therefore not be influenced much via ferromagnetic parameters.

Comparative measurements on Fender E_2-strings of the types 150 (nickel-wrap), 250 (nickel plated steel-wrap), and 350 (stainless steel-wrap) yielded comparable voltage levels for the 150 and 350 types, with the type-250-string generating 1 dB more relative to this. About half of this efficiency increase could be attributed to the slightly larger core diameter. An unobtainably high precision would have been required to exactly research the underlying reasons: for a measurement accuracy of 0,1 dB, the core diameter would have to be determined (and maintained) with a precision of 0,6% – for a core diameter of 0,4 mm this implies a tolerance of 2,4 µm! Furthermore, the distance between string and magnet would have to be adjusted with a precision of 40 µm. While the latter requirement appears doable, it is certainly not trivial given a test bench made entirely of plastic components. Therefore, **tolerances** of some 10^{ths} of a dB have to be expected for all statements regarding levels.

The pickup-industry has already early on attended to the variations on string gauges; adjustable or different-length magnets were included in the pickups (**staggered Magnets**, Chapter. 5.4.6). However, apparently the differences are judged to be more on the insignificant side, because in many magnetic pickups the 6 magnets protrude to the same extent from the pickup housing. Be warned about unauthorized modifications, though: it is not advisable to move the magnets in old Fender pickups – the fine-as-a-hair winding wire is in direct contact with the magnet and can be damaged very easily. In modern pickups with a plastic bobbin, shifting the magnets should be possible but even in this case a consultation call with the manufacturer might be a wise idea.

Supplementing the measurement with the shaker, the levels of the strings were also subject to an auditory evaluation. A well-versed guitarist played a Stratocaster (flush pole-pieces) fitted with Fender 150 strings and did his best to pick the individual strings with equal force. With much effort, it was possible to detect any significant difference in the **overall level** between the D- and the G-string: the level of the G-string was about 4 dB higher relative to the D-string. Due to a lack of reproducibility, the level differences of the remaining strings could not be determined with sufficient accuracy. When playing regular lead and rhythm, differences were not noticeable. The D/G-difference was just about detectable – if one really concentrated on the task. However, as soon as the player directed his attention to the music to be played (this would be have to be seen as the normal approach), the differences between the strings did not stand out anymore. We did not further investigate whether there was any compensatory action in a senso-motoric control circuit, of whether the perceptional threshold had shifted.

3.3 Magnetic parameters of the strings

When it comes to strings, manufacturers swiftly turn into poets: "*Gleaming nickel squiggles around Swedish hex-steel and guarantees brilliant* (sic!) *tone with never-ending sustain. These are your weapons of choice to deal with any degree of overdrive and get assertive solo-sounds with bite at absolutely unbelievable killer distortion. Hotter'n Hell!*", opines **Gibson sales**. Which one of those not-so-few-anymore and probably not-quite-resting-in-peace deceased 6-string-slingers will have signaled this under-worldly temperature assessment to the ground floor?

It would appear that the required high breaking stress cannot leave a lot of latitude for differences in the magnetic parameters. The solid strings and the core wires of the wound strings differ only little when it comes to magnetics. Even the effects of different winding wires remain unspectacular: measurements with nickel-wound string (Fender 150) and steel-wound strings (Fender 350) show no difference when subjected to the shaker-equipped test bench. The string wound with nickel-coated steel wire yielded a level higher by 1 dB … but half of that effect is due to the somewhat thicker core wire. That does not mean that these strings must sound the same: the mechanical vibration-behavior may well differ – but the magnetic properties are still very similar, even if nickel and steel show different hysteresis curves. The core-characteristics are equal in all three string-types, and together with pre-magnetization- and saturation-effects this leads to similar magnetic parameters.

To measure these magnetic characteristics is not easy but still just about doable with sufficient precision – and with justifiable effort. Since every measurement process includes inadequacies inherent in the system, we will present – in the following paragraphs – several methods of analysis to gather the magnetic data of strings. An extensive presentation of electromagnetic fields follows in Chapter 4.

3.3.1 Measurements with the string-ring

Measuring magnetic parameters is complicated: the magnetic field in not homogenous, and there is a non-linear relationship between the field strength H and the flux density B. A substantial simplification can be obtained if the field-geometry can be shaped in such a way that it can approximately be seen as homogenous. An annulus-shaped (torus-like) examination piece that is completely wound with copper wire on its lateral surface will generate an azimuthal circulatory magnetic field. When described using cylinder coordinates, this field may be seen – in the space within the examination piece – as location-independent … at least as long as DC-current flows though the copper wire. Two challenges need to be mastered in this scenario: manufacturing a ring made of steel as it is used for strings, and the measurement of the magnetic flux density.

For the following measurements, guitar strings were wound to form a ring. Winding a string of a length of 85 cm into 6 turns yields a "string-ring" with a diameter of 4,5 cm. Start and end of the string should join up as much as at all possible to minimize the effects of the unavoidable air gap. The magnetically effective cross-sectional area of this ring is the 6-fold of the cross-sectional area of the individual string – in the case of a 17-mil-string this will give us an overall area of 0,9 mm^2. The ring as a whole is wound – along its 14-cm-long "core" – with a single layer of enameled copper wire ($\varnothing$ = 0,5 mm); in the present experiment, 239 turns were required.

The azimutal magnetic field strength H in the interior of this annular coil amounts to:

$$H = \frac{N_1 \cdot I}{\pi \cdot D}$$ Field strength in the annular coil

In this formula, N_1 is the number of turns of the primary coil (in our example 239), I is the excitation current, and D represents the diameter of the ring (45,8 mm). Given I = 5 A, we calculate H = 8,3 kA/m – this is a value sufficiently high for string-steel. In order to measure the magnetic flux density, a second winding is wound – as a secondary coil – onto the first one. In our example this has N_2 = 100 turns. Using AC-operation, an AC-voltage is induced into the secondary coil. This voltage depends – among other factors – on the change of the flux density B (law of induction, Chapter 4.10).

The voltage induced into the N_2 windings is $U = N_2 \cdot d\Phi / dt$. The flux Φ is calculated from the product of flux density and surface area. Because the string is – compared to air – the much better conductor for magnetic fields, we need to use (in this example) not the cross-sectional area of the coil but six-fold the cross-sectional area of the string used. For the sake of completeness is should be mentioned that this simplification reaches its limits as the magnetization approaches saturation. **Fig. 3.4** presents measuring results from a 17-mil-string. On the left we see the sinusoidal current (f = 10 Hz) and the impulse-shaped induction voltage. Since this voltage is the time-derivative of the flux density, it may be integrated to obtain B (right-hand graph). Clearly visible is the almost square-shaped B-curve that points to a pronounced saturation.

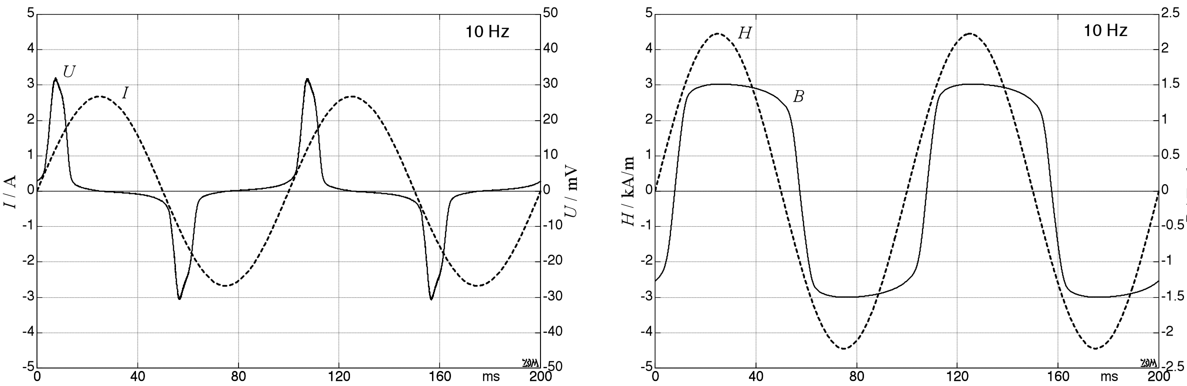


Fig. 3.4: Excitation current I and induction voltage U (left); Fields strength H and flux density B (right).

As we vary the **frequency** of the excitation current, shape and phase of the B-curve change, as well: evidently there are delays in the build-up of the magnetic field that could not be really expected given the low frequencies at work here. The reason for the delays is the **skin effect**: eddy currents weaken the H-field, and only as they decrease, the field can be built up to strength. The H-field reacts to changes in the current in a delayed fashion, and therefore the magnetic flux also reacts with a delay to such current changes (Chapters 3.3.2 and 4.10.4). To minimize the effect, all string-rings used were fashioned using lacquered strings – that way, eddy currents can circle only within the individual string (Figs. 3.5 and 5.9.17). To measure the hysteresis, eddy currents do not need to be determined quantitatively: it is sufficient to decrease the frequency in successive measurements until the differences become smaller than the envisaged measuring error. For this, imprinted voltage is more purposeful than imprinted current.

3.3.2 Skin effect in steel strings

As a string moves within the magnetic field of a pickup, its position relative to the pickup magnet changes. As a consequence, field strength and flux density within the string also change. A variation in the flux density will induce, in the electrically conductive string, an eddy current (**Fig. 3.5**), that itself generates its own magnetic field in opposite direction to the primary field. Because the strength of the eddy current depends on the *change* of the primary field, the primary field is more and more squeezed out of the string as the frequency increases. At high frequencies, a substantial magnetic flux is left merely in a thin outer layer (i.e. the skin) of the string. Therefore, the magnetic conductivity decreases with increasing frequency. This so-called skin effect is dependent on the basic magnetic conductivity of the material (a large μ results in a large B), and on the electrical conductivity (a large σ results in a large I). An extensive discussion of the skin effect will follow in Chapter 4.10.4.

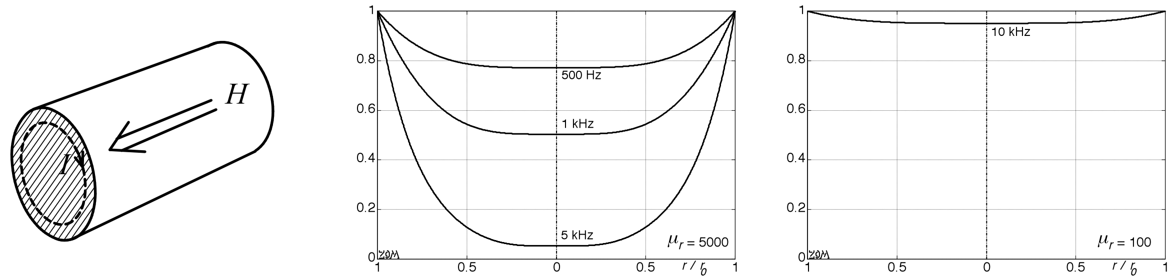


Fig. 3.5: Metal cylinder permeated axially by the magnetic field *H*, with eddy current *I* (left); radial distribution of the magnetic flux density in a 17-mil-string (middle). For μ_r = 100 (right), there is almost no field distribution: the magnetic flux density is practically independent of the location. Approximation: μ_r is constant.

Given a sinusoidal vibration, the temporal change of the flux density is particularly strong at the zero-crossing. At these instants, the magnetic field will therefore not be able to permeate the complete string material – there will be delay in the build up of the field. In the left-hand-section of **Fig. 3.6**, the hysteresis loop measured at 1 Hz is depicted; on the right we see the broadening at increased frequency. The skin effect is relevant if the whole hysteresis reaching into saturation is measured. Given the string vibrating over a pickup magnet, we find other conditions, though: within the string there is a strong DC-field with a rather small change superimposed. In this case it is not the differential permeability that is important, but the reversible permeability, the latter being much smaller in steel string than the differential permeability (μ_{rev} < 70, Chapter. 3.3.3).

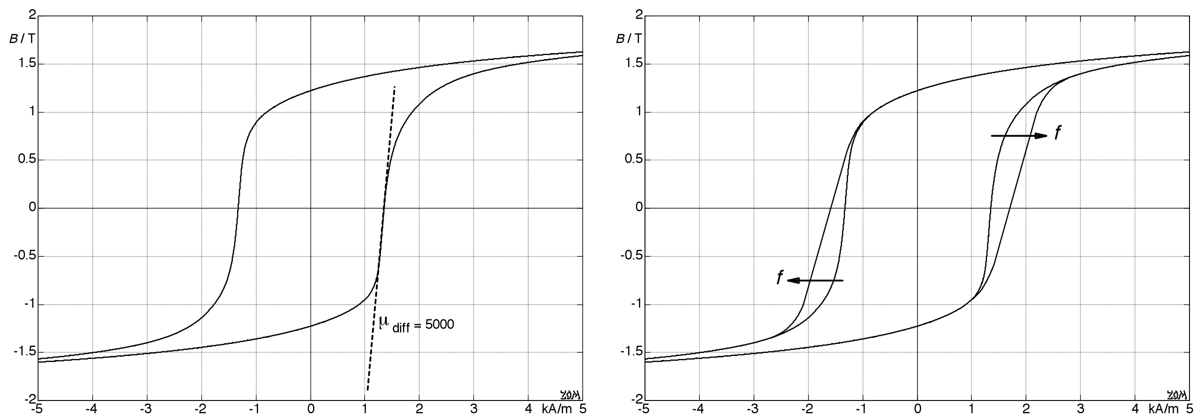


Fig. 3.6: Hysteresis loop, maximum inclination (left); frequency dependent broadening (right).

3.3.3 Reversible permeability

The connection between magnetic field strength and magnetic flux density is a non-linear one, and it is also dependent on the previous history: the hysteresis loop shown in Fig. 3.6 can only be run through clockwise (see also Chapter 4.3). However, for small variations around an operating point (DC-field and superimposed AC-field), the changes do not happen along a small section of the hysteresis curve, but on much more shallow curves. Their much less pronounced inclination (dB/dH) is the reversible permeability $\boldsymbol{\mu_{\mathrm{rev}}}$. **Fig. 3.7** shows measurement results determined with a string ring. A low-frequency sinusoid (1 Hz) forms the large drive signal, with a weak 266-Hz tone superimposed. The B-field does not follow the reversals in the drive signal on the large hysteresis but on the flat small lines (that in fact are lance-leave-shaped loops, as magnification would reveal). The gradient of these flat lines is highest for the flux density approaching zero and decreases as the magnitude of the flux density increases.

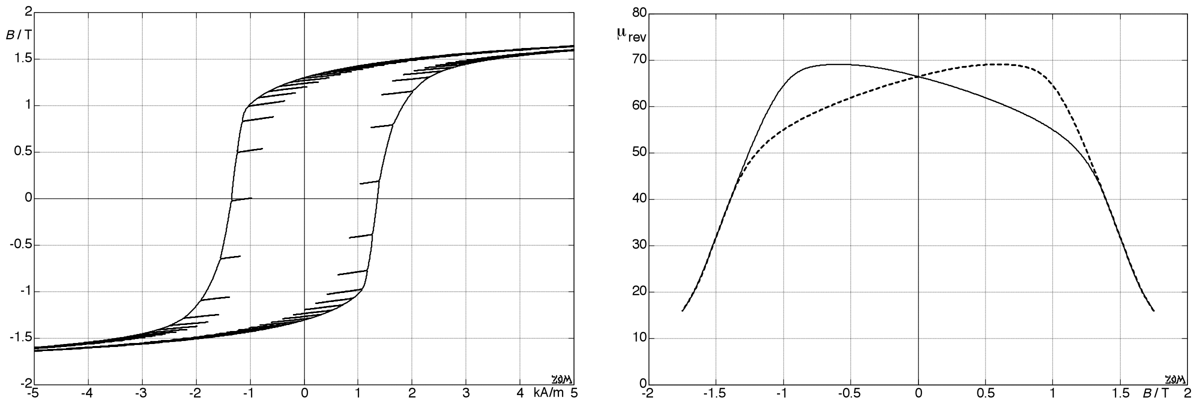
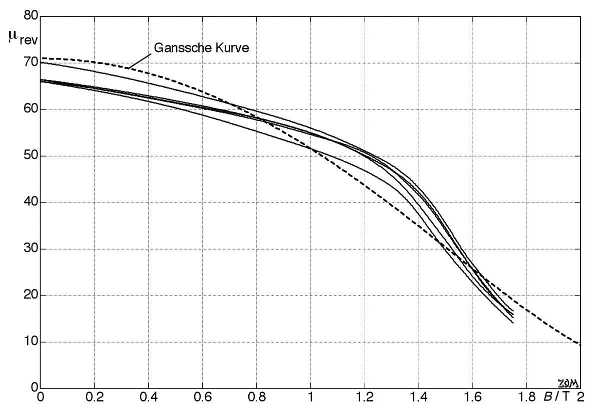


Fig. 3.7: Hysteresis curve, determined with a two-tone signal (1 Hz @ 0 dB; 266 Hz @ -32 dB). Right: slope of the flat lines shown dependent on the flux density, i.e. this is the reversible (relative) permeability. The dashed curve holds for the 'reversal' of the hysteresis i.e. for the upper branch of the hysteresis.

Already early on, R. Gans published a formula connecting B and μ_{rev}[♣]. It turns out, however, that this "**Gans-sian curve**" may only be regarded as a rough orientation; even the supposed independence of H is not present[⊗]. **Fig. 3.8** shows corresponding measurements taken with 5 solid strings in comparison to the "Gans-sian curve".



$$\frac{\chi_{\mathrm{rev}}}{\chi_{\mathrm{A}}} = 3 \cdot \left(\frac{1}{x^2} - \frac{1}{\sinh^2(x)} \right)$$

$$\frac{J}{J_{\mathrm{sat}}} = \coth(x) - \frac{1}{x}$$

'Gans-sian curve'; compare to Chapter 4.10.3

Fig. 3.8: Measured relative permeability, calculated "Gans-sian curve" (= "Ganssche Kurve").

♣ R. Gans, Annalen der Physik, **23**, p. 399; 1907.

⊗ H. Jordan, Annalen der Physik, **21**, S. 405; 1934.

3.3.4 Measurements with the yoke

Putting together a string-ring wound with two coils is highly time-consuming. For any investigations into the market, simpler measurement approaches would thus be desirable. In the following, the **test-bench utilizing a yoke** will be introduced: it employs a ring-shaped electromagnet with an air gap. The string to be measured is inserted into the latter.

To measure magnet parameters, advantage is taken of the **continuity conditions** that appear at boundaries as the field permeates them [e.g. 7]. At the string/air-boundary, the tangential component of the field strength H is continuous. Therefore, if the axis of the string is directed in parallel with the field lines within the air gab, the field strength H_i internal to the string corresponds to the field strength H_L within the adjacent air layer. The field strength in the interior of the string can therefore be determined without having to actually enter the string. To **measure** H_L, two coils of different diameter are wound around the string: a tightly fitting inner coil with the diameter D_1, and coaxially a second coil with the diameter $D_2 > D_1$. As a sinusoidal AC-flux Φ flows through the string, induction voltages are generated in both coils. These voltages depend on Φ, on the frequency f, and on the turns-numbers. If both coils feature the same number of turns N, opposite-phase connection makes it possible to compensate for and cancel out the part of the voltage that results from the magnetic flux flowing through the inner coil. As a consequence, the combination of the two coils measures only the magnetic flux in the *ring-shaped range* between the two coil surfaces. Using this approach, the field strength H_L in air can be determined via μ_0 (the known permeability of air). H_L corresponds to the axial field strength in the string (provided there is homogeneity).

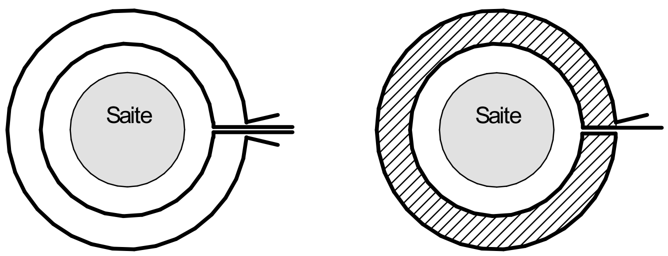


Fig. 3.9: Coaxial annular coil.
Left: two windings with 1 turn each.
Right: ring-winding for measuring H.

Fig. 3.9 presents a cross-section of the measurement setup. The magnetic field generated by an electromagnet (not shown in the figure) is directed perpendicularly to the viewing-plane. It runs in parallel to the string axis and permeates two coil-windings concentrically surrounding the string. The overall cross-sectional area is designated S_S, the cross-sectional area of the inner winding is S_1, and that of the outer winding is S_2. For reasons of clarity, each winding consist of merely one turn in the figure; in practice about 100 turns each yields a good compromise between sensitivity and (small) size. The number of turns of the two coils should be exactly the same♣; they are connected in opposite phase. Given this setup, only the magnetic field flowing between the two windings into the ring-surface forms a contribution to the induced voltage. In the right-hand part of Fig. 3.9, two ends of the windings are connected such that a winding W_H encompassing the ring surface ($S_2 - S_1$) results. The voltage induced in W_H depends, according to the law of induction, on the turns number N, and on the temporal change of the magnetic flux Φ_{Ring} permeating the ring surface. This flux is again a product of ring surface, magnetic field strength H, and the permeability of air μ_0.

♣ If both coil-voltages are recorded separately, correction can also be achieved via post-processing.

From the ring induction voltage U_H, the **field strength at the ring surface** H can be calculated:

$$U_1 = N \cdot d\Phi_1 / dt; \qquad U_2 = N \cdot d\Phi_2 / dt; \qquad U_H = U_2 - U_1 = N \cdot d\Phi_{Ring} / dt$$

$$\boxed{H = \int \frac{U_H}{N \cdot \mu_0 \cdot (S_2 - S_1)} dt}$$

$S_2 - S_1$ = ring surface, $\mu_0 = 4\pi \cdot 10^{-7}$ Vs/Am

Prerequisite for exact measurements is a homogenous H-field; with pole shoes of high magnetic conductance this can be generated with sufficient accuracy. The measuring coils can be wound with very thin wire, making small dimensions possible. About 100 turns will generate induction voltages in the range of 10 – 100 µV, which is comfortably measurable with a low-noise amplifier. Highly significant is avoiding measurement of external interfering fields (connecting lines, shielding, grounding!). In case not the voltage of the ring winding is recorded, but rather the individual voltages of the two coaxial coils, particularly high precision is required: the ring-voltage U_H results from the difference of two voltages that may potentially differ by a factor of 100. Any imbalance between the measurement channels (even as small as in the ‰-range) may lead to unacceptable errors.

On top of the **field strength at the ring surface** H (that approximately corresponds to the axial field strength of the string), the axial **flux density of the string** needs to be measured as the second field quantity. Magnetic flux in the string Φ and flux density in the string can be determined via the voltage U_1 induced in the inner coil. However, this involves a systematic error because the inner coil will not directly touch the string in a test-bench suitable for various string diameters. Instead of measuring only the part of the flux that flows through the string, a part of the flux that flows through the surrounding air is measured in addition. Given high permeability of the string, this error would possibly be negligible – but in the saturation range the string-permeability is precisely NOT high anymore, and the error would be inacceptable. Still, there is an elegant way to directly measure the magnetic **polarization** J of the string. J may be imagined as "material-bound part of the flux density". Given imprinted field strength H, the flux density $B_0 = \mu_0 \cdot H$ results in air. Introducing ferromagnetic material into this H-field will increase the flux density to $B = \mu_r \cdot B_0$. This is transformed to:

$$B = (\mu_r - 1) \cdot B_0 + B_0 = J + B_0 \qquad J = B - B_0$$

J = magnetic polarization

Thus J is the share by which the flux density increases (from B_0 to B), depending on the given material. Now the voltages induced into the windings W_1 and W_2 can be rearranged into:

$$U_1 = S_1 \cdot N \cdot \dot{B}_0 + S_S \cdot N \cdot \dot{J} \qquad U_2 = S_2 \cdot N \cdot \dot{B}_0 + S_S \cdot N \cdot \dot{J}$$

$S_1 \cdot N \cdot \dot{B}_0$ is the part of the voltage that would be induced into the inner coil if there were no string present. The part of the voltage delivered by the string is added as the second summand $S_S \cdot N \cdot \dot{J}$. In both voltage equations, $\dot{B}_0$ may be eliminated, and J can be calculated♣:

$$\boxed{J = \int \frac{U_2 - kU_1}{(k-1) \cdot N \cdot S_{\text{string}}} dt} \qquad k = S_2 / S_1$$

Given known geometry of the coils, the field strength and the polarization in the string can now be determined from the two coil-voltages U_1 und U_2.

♣ The letter J is – loco citato – also used for the electrical current density!

The accuracy for the *H*-measurement is determined by the ring voltage U_H and the area of the ring. The potential problems with forming the difference have already been noted. The ring area should be very small in order to capture exclusively the field in the air directly next to the string; this makes the precise determination of area difficult, though. A solution is the use of Helmholtz-coil enabling us to generate a highly accurate magnetic field and to calibrate the *H*-measurement that way. For establishing the value of *J*, especially the area-ratio $k = S_2 / S_1$ needs to be precisely known. Calibration is done without string: the value of *k* is corrected as necessary until *J* reaches zero. For the integration (which advantageously is performed with digitized signals in a simple manner), attention needs to be paid to extremely precise offset-compensation. If errors occur here, the hysteresis curve fail to close in the case of multiple revolutions; it will rather diverge, or be represented with the wrong width.

Fig. 3.10 shows measurement results of a “no-name” string that were gathered with the measurement setup as described above. The *H*/*J*-relation is typical for metals that are magnetically hard to a lesser degree. We obtain $_JH_C$ = 1,6 kA/m for the **coercitivity**, and we get J_R = 1,4 Tesla for the **remanence**. A comparison with “name products” (**Fig. 3.11**) indicates small differences regarding the magnetic parameters. The sources of these differences cannot be clarified unequivocally – it may be assumed, though, that the tolerances due to the test bench are in a similar order of magnitude. Let us therefore remind ourselves that measuring magnetic parameters requires much effort, and despite this effort they can only achieve a modest accuracy.

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Fig. 3.10: Hysteresis-loops measured on the yoke-test-bench for a “no-name” string (∅ = 0,43mm, plain). The measurement frequency (2 Hz) is sufficiently low for individual strings.

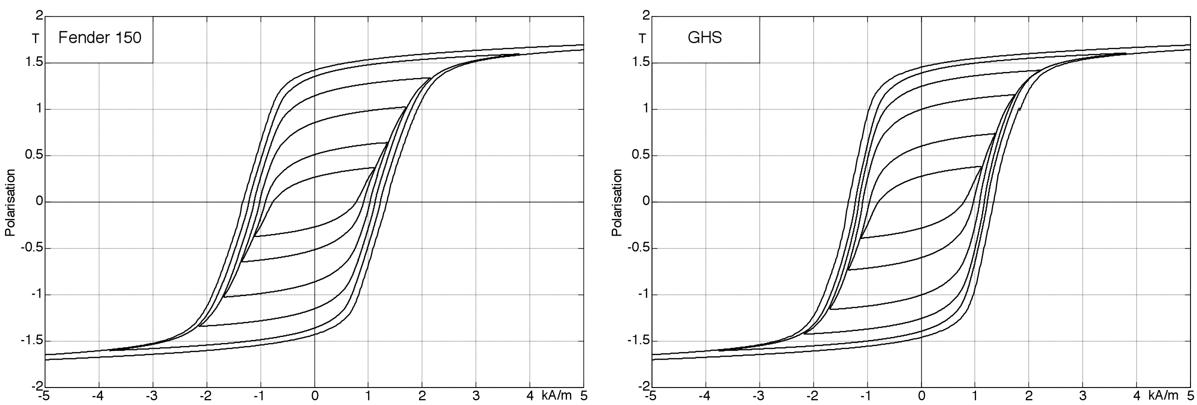

Fig. 3.11: Hysteresis-loops measured on the yoke-test-bench for “name” strings (∅ = 0,43mm, plain).