8.1.4 Intervals in the equal temperament

The interval (inter vallum = space in between) is the distance of two notes; expressed numerically by the relation (ratio) of the frequencies of the corresponding tones. The names of the intervals are derived from the place numbers within the scale – for the C-major-scale, this implies: C = prime, D = second, E = third, F = fourth, G = fifth, A = sixth, B = seventh, C' = octave. Between the 3rd and 4th notes, and between the 7th and 8th notes, we find a half-step, all other notes are a whole-step apart each. In the equal-temperament tuning, a **whole-step** consists of two equal-size **half-step** (**HS**). All intervals can be represented by multiples of a HS:

Distance between notes (intervals) in the diatonic scale, represented by half-steps:
C-C = 0, C-D = 2, C-E = 4, C-F = 5, C-G = 7, C-A = 9, C-B = 11, C-C' = 12.

Intervals are not just definable as HS-multiples in their relation to the root note C of the C-scale, but also between all notes: e.g. D-E = 2 HS, G-H = 4 HS, F-A = 4 HS.

By the subdivision of the whole-step into two half-steps, new notes are obtained; they are designated by the chromatic sign relative to their neighbors: C# = C-augmented-by-one-HS, and (in the equal-temperament tuning) identical to the D*b* = D-diminished-by-one-HS. Corresponding: D# = E*b*, F# = G*b*, G# = A*b*, A# = B*b*. Equating the diminished notes and the augmented notes (e.g. C# = D*b*) is called the **enharmonic equivalent** (or enharmonic ambiguity). Out of experience, it appears that guitar players are more familiar with the augment-sign (#) than with the diminish-sign (*b*), and therefore we will give the former priority in the following. From the 7-step diatonic scale (C-D-E-F-G-A-B), a 12-step chromatic scale emerged:

C – C# – D – D# – E – F – F# – G – G# – A – A# – B   chromatic scale

Each hyphen in this sequence represents a HS; the size of an interval can therefore be easily accounted for as HS-multiples. The regular numerals (second, third, fourth, fifth, etc.) are, however, already used (up) for the 7-step major (diatonic) scale, and this led to a somewhat confusing nomenclature: unison (0 HS, also called keynote or root), fourth (5 HS), fifth (7 HS) and octave (12 HS) are designated as **"perfect" intervals**, even if their tuning is not "pure" and free of beats! Caution is advised: C-G, for example, is designated a "perfect fifth" even in equal-temperament tuning. All other intervals within the major scale are "**major**" and thus: C-C = (perfect) unison, C-D= major second, C-E = major third, C-F = perfect fourth, C-G = perfect fifth, C-A = major sixth, C-H = major seventh, C-C' = perfect octave.

Reducing a large (major) interval by a HS results in a small (**minor**) interval. To get there, two possibilities exist: either the higher note is pushed down by a HS, or the lower note is pushed up by a HS: C-D*b* = C#-D = minor second, C-E*b* = C#-E = minor third, C-A*b* = C#-A = minor sixth, C-B = C#-H = minor seventh. If a perfect (or major) interval is enlarged by a HS we have an **augmented** interval; if a perfect (or major) interval is reduced by a HS we have a **diminished** interval. This results in two schemes:

diminished – minor – major – augmented   (second, third, sixth, seventh)
diminished – perfect – augmented   (unison, fourth, fifth, octave)

C-D# therefore represents an augmented second; in the sense of the enharmonic equivalent within the equal-temperament tuning, however, it also corresponds to the minor third C-E*b*. Purists turn away in horror, but the pragmatist just deals with it in everyday life: "C-D# is a minor third." Indeed, it is without purpose to ponder the differences between C# and D*b* when working with equal-temperament tuning. Of course, singers or violinists (as an example) will tend to intonate the augmented notes (#) slightly higher and the diminished notes (*b*) slightly lower, but that is then outside of equal-temperament tuning. When playing chords, the guitar player (and we are concerned with the associated instrument here, after all) has hardly any possibility to modify individual notes within the chord in their pitch. When playing single-note melody, higher-order knowledge of harmony could be put to use – unless the keyboard player in the band with his/her equal-temperament tuning shoots that down.

The following list gives an overview for **all intervals**, in this case referenced to C; with these representations: p = perfect, d = diminished, mi = minor, ma = major, a = augmented:

d-octave: C-C'*b*	p-octave: C-C'		a-octave: C-C'#
d-seventh: C-B*bb*	mi-seventh: C-B*b*	ma-seventh: C-B	a-seventh: C-B#
d-sixth: C-A*bb*	mi-sixth: C-A*b*	ma-sixth: C-A	a-sixth: C-A#
d-fifth: C-G*b*	p-fifth: C-G		a-fifth: C-G#
d-fourth: C-F*b*	p-fourth: C-F		a-fourth: C-F#
d-third: C-E*bb*	mi-third: C-E*b*	ma-third: C-E	a-third: C-E#
d-second: C-D*bb*	mi-second: C-D*b*	ma-second: C-D	a-second: C-D#
d-unison: C-C*b*	p-unison: C-C		a-unison: C-C#

This way, and given the enharmonic equivalent, every tone of the chromatic scale may exist in two different interval relationships to the keynote (in this case C):

C	perfect octave	12	augmented seventh	octave
B	major seventh	11	diminished octave	major-7^{th}
B*b*	minor seventh	10	augmented sixth	seventh (mixo)
A	major sixth	9	diminished seventh	sixth (dorian)
G#	minor sixth	8	augmented fifth	#5
G	perfect fifth	7	diminished sixth	fifth
F#	augmented fourth	6	diminished fifth	tritone (lydian)
F	perfect fourth	5	augmented third	fourth
E	major third	4	diminished fourth	major 3^{rd}
D#	minor third	3	augmented second	minor 3^{rd}
D	major second	2	diminished third	whole step
C#	minor second	1	augmented unison	half step (phrygian)
C	perfect unison	0	diminished second	root

The *first* column in this table holds the designations of the note, the *second* column the preferred interval designations. The *third* column represents the half-note intervals relative to the keynote, and the *fourth* column represents the alternate designations. In the *fifth* column, some abbreviations customarily used by musicians are found (there may be others, of course). Again: this is based on equal-temperament tuning including enharmonic equivalents. Classical harmony theory finds reasons for a further differentiation; however, this is beyond the aim of the present elaborations [see secondary literature].

The below table indicates the numerical differences between just tuning and equal-temperament tuning. The deviation is just tuning vs. equal temperament tuning.

Interval name	no. of HS	notes	frequency relation	cents	deviation
Perfect octave	12	C-C'	1\2	1200,00	0,00
Major seventh	11	C-B	8\15	1088,27	-11,73
Minor seventh	10	C-B*b*	9\16	996,09	-3,91
Major sixth	9	C-A	3\5	884,36	-15,64
Minor sixth	8	C-G#	5\8	813,69	+13,69
Perfect fifth	7	C-G	2\3	701,96	+1,96
Tritone	6	C-#	32\45	590,22	-9,78
Perfect fourth	5	C-F	3\4	498,05	-1,95
Major third	4	C-E	4\5	386,31	-13,69
Minor third	3	C-D#	5\6	315,64	+15,64
Major second	2	C-D	8\9	203,91	+3,91
Minor second	1	C-C#	15\16	111,73	+11,73
Perfect unison	0	C-C	1\1	0,00	0,00
					cent

Table: Frequency relations of octave-internal intervals for just tuning. The deviations refer to the corresponding interval in equal-temperament tuning. Specifications in 1/100th cents should be in practice rounded off to whole cent-values. Compared to the major third in equal-temperament tuning, the major third in just tuning is to low by 14 cents. The other way round: compared to the just-intonated major third, the major third in equal-temperament tuning is too high by 14 cents. A deviation of 1 cent corresponds to a frequency difference of 0,058%.

We can see the frequency relations for different tunings in the following **Fig. 8.9**. The abscissa is a logarithmically divided frequency axis.

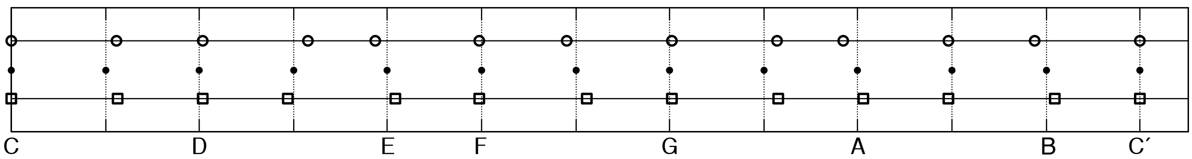

Fig. 8.9: Pythagorean (□), just-intonated (•), und perfect (o) intervals.

Since the half-step intervals are all equal in equal-temperament tuning, changing key (i.e. moving to a scale with a different keynote) does not represent a problem. For example, referencing to E results in the following intervals: E-F# = 2 HS = major second, E-A = 5 HS = perfect fourth. The reference to a particular key may now be omitted, because every interval is unambiguously defined by the number of its half-steps (HS).

Further interval designations exist **beyond the octave space**, as well: minor ninth (13 HS), major ninth (14 HS), minor tenth (15 HS), major tenth (16 HS), perfect eleventh (17 HS), augmented eleventh = diminished twelfth (18 HS), perfect twelfth (19 HS), minor thirteenth (20 HS), major thirteenth (21 HS); the half-step distances are given in brackets.

8.1.5 Typical detuning in guitars

Every guitarist will have experienced days when his/her guitar would just not tune properly. It typically gets really bad when we try to re-tune individual notes within *chords*. Even with a perfectly fretted neck and premium strings, this problem may occur – the most likely reason for which is the difference between just intonation and equal-temperament intonation. While the **fifth** tuned according to the latter is, with a deviation of 2 cents, really close to the perfect fifth tuned with just intonation, we find a much larger deviation for the **third**: that would be +13,7 cents for the major third, and as much as -15,6 cents for the minor third! Such detuning is already well audible, and the guitarist simply has to live with it. Trying to chord-specifically retune individual strings (towards just intonation) may easily generate deviations of as much as 29 cents for notes in other chords – and with that it now gets really shoddy. As an example:

An A-major chord (played without barré) consists of the notes [e-a-e-a-c#-e]. Given that all notes are tuned to equal temperament, it is in particular the C# played on the B-string that creates problems: it is sharp by 14 cents compared to a justly intonated C#. If we now retune the B-string by -14 cents (7,9 ‰), this A-chord will sound perfect. However: if, with the same retuning, e.g. an E-chord [e-h-e-g#-b-e] is played, the resulting sound is atrociously off. What happens is that the down-tuned B-string sounds a flat fifth – while the major third in that E-chord (the G# played on the neighboring G-string) is sharp. The interval between these two strings (3 half-steps G#-B) is too small by 29 cents! Changing from that re-adjusted A-major chord to a D-major chord creates a similar disaster: the down-tuned B-string now sounds too flat a D. The major third (F#) played on the neighboring E-string is already anyway too sharp by 13,7 cents and now sounds doubly out-of-tune relative to the tonic (that is lowered by 13,7 cents).

There may always be special cases when – given a limited selection of chords – a special detuning creates advantages. For example, it does not sound bad at all to slightly lower the tuning of the G-string for E and A7. E-major has [e-b-e-g#-b-e], and A7 has [e-a-e-g-c#-e]. In the E-major chord, the third profits, and in the A7 chord the diminished seventh – both are sharp in equal temperament relative to just tuning so that this detuning makes sense. For the same reason, the same detuning works well with the B7-chord [f#-b-d#-a-b-f#]. But don't you dare now changing to C or G … Thus, for universal deployment it is the equal-temperament tuning (executed as perfectly as possible) that remains a workable solution.

8.1.6 Stretched tuning

Piano tuners are known to tune not exactly according to equal temperament but in a slightly stretched-out fashion. In particular, in the very high and very low ranges, deviations of up to 30 cent can result. A spreading-out of partials, and in addition a narrowing of the pitch perception, are given as justification. In the guitar-relevant pitch range, however, the effect (merely 2 cents per octave) is rather weak, and the (compared to guitar strings) much heavier piano strings are no adequate equivalent. "Buzz" Feiten has obtained a US-patent for the stretched tuning – see Chapter 7.2.3). Fender, on the other hand, recommends adjusting the octave at the 12th fret with no more than 1 cent error – no spreading. To each his/her own …

