8.2 Frequency and pitch

Frequency is a quantity from the realm of physics, while **pitch** – as a sensory perception-quantity – belongs with the auditory event. Usually, frequency is measured in Hz, representing the numbers of oscillations per second. The unit Hertz (abbreviated Hz) is named after the physicist Heinrich Hertz. The inverse of the frequency is the period (short for duration of periodicity of a cycle). A period of $T = 2$ ms corresponds to a frequency $f = 500$ Hz. The **pitch** may either be determined via self-experiment (introspection), or indirectly via evaluation of the reaction of a test-person (a "subject"). Although the pitch is a subjectively rated quantity, it can be measured numerically. **Measuring** means in this context to allocate numbers to an object-set according to predetermined rules, with these numbers being reproducible within purposeful error margins. Now, what one considers purposeful – that again is a rather subjective decision♣. Most psychometric experiments yield intra- and inter-individual **scatter**: one and the same test person may give different evaluations when carrying out the same experiment a number of times (intra-individual scatter), and the assessments of different test persons may vary when an experiment is presented once for each person (inter-individual scatter).

8.2.1 Frequency measurement

Simple measurement devices for frequency count the number of oscillations occurring per time-interval: 5 oscillations per 0,1 s yields 50 Hz, for example. 'Oscillation' always implies a whole period. For a string, this means: swinging from the rest-position in one direction, reversal at the crest (apex), swinging (across the rest-position) fully to the other apex, reversal at the latter, and swinging back to the rest-position. Given an ideal oscillation, terms such as frequency or period are thus easily definable – real oscillations are, however, not ideal. Signal theory defines a **periodic** process as *infinitely repeated in identical form*. Thus, a sine tone is periodic and has one single frequency. A sound composed of a 100-Hz-tone and a 200-Hz-tone (in music this would be called a note) would be periodic, as well (**Fig. 8.10**). However, since more than one frequency appears here (i.e. 100 Hz and 200 Hz), we need to distinguish between **frequency of the partial** and the **fundamental frequency**. Now, the fundamental frequency is not necessarily that of the lowest partial, but the reciprocal of the period. The oscillation-pattern of a sound comprised from sine components at 200 Hz, 300 Hz, and 400 Hz repeats after 10 ms; the fundamental frequency therefore is 100 Hz although there is no actual partial found at 100 Hz within that sound. Generally speaking, the fundamental frequency is the largest common denominator of the frequencies of the partials, and the period is the least common multiple of all periods of the partials.

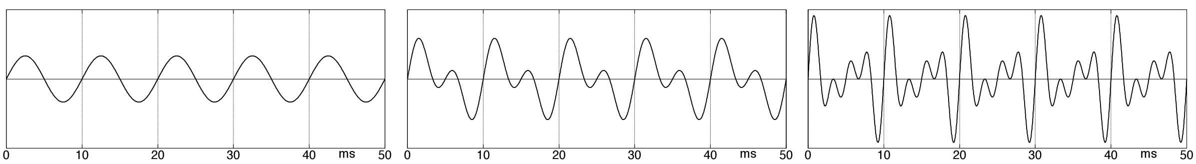

Fig. 8.10: Sine tone (100Hz), two-tone sound (100|200Hz), three-tone sound (200Hz|300Hz|400Hz); 0–50 ms each.

♣ A driver of a vehicle that has just reflected a high-frequency radar-beam may possibly demand a larger margin of error than what the municipal administration profiting from motoring fines would see as appropriate.

Evidently, a tone does not need to be of mono-frequent characteristic to feature *one* frequency (more exactly: one single fundamental frequency). In theory, there even may be an infinite number of partials (as is the case for an ideal square-wave sound). However, the partials have to be **harmonic** i.e. their frequencies need to be integer multiples of the fundamental frequency. This condition cannot be met e.g. for irrational numbers such as $\sqrt{2}$ und $\sqrt{3}$. In practice, though, frequencies can be specified only to a finite number of decimals, e.g. 1,414 Hz, or 1,732 Hz. If these examples would be rounded-off roots, a specification of "the fundamental frequency is 0,001 Hz" would be very arbitrary. Nor would it be within the meaning of the largest common denominator; 0,002 Hz, at least, would be a common denominator. It should be noted that the issue with the irrational numbers is of a less academic nature than one would think. This is because string vibrations are never of an exactly harmonic nature. The decay process gives every "period" different amplitudes, and the partials are not actually in a strictly harmonic relationship (i.e. they are **un-harmonic**), due to bending stiffness, and to the dispersive wave propagation connected to it. Let us assume that the decay process is so slow that its effects on the spectral purity may be disregarded. Let us further assume that the analysis of a guitar tone has yielded four components (partials) at the frequencies of 100 Hz, 201 Hz, 302 Hz, and 404 Hz. What would be the frequency of this tone? It makes no sense to specify 1 Hz as the fundamental frequency, and to call the partials the 100^{th}, 201^{st}, 302^{nd}, and 404^{th} harmonic, respectively. What remains is the sobering insight that **a guitar tone has no fundamental frequency**. It does, however, have a pitch! Determining that pitch shall be reserved for a later paragraph – first we still have to clarify what a **tuning device** is in fact doing given the above finding, and why a string may be tuned – despite all this.

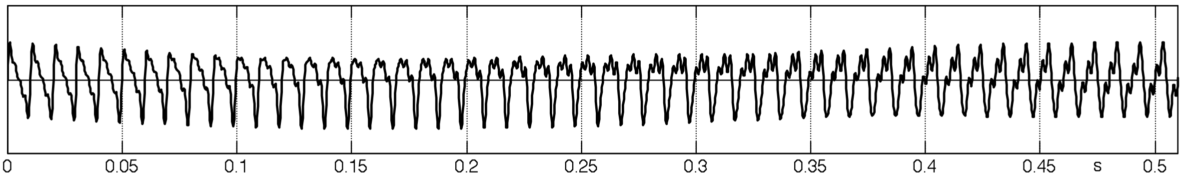

Fig. 8.11: 4-partials sound, f_1 = 100Hz, f_2 = 201Hz, f_3 = 302Hz, f_4 = 404Hz. $1/f$–envelope; t = 0 – 0,5 s.

Fig. 8.11 depicts the first 0,5 seconds of a sound comprised of the frequencies mentioned above. How many periods appear during that time interval? Trying to count the maxima, we get into a bit of trouble approaching the mid-section of the figure, but we can make it to the right-hand end with the finding that there will be about 49 and $3/4^{th}$ periods. But what is that in this case: a "period"? To deliver a *visual* evaluation, our optical sense seeks to – as well as possible – perform visual smoothing (i.e. filtering!) and locally limited auto-correlations. What else could a visual system do in the first place. But will that be helpful in the context of an acoustical signal? What could an exact algorithm be? Simple measurement devices determine the upward (or downward) zero-crossing. Given the above signal, that will make for considerable problems between 0,15 and 0,2 s, and between 0,35 and 0,45 s. Of course, **smoothing** (i.e. low-pass filtering) is a solution, but with it the frequency of the *filtered signal* will be determined. In the extreme case, the filtering will pass on merely the 100-Hz-oscillation – with that, the frequency-measurement certainly is most straightforward. Presumably most tuning devices (electronic tuners) have a built-in low-pass filter that filters string-specifically, or at least instrument-specifically. Also, they will accept small deviations from the nominal value. It may still happen that the display sways back and forth between correct and incorrect. The well-versed guitar player will then turn down the tone control (low-pass filter) or relinquish any high demands on accuracy. Some may celebrate an act of the gripping drama: "Guitarists never stop tuning, guitars eternally refuse to be correctly tuned".

The frequencies on which Fig. 8.11 is based show the fundamental problem but do exaggerate the situation. The spreading of the partials found with electric guitars amounts to about 0,2% for the E_2-string at 500 Hz, and to about 1% at 1 kHz. Still: if the 12th partial of the low E-string is represented with substantial level in the overall signal, a possibly annoying discrepancy of about 9 Hz between ideal $(12 \cdot 82{,}4\,\text{Hz} = 988{,}8\,\text{Hz})$ and real (997,7 Hz) may result. Such an error would be inacceptable for precise tuning. However, the amplitudes of the higher partials usually decay much faster than those of the lower partials, and thus most electronic tuners achieve an acceptable reading, especially since the guitarist will pluck the string rather softly so as not to emphasize the harmonics too much. For the lower partials of the low E-string, the inharmonicity will then be rather unproblematic with 0,02% for the third harmonic, and 0,05% for the fourth. There will be even less of an issue for the higher guitar strings: due to the smaller string diameter, the bending stiffness plays not as big a role, and the number of the possibly interfering harmonics decreases due to the low-pass character of the pickup.

As a summary, we may therefore note: even though the string vibration is comprised of inharmonic partials and therefore in theory has no fundamental frequency, electronic tuners will in practice detect the frequency of a "practical" fundamental, or a value that is very close to it. Whether our hearing system arrives at the same conclusion is, however, an entirely different question (see Chapter 8.2.3).

8.2.2 Accuracy of frequency and pitch

Following a chapter on frequency measurements, it would seem natural to explain pitch determination in more detail. First, however, desired accuracy and measurement errors shall be looked into. This way it will be easier to assess the properties of the hearing system that will be the focus in the subsequent chapter.

The frequency of a strictly periodic tone can be measured with an accuracy that is more than adequate for musicians. Precision frequency counters feature relative measurement errors in the range of 10^{-5}, and 10^{-6} is not impossible, either. In a watch, for example, an error of 10^{-5} leads to an inaccuracy of 1 second / day. The problem does not lie in the underlying reference (oven-stabilized quartz generators are extremely accurate) but in the signal to be measured. Measuring does become tricky if this signal does not have exactly identical periods. Given a known shape of the signal, frequency measurement is simple and quick: three points on a sine curve (excluding a few special points such as the zero crossing) suffice to determine the three degrees of freedom: amplitude, frequency, and phase. In theory, the three points may succeed one another very quickly, and thus achieving both high measurement precision and a short measuring time is not a contradiction. These highly theoretical findings based on function analysis do not help for measuring the frequency, though. This is because the shape of the signal is not known, and with that the rule holds that the **duration of the measurement** and the **accuracy of the measurement** are reciprocal to each other. If the frequency measurement is based on counting periods of the signal, a measurement of a length of 10 s is required in order to achieve an accuracy of 0,1 Hz. Interactive tuning would be impossible given such long durations. Frequency-doubling or half-period-measurements could be advantageous, but requires that the duty-factor of the signal is known – which is not the case with sounds of musical instruments. What remains is to determine the frequency of individual partials. Presumably, most tuning devices will indentify the frequency of the fundamental, and – in the case of the guitar – will indicate that as the frequency of the string.

