The frequencies on which Fig. 8.11 is based show the fundamental problem but do exaggerate the situation. The spreading of the partials found with electric guitars amounts to about 0,2% for the E_2-string at 500 Hz, and to about 1% at 1 kHz. Still: if the 12th partial of the low E-string is represented with substantial level in the overall signal, a possibly annoying discrepancy of about 9 Hz between ideal $(12 \cdot 82{,}4\,\mathrm{Hz} = 988{,}8\,\mathrm{Hz})$ and real (997,7 Hz) may result. Such an error would be inacceptable for precise tuning. However, the amplitudes of the higher partials usually decay much faster than those of the lower partials, and thus most electronic tuners achieve an acceptable reading, especially since the guitarist will pluck the string rather softly so as not to emphasize the harmonics too much. For the lower partials of the low E-string, the inharmonicity will then be rather unproblematic with 0,02% for the third harmonic, and 0,05% for the fourth. There will be even less of an issue for the higher guitar strings: due to the smaller string diameter, the bending stiffness plays not as big a role, and the number of the possibly interfering harmonics decreases due to the low-pass character of the pickup.

As a summary, we may therefore note: even though the string vibration is comprised of inharmonic partials and therefore in theory has no fundamental frequency, electronic tuners will in practice detect the frequency of a “practical” fundamental, or a value that is very close to it. Whether our hearing system arrives at the same conclusion is, however, an entirely different question (see Chapter 8.2.3).

8.2.2 Accuracy of frequency and pitch

Following a chapter on frequency measurements, it would seem natural to explain pitch determination in more detail. First, however, desired accuracy and measurement errors shall be looked into. This way it will be easier to assess the properties of the hearing system that will be the focus in the subsequent chapter.

The frequency of a strictly periodic tone can be measured with an accuracy that is more than adequate for musicians. Precision frequency counters feature relative measurement errors in the range of 10^{-5}, and 10^{-6} is not impossible, either. In a watch, for example, an error of 10^{-5} leads to an inaccuracy of 1 second / day. The problem does not lie in the underlying reference (oven-stabilized quartz generators are extremely accurate) but in the signal to be measured. Measuring does become tricky if this signal does not have exactly identical periods. Given a known shape of the signal, frequency measurement is simple and quick: three points on a sine curve (excluding a few special points such as the zero crossing) suffice to determine the three degrees of freedom: amplitude, frequency, and phase. In theory, the three points may succeed one another very quickly, and thus achieving both high measurement precision and a short measuring time is not a contradiction. These highly theoretical findings based on function analysis do not help for measuring the frequency, though. This is because the shape of the signal is not known, and with that the rule holds that the **duration of the measurement** and the **accuracy of the measurement** are reciprocal to each other. If the frequency measurement is based on counting periods of the signal, a measurement of a length of 10 s is required in order to achieve an accuracy of 0,1 Hz. Interactive tuning would be impossible given such long durations. Frequency-doubling or half-period-measurements could be advantageous, but requires that the duty-factor of the signal is known – which is not the case with sounds of musical instruments. What remains is to determine the frequency of individual partials. Presumably, most tuning devices will indentify the frequency of the fundamental, and – in the case of the guitar – will indicate that as the frequency of the string.

It is not only the measurement process that requires us to consider the measurement duration, but also the fact that the signal to be measured is **time-variant**. The amplitudes of the partials decay with different speed as a function of time, and moreover the **frequencies of the partials** will change. This is connected to the string being elongated and thus stretched more as it moves from its rest-position: the larger the vibration-amplitude, the higher the frequency. Further, it needs to be considered that real string oscillations are never limited to remain in exactly **one single plane**. During the decay process, the plane of oscillation rotates; this can be seen as the superposition of two orthogonal vibrations. Due to direction-dependent bearing-impedances, these two vibrations may differ slightly in their frequencies, and consequently there will be changes in amplitude and frequency over time.

A (non-representative) field experiment shall give some indications of how accurate the frequencies of strings can be measured despite all these issues. From the many digital **electronic tuners** on the market, three were selected and checked using a sine generator and a precision frequency counter. The ranges within which the devices registered a "correct tuning" measurement were ±1,6‰, ±2,0‰, and ±2,3‰, i.e. on average **±2‰**. This corresponds to **±3,5 cent**. To be clear: "correct tuning" in this context means that, for example, the device under test evaluated all frequencies between 439,4 Hz and 440,7 Hz as *correctly tuned to A*. The width of that tolerance interval is a compromise between high precision (possibly never achievable due to the aforementioned issues) on the one hand, and more easily achievable "kind-of-in-tune" state (that may not be accepted due to audible deviation from the ideal value) on the other hand.

Fig. 8.12 shows the progress over time of such a measurement. Using a tuning device (Korg GT-2), the tuning of two guitars was assessed; depicted are the deviations of the value indicated by the tuner from the reference value (during 8 seconds of a measuring time; for each string). The string was plucked with regular strength at t = 0; all non-involved strings were damped in order to avoid interferences. For the measurement with the Gretsch Tennessian, the stronger decrease of the pitch during the first seconds stands out. This effect was not further investigated; a cause may be found in the relatively thin strings: their average tensile stress is increased with strong vibration. Towards the end of the shown measuring time, the deviations increase; this is due to the decreasing signal level. The Ovation (with the signal of the piezo pickup measured) also caused some fluctuations during the measuring time; the causes for these were looked into in more detail.

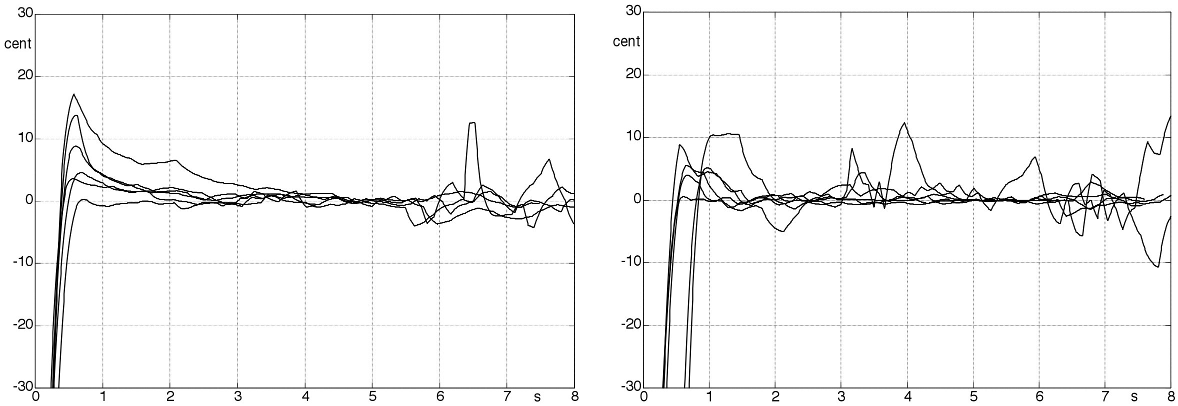

Fig. 8.12: Pitch measurement with the electronic tuner Korg GT-2. Tennessean (left), Ovation SMT (right).

In **Fig. 8.13**, the measured pitch is compared to the level of the fundamental over time. The signal generator is in both cases the plucked B-string of the Ovation SMT. At 3,5 s we see a minimum of the level of the fundamental. Assuming a time lag of around 0,5 s due to the processing, pitch-fluctuations at about 4 s can be explained; the other fluctuations cannot be attributed to anything specific with any certainty.

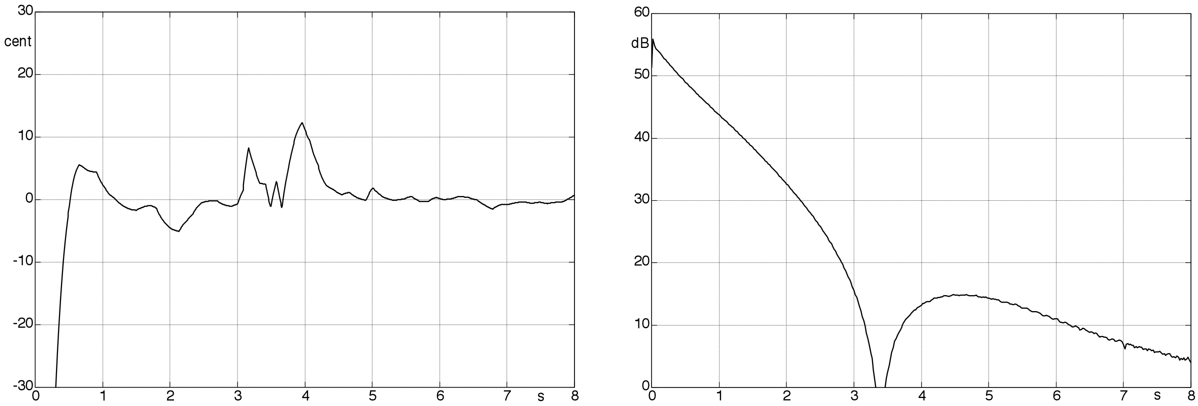

Fig. 8.13: Measured pitch-deviation, level of fundamental; Ovation SMT, B-string plucked at t = 0.

The measurements show that – despite alleged digital precision – considerable fluctuations in the display value are to be expected. Since the electronic tuning shows a highly accurate display without any noteworthy fluctuation when a precision generator serves as input, only the guitar tone itself can be the reason. The more "lively" this tone is, the larger the fluctuations in the measurement result will be, and the larger the variations in pitch.

At this point, a short digression into **thermodynamics** makes sense. The linear thermal expansion coefficient describes how dimensions change dependent on temperature. If the dimensions are "imprinted" (forced), the mechanical stress will vary as the temperature changes. This implies for steel strings: the un-tensioned string will experience an elongation by a factor of 16x10^{-6} for a temperature increase of +1°C. While this appears insignificant compared to the 2‰ mentioned above, we need to consider that for the change of the string frequency, the relative change in *stress* is the influential factor. Typically, an E_2-string needs to experience an elongation (strain) of about 1,5 mm for correct intonation. It is this 1,5-mm-strain that needs to be seen in connection with the change in length caused by the temperature change. The relative frequency change corresponds to half the relative change in strain (square-root in action here!). For our example, this means: **with 1°C temperature change, the string frequency changes by 5,3‰**. Here we assumed that the dimensions of the neck and body of the guitar remain constant; given the highly different thermal time constants over short time-periods, this is justified. Confirmation was provided by an experiment: taking a correctly tuned guitar (Gibson ES-335) from a room to the outside (cooler than in the room by a few degrees) raised the frequency of the E_2-string within a few seconds by 12‰. Conversely, it follows: if you seek to keep the tuning of a guitar constant within 1‰, you need to demand that short-term temperature fluctuations remain within 0,2°C ☺.

We have saved the most important question for last: **how precise actually is the hearing system?** In the terminology of psychoacoustics: how large is the threshold of pitch discrimination? You will find quite different answers – it depends on the experimental methodology. Fundamentally, we need to distinguish between a **successive pair** (2 tones follow each other in time) and the **dyad** (two-tone complex; two tones are sounded at the same time)

When concurrently presenting two tones, the smallest of differences between frequencies may be noticeable – depending of the circumstances. For example, if two 1-kHz-sine-tones are detuned by 0,1 Hz with respect to each other, a beating results: i.e. a tone is gradually getting louder and becoming softer again, with its amplitude reaching its maximum value every 10 seconds. The latter duration is short compared to the average life expectancy, and also small relative to the tolerance-span of the test persons (subjects) – therefore it is well observable. For the same reasons, a periodicity of 0,01 Hz would still be observable – but with 0,001 Hz the limited patience of the subject might become a problem. Relative to 1000 Hz, 0,001 Hz already represents is factor of 10^{-6}. However, to conclude that the frequency resolution of the auditory system would always be 0,001‰ – that would be nonsense. The result is only usable in the given experimental context.

Clearly, a large part of music consists of sounds comprising two or more tones – so: what gives? The answer will necessarily remain unsatisfactory because music is diverse, but there are rough guidelines. A first borderline is defined by the duration of sounds. If a sound consisting of two tones lasts only for a second, a frequency deviation between the two tones of 0,1 Hz will not be detected. Sounds of longer duration generally facilitate recognizing frequency differences. Still: long sustained notes are often played with **vibrato** (for the terminology see Chapter 10.8.2), and in this case a small detuning will be noticed less. Pitch vibrato, however, cannot be generated on every type of instrument – but then a polychoral design will make for audible modulations already in single notes. On the piano, for example, most notes are generated by two or three very closely tuned strings; beats will be inherent in the system here. Even when trying to tune all strings of one piano note to exactly equal pitch, the overcritical coupling of the strings will result in beating. Besides the beats audible in the single note, additional beating between different notes may be audible as a separate characteristic – but this will depend on too many factors to make an analysis with simple algorithms feasible. Looking at the distribution of how often musical notes of certain durations occur, and considering the auditory fluctuation assessment, we may cautiously estimate the following: upwards of an envelope-period of about 1 s, beats loose their sensory significance. This corresponds to a frequency resolution of about 1 Hz.

Given a **sequential presentation** of tones, beating is excluded. Or so many psychoacousticians believe. However, of significance is not which sounds are generated, but which sounds actually arrive at the ears of the subjects. Presentations of sounds in a room are always accompanied by **reflections** – if these occur in great numbers, they are called **reverb**. If the pause between sequentially presented tones is too short, there may still occur a short beating at the transition, and this beating may be perceived depending on the circumstances. Such experiments should therefore exclusively be carried out using headphones. A room as a transmission system has other issues, as well: due to the superposition to interleaved reflections, the impulse response is lengthened. The Fourier transform (the transmission function) obtains selective minima and maxima, and between these includes steep flanks. A frequency change of 1 Hz that is inaudible as such may now receive a change in level of several dB. This will be audible – however, although the original cause is a frequency change, it is the threshold of the hearing system for amplitude discrimination that is decisive for the detection.

For sine tones of a duration of no less that 0,2 s (sequentially presented via headphones), the **threshold for frequency discrimination** is about 1 Hz in the frequency range below 500 Hz. Above 500 Hz, this threshold is not constant anymore, but about ca. 2‰ of the given frequency. With shorter duration (< 0,2 s), the discrimination threshold deteriorates. These data are averages from a large number of psychoacoustical experiments.

For a sine tone, it is easy to assess whether it ties in with the 1-Hz-criterion, or with the 2‰-critrerion: the limit is at 500 Hz, with a transition from one limit value to the other♣. For sounds comprising several partials, this decision is not so simple anymore. Given an E_2-string, the first 6 partials are below 500 Hz, all further partials are above that limit. In such cases the following holds: frequency changes become audible if for at least one (audible) partial the threshold or frequency discrimination is surpassed. For the E_2-string it thus is not the 1 Hz / 82,4 Hz ≙ 12‰ criterion that forms the basis for the decision but the 2‰-harmonics-criterion. This is a good match to the tolerance range we found in electronic tuners. With the conversion into the unit cent that is customary among musicians, the tolerance range is **3 – 5 cent** (with 1 cent = 1/100th semi-tone interval ≙ 0,58‰). The 1-cent-accurracy that is sometimes demanded is exaggerated: on the guitar, the temperature of the strings would have to be kept constant within 0,1°C (which may be difficult when playing your hot grooves, as cool as they may feel). If the guitar can be tuned with an accuracy of ±2‰, we are on the safe side. This does not mean, though, that every larger deviation will immediately sound out-of-tune. Our hearing system can be quite forgiving and ready to generously compromise in certain individual situations.

8.2.3 Pitch perception

It has already been noted above that pitch and frequency are different quantities. Our auditory system determines the pitch according to complex algorithms – an associated comprehensive discussion would go beyond the scope of this book (specialist literature exists for this). A first important processing step is the frequency/place transformation in the inner ear (cochlea): a travelling wave runs within the helical cochlea, with the wave-maximum depending on amplitude and frequency of the sound wave. Tiny sensory cells react to the movement of this travelling wave; they transmit nerve impulses among various nerve fibers to the brain. The latter performs further advanced processing. A regularly plucked guitar sound consists of a multitude of almost harmonic partials. Round about the first 6 – 8 of these partials result in distinguishable local travelling-wave maxima, the higher partials are processed grouped together.

Normally, we cannot hear the individual partials when a string is plucked. Rather, we hear a complex tone with *one single* pitch. With a little effort, however, these individual partials may be heard, after all. To do this, we first suppress a given partial using a notch-filter, and then switch off the filter-effect so that the original signal is reproduced. From the moment the filter is switched off, the partial in question will be audible for a few seconds, and then merge again with its colleagues to form the integral sound experience that was originally audible. A sufficient level of the partial is a requirement; the partial may not be masked to such an extent by its spectral neighbors that it does not contribute at all to the aural impression. How the single elements are grouped and combined together – that has long been a topic of research for the Gestalt-psychologists. This topic resulted first of all in the **Gestalt laws** for the visual system (see Chapter 8.2.4). In particular, it is the “principle of common fate” that also plays a role in the auditory system if the issue is to group the individual partials of a complex sound event, attributed them to sound sources, and to assign to the latter characteristics such as e.g. a pitch.

♣ Both “1 Hz” and “2‰” are to be taken as approximate values that are subject to individual variations.

